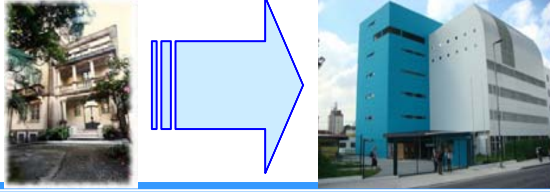
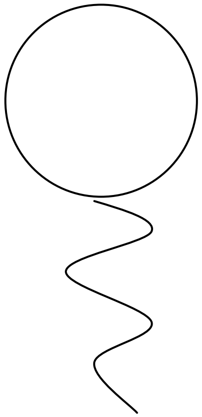




DO TADPOLES VANISH IN THE LIGHT FRONT?

Alfredo Takashi Suzuki

Jorge Henrique de Oliveira Sales,

Juliana Diniz Bolzan

and

Luís Alberto Soriano Carrillo

Instituto de Física Teórica – UNESP

Rua Dr. Bento Teobaldo Ferraz, 271 – Bloco II – 01140-070 – São Paulo, SP – Brazil

OUTLINE

- **Introduction**
- **Hands on the specific problem**
- **Conclusions**

INTRODUCTION

In covariant gauges, e.g. Feynman gauge...

$$T_{\text{cov.}} \doteq \int \frac{d^{\mathrm{D}}q}{(p-q)^2} = \int \frac{d^{\mathrm{D}}q}{q^2} \equiv 0$$

INTRODUCTION

Actually, ...

$$T_{\mathrm{cov.}} \doteq \int \frac{d^{\mathrm{D}}q}{(p-q)^2} = \int \frac{d^{\mathrm{D}}q}{q^2}$$

$$= divergent\,(\infty)!$$

INTRODUCTION

One may work out a consistency check, such as ...

$$T_{\text{cov.}} \doteq \int \frac{d^{\mathrm{D}}q}{(p-q)^2} = \int \frac{d^{\mathrm{D}}q\, q^2}{q^2 (p-q)^2}$$

$$= \iint \frac{d^{\mathrm{D}-1}\mathbf{q}\; dq^0 \left(q_0^2 - \mathbf{q}^2\right)}{q^2 (p-q)^2}$$

$$= 0$$

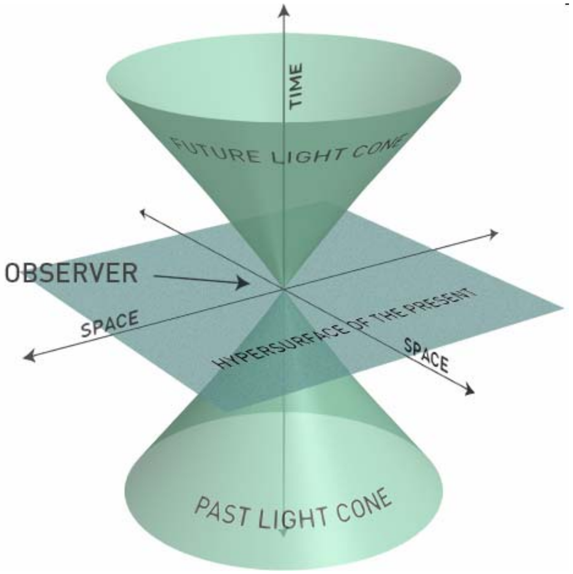
INTRODUCTION

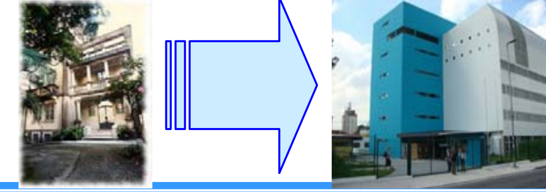
In the light front gauge ...

$$x^{\pm} \doteq \frac{x^0 \pm x^3}{\sqrt{2}}$$

$$x^2 = 2x^+ x^- - \hat{\mathbf{x}}^2$$

$$\hat{\mathbf{x}}^2 \doteq x_1^2 + x_2^2$$





INTRODUCTION

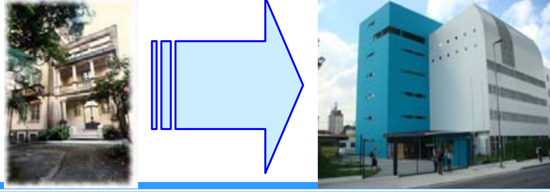
In the light front gauge ...

