Scattering Vector Mesons in D4/D8 model

Marcus A. C. Torres C. A. Ballon Bayona H. Boschi-Filho N. Braga

Instituto de Física
Universidade Federal do Rio de Janeiro

Light-Cone 2009: Relativistic Hadronic and Particle Physics

Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors
2. D4/D8 Brane Model
   - Holographic QCD
3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius
4. Summary

Outline

1 Motivation
- Gauge/String duality vs. the Real QCD
- D4/D8 pion form factors

2 D4/D8 Brane Model
- Holographic QCD

3 Work
- Scattering Vector Mesons
- Wave functions
- Quadratic radius

4 Summary

From Gauge/String duality towards Real QCD

- AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.
- To do:
  Break SUSY, add quarks, masses, $\not{\chi}$, confinement scale.

  $\longrightarrow$ Holographic QCD
  D4-D8 brane model

From Gauge/String duality towards Real QCD

- AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.
- To do:
  Break SUSY, add quarks, masses, $\not\chi$, confinement scale.

$\longrightarrow$

Holographic QCD
D4-D8 brane model

From Gauge/String duality towards Real QCD

- AdS/CFT provides means to reach strong coupling gauge theory, but it is not a QCD.
To do:
Break SUSY, add quarks, masses, $\not\chi$, confinement scale.

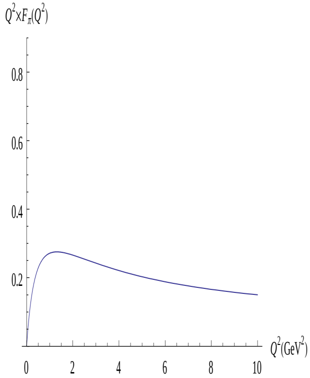
  $\longrightarrow$

Holographic QCD
D4-D8 brane model

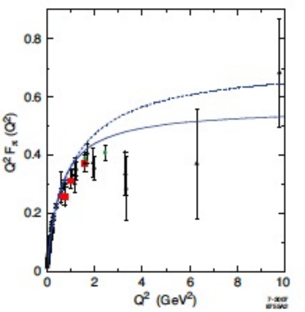
Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors
2. D4/D8 Brane Model
   - Holographic QCD
3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius
4. Summary

D4/D8 pion form factors



(a) $Q^2 \times F_\pi(Q^2)$ plot from D4/D8 model



(b) $Q^2 \times F_\pi(Q^2)$ prediction from hardwall (dashed line), softwall (continuous line) [5] models and data

Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors
2. D4/D8 Brane Model