⇨ “Genuine” tadpoles

$$S_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{q^2 q^+}$$

$$= \iiint \frac{d^{\mathrm{D}-2}\hat{\mathbf{q}}\, dq^+ dq^-}{\left[2q^+ q^- - \hat{\mathbf{q}}^2\right] q^+}$$

$$\equiv 0\ ?$$



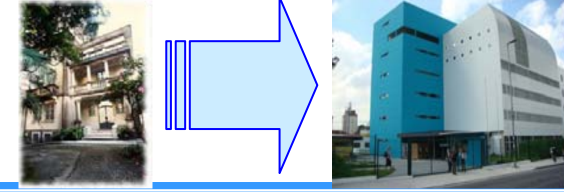
INTRODUCTION

In the light front gauge ...

⇨ "Genuine" tadpoles ➔ Can we do consistency check?

$$S_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{q^2 q^+} = \int \frac{d^{\mathrm{D}}q\,(p-q)^2}{q^2\,(p-q)^2\,q^+}$$

$$= \int \frac{d^{\mathrm{D}}q\,[2(p^+ - q^+)(p^- - q^-) - (\hat{\mathbf{p}} - \hat{\mathbf{q}})^2]}{q^2\,(p-q)^2\,q^+}$$



INTRODUCTION

In the light front gauge ...

⇨ Tadpole-like integrals ➔ Actually volume divergent

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{(p-q)^2 q^+}$$

$$= \iiint \frac{d^{\mathrm{D}-2}\hat{\mathbf{q}}\, dq^+ dq^-}{\left[2\left(p^+ - q^+\right)\left(p^- - q^-\right) - (\hat{\mathbf{p}} - \hat{\mathbf{q}})^2\right] q^+}$$

$$\equiv 0\ ?$$

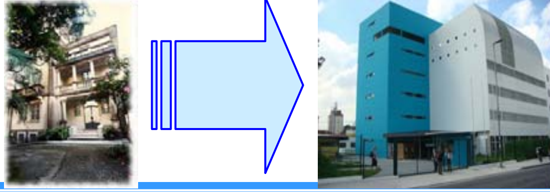
INTRODUCTION

In the light front gauge ...

⇨ Tadpole-like integrals ➔ Can we do consistency check?

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{(p-q)^2 q^+} = \int \frac{d^{\mathrm{D}}q\, q^2}{q^2 (p-q)^2 q^+}$$

$$= \int \frac{d^{\mathrm{D}}q\,(2q^+q^- - \hat{\mathbf{q}}^2)}{q^2 (p-q)^2 q^+}$$



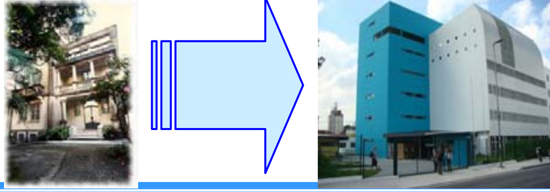
INTRODUCTION

In the light front gauge ...

- Some subtleties…

1. Which integration do we perform first?
2. Are integration results independent of order of integrating?
3. What do we do with non manageable integrals such as…

$$\int_{-\infty}^{+\infty} \frac{dq^- \, q^-}{q^- - \Lambda + i\varepsilon}$$



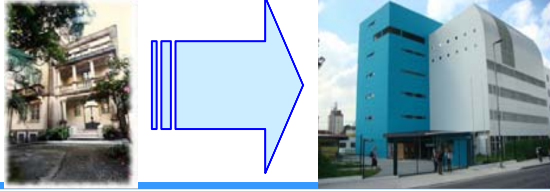
INTRODUCTION

In the light front gauge ...

- Some subtleties…

1. Which integration do we perform first? ⇒ Light – front time q^- (?)
2. Are integration results independent of order of integrating?
3. What do we do with non manageable integrals such as…

$$\int_{-\infty}^{+\infty} \frac{dq^- \, q^-}{q^- - \Lambda + i\varepsilon}$$



INTRODUCTION

In the light front gauge ...

- Some subtleties…

  1. Which integration do we perform first? ⇨ Light – front time q^- (?)
  2. Are integration results independent of order of integrating? ⇨ Often assumed so, but is it really (?)
  3. What do we do with non manageable integrals such as…

$$\int_{-\infty}^{+\infty} \frac{dq^- \, q^-}{q^- - \Lambda + i\varepsilon}$$

INTRODUCTION

In the light front gauge ...

- Some subtleties…

1. Which integration do we perform first? → Light – front time q^- (?)

2. Are integration results independent of order of integrating? → Often assumed so, but is it really (?)

3. What do we do with non manageable integrals such as…

$$\int_{-\infty}^{+\infty} \frac{dq^- \, q^-}{q^- - \Lambda + i\varepsilon}$$

→ Here Cauchy complex integration is powerless

INTRODUCTION

In the light front gauge ...

- Some subtleties…

- What do we do with the “bilinear” constraint?

INTRODUCTION

In the light front gauge ...

- Some subtleties…
  - What do we do with the “bilinear” constraint?

$$2q^{+}q^{-} = q^{2} + \hat{\mathbf{q}}^{2}$$

INTRODUCTION

In the light front gauge ...

- Some subtleties…
  - What do we do with the “bilinear” constraint?


INTRODUCTION

In the light front gauge ...

- Some subtleties…

- What do we do with the “bilinear” constraint?

What we can do is ...

1. Pretend it is not there
2. Take it seriously

INTRODUCTION

In the light front gauge ...

- Some subtleties…

- What do we do with the "bilinear" constraint?

What we can do is ...

1. Pretend it is not there → See where it leads to
2. Take it seriously

INTRODUCTION

In the light front gauge ...

- Some subtleties…

  - What do we do with the “bilinear” constraint?

What we can do is ...

1. Pretend it is not there → See where it leads to
2. Take it seriously → But no idea how to treat it yet

INTRODUCTION

In the light front gauge ...

- Some subtleties…

- What do we do with the “bilinear” constraint?

What we can do is ...

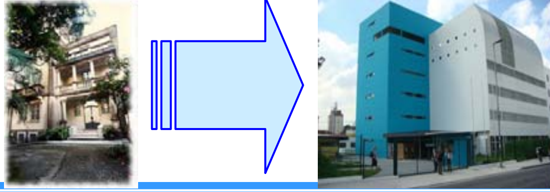
1. Pretend it is not there → See where it leads to
2. Take it seriously → But no idea how to treat it yet

HANDS ON THE SPECIFIC PROBLEM

- The tadpole-like Feynman integral ...

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon] q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int\int\int \frac{d^{\mathrm{n}} \hat{\mathbf{q}}\, dq^+ dq^-}{[2(p^+ - q^+)(p^- - q^-) - (\hat{\mathbf{p}} - \hat{\mathbf{q}})^2 + i\varepsilon]\, q^+}$$



HANDS ON THE SPECIFIC PROBLEM

❶ Do the q^- integration first ...

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2 + i\varepsilon]q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= -\int d^{\mathrm{n}}\hat{\mathbf{q}} \int \frac{dq^+}{2(p^+ - q^+)q^+} \int_{-\infty}^{+\infty} \frac{dq^-}{(q^- - \Lambda - i\xi)}$$

$$\Lambda \doteq p^- - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^+ - q^+)};$$

$$\xi \doteq \frac{\varepsilon}{2(p^+ - q^+)}$$

HANDS ON THE SPECIFIC PROBLEM

❷ Change order of integration

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon] q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int \frac{dq^+}{2(p^+ - q^+) q^+} \int d^{\mathrm{n}} \hat{\mathbf{q}} \int_{-\infty}^{+\infty} \frac{dq^-}{(q^- - \Lambda - i\xi)}$$

HANDS ON THE SPECIFIC PROBLEM

❸ Perform l.f. “energy” integration via residues

➔ Two distinct regions for q^+ integration:

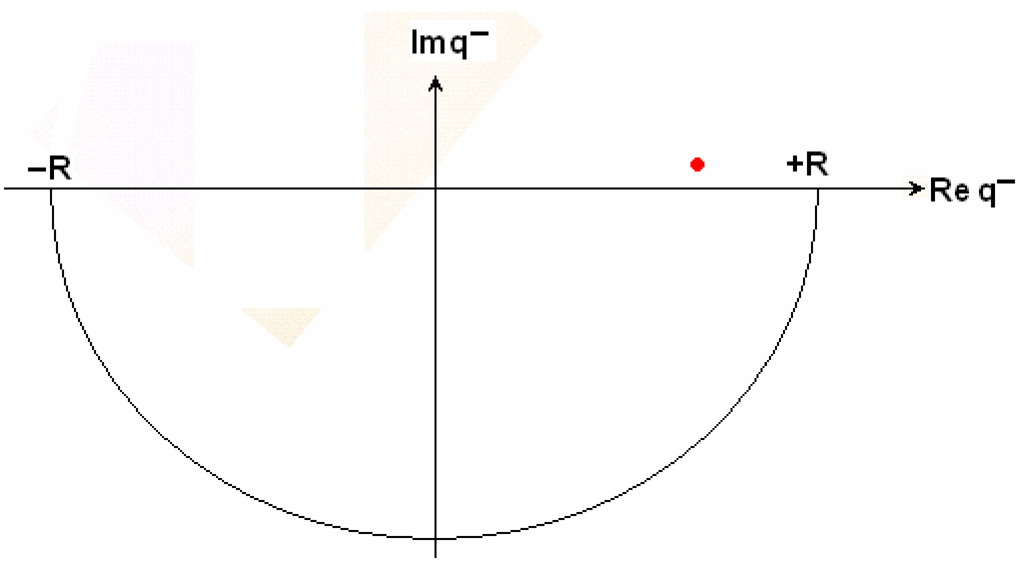
$$1.\ \ 0 < q^+ < p^+$$

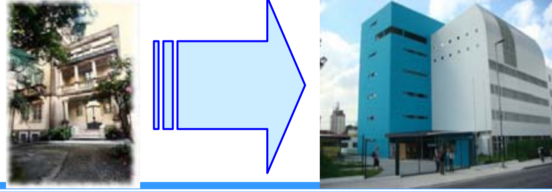
$$2.\ \ p^+ < q^+ < 0$$

HANDS ON THE SPECIFIC PROBLEM

❸ Perform l.f. “energy” integration via residues

➔ For $1.\ \ 0 < q^{+} < p^{+}$

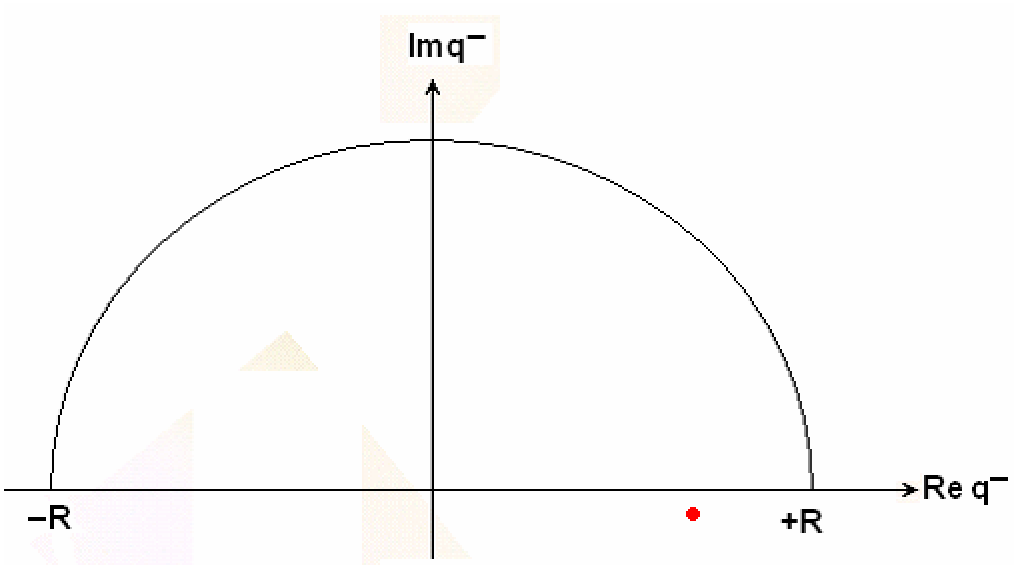




HANDS ON THE SPECIFIC PROBLEM

❹ Perform l.f. “energy” integration via residues

➔ For $2.\ \ p^{+} < q^{+} < 0$

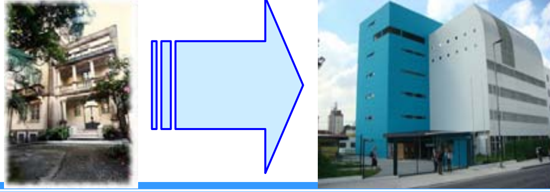


HANDS ON THE SPECIFIC PROBLEM

❺ For any of the contours chosen we have:

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon] q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int \frac{dq^+}{2(p^+ - q^+) q^+} \int d^{\mathrm{n}} \hat{\mathbf{q}} \boxed{[i\pi]}$$



HANDS ON THE SPECIFIC PROBLEM

❺ For any of the contours chosen we have:

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon]\, q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int \frac{dq^+}{2(p^+ - q^+)\, q^+} \int d^{\mathrm{n}} \hat{\mathbf{q}} \, [i\pi]$$

Log divergence

Volume divergence

HANDS ON THE SPECIFIC PROBLEM

❻ So ...

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2 + i\varepsilon]q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$\equiv 0 \quad Eureka!?$$

HANDS ON THE SPECIFIC PROBLEM

7. With consistency check

(same order of integrations used)

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{(p-q)^2 q^+} = \int \frac{d^{\mathrm{D}}q\, q^2}{q^2 (p-q)^2 q^+}$$

$$= \int \frac{d^{\mathrm{D}}q\,(2q^+ q^- - \hat{\mathbf{q}}^2)}{q^2 (p-q)^2 q^+}$$

$$= 0 \quad !!!$$


HANDS ON THE SPECIFIC PROBLEM

① Do the q^+ integration first

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon] q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= -\int d^{\mathrm{n}} \hat{\mathbf{q}} \int \frac{dq^-}{2(p^- - q^-)} \int_{-\infty}^{+\infty} \frac{dq^+}{(q^+ - \Gamma - i\zeta)\, q^+}$$

$$\Gamma \doteq p^+ - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^- - q^-)};$$

$$\zeta \doteq \frac{\varepsilon}{2(p^- - q^-)}$$

HANDS ON THE SPECIFIC PROBLEM

① Do the q^+ integration first

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon] q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= -\int d^{\mathrm{n}} \hat{\mathbf{q}} \int \frac{dq^-}{2(p^- - q^-)} \int_{-\infty}^{+\infty} \frac{dq^+}{(q^+ - \Gamma - i\zeta) q^+}$$

$$\Gamma \doteq p^+ - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^- - q^-)};$$

$$\zeta \doteq \frac{\varepsilon}{2(p^- - q^-)}$$

HANDS ON THE SPECIFIC PROBLEM

② Change order of integration …

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}} q}{[(p-q)^2 + i\varepsilon]q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= -\int \frac{dq^-}{2(p^- - q^-)} \int d^{\mathrm{n}} \hat{\mathbf{q}} \int_{-\infty}^{+\infty} \frac{dq^+}{(q^+ - \Gamma - i\zeta)\, q^+}$$

$$\Gamma \doteq p^+ - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^- - q^-)};$$

$$\zeta \doteq \frac{\varepsilon}{2(p^- - q^-)}$$

HANDS ON THE SPECIFIC PROBLEM

③ Perform l.f. q^{+} integration via residues

➔ Two distinct regions for q^{-} integration:

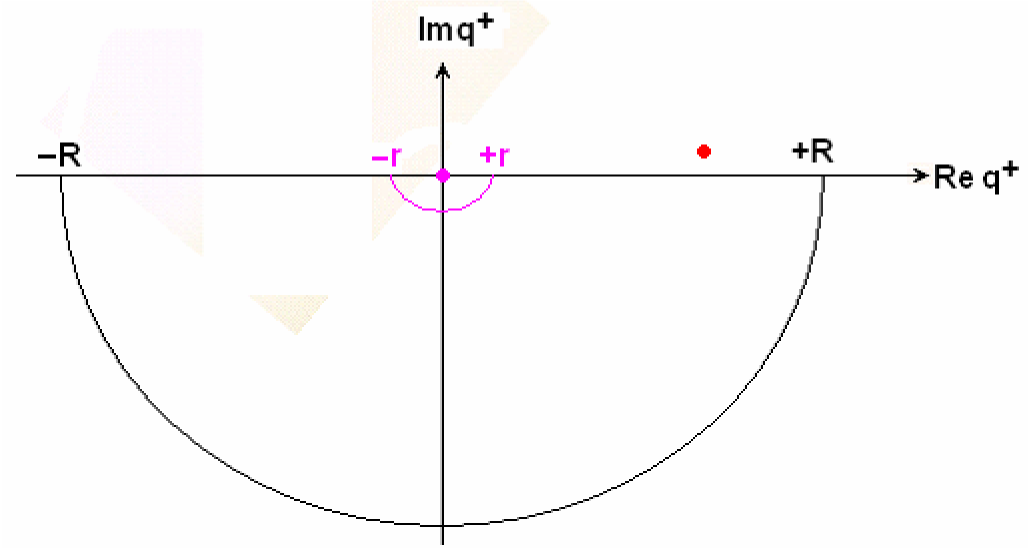
$$1.\ \ 0 < q^{-} < p^{-}$$

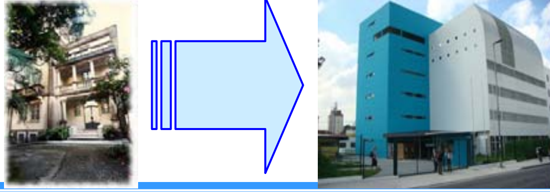
$$2.\ \ p^{-} < q^{-} < 0$$

HANDS ON THE SPECIFIC PROBLEM

③ Perform l.f. q^+ integration via residues

➔ For $1.\ \ 0 < q^- < p^-$

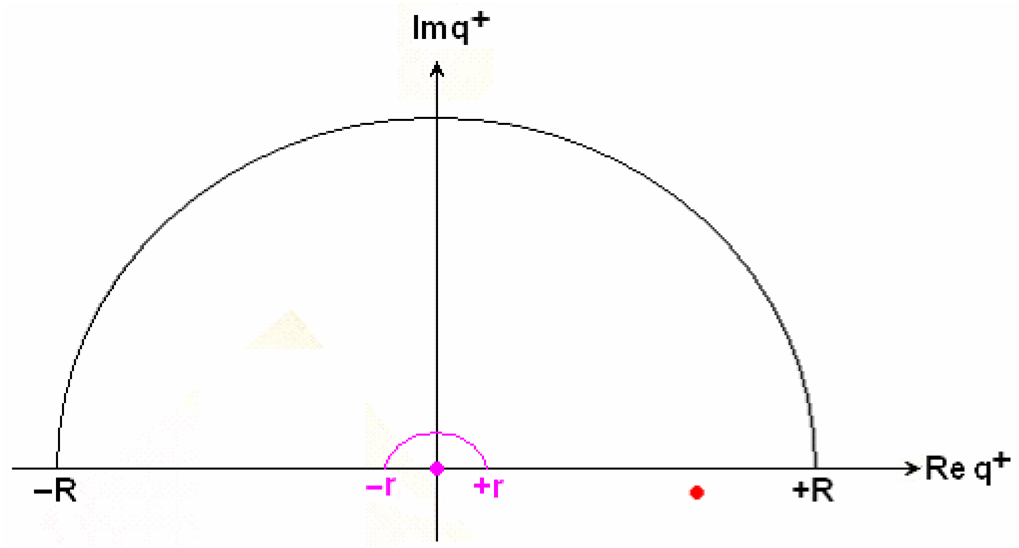




HANDS ON THE SPECIFIC PROBLEM

④ Perform l.f. q^{+} integration via residues

➔ For $2.\ \ p^{-} < q^{-} < 0$



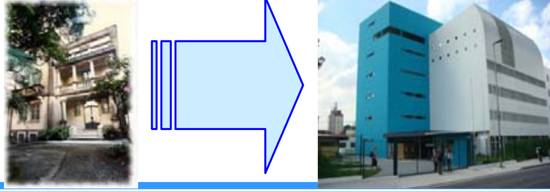
HANDS ON THE SPECIFIC PROBLEM

⑤ For any of the contours chosen we have:

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2 + i\varepsilon]q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int_0^{p^-} \frac{dq^-}{2(p^- - q^-)} \int d^{\mathrm{n}}\hat{\mathbf{q}} \left[\frac{i\pi}{\Gamma}\right]$$

$$\Gamma \doteq p^+ - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^- - q^-)}$$



HANDS ON THE SPECIFIC PROBLEM

⑤ For any of the contours chosen we have:

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2+i\varepsilon]q^+} \quad ; \mathrm{D} = 2 + \mathrm{n}$$

$$= \int_0^{p^-} \frac{dq^-}{2(p^- - q^-)} \int d^{\mathrm{n}}\hat{\mathbf{q}} \left[\frac{i\pi}{\Gamma} \right]$$

$$\Gamma \doteq p^+ - \frac{(\hat{\mathbf{p}} - \hat{\mathbf{q}})^2}{2(p^- - q^-)}$$

HANDS ON THE SPECIFIC PROBLEM

⑥ So…

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2 + i\varepsilon]q^{+}} \quad ; \mathrm{D} = 2 + \mathrm{n} = 2\omega$$

$$= i\,p^{-}\left(-\pi\right)^{\omega}\left(2p^{+}p^{-}\right)^{\omega-2}\frac{\Gamma(2-\omega)\Gamma(\omega-1)}{\Gamma(\omega)}$$

HANDS ON THE SPECIFIC PROBLEM

⑦ Finally …

$$T_{\mathrm{LF}} \doteq \int \frac{d^{\mathrm{D}}q}{[(p-q)^2+i\varepsilon]q^+} \neq 0 \quad !!!$$

Not of volume divergence type!

HANDS ON THE SPECIFIC PROBLEM

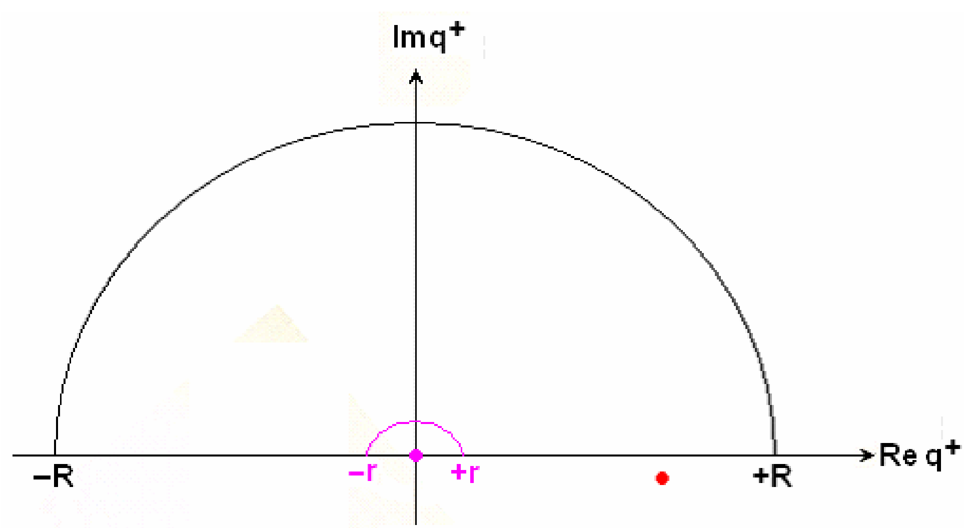
To summarize …

- Order of integration **does** matter (?);
- With q-plus integration first, not a volume divergence (?);
- Rechecking, with

STEP 4

④ Perform l.f. q^+ integration via residues

➔ For $2.\ \ p^- < q^- < 0$



Implies a contribution coming from the "anti-particle" sector of the Fock space. Formal answer is the same, but sign of result is different. Thus sum of both yields zero!

CONCLUSIONS

- Order of integration does not matter when both sectors – particle and anti-particle – are taken into account;

- It is consistent with covariant dimensional regularization setting of tadpoles to zero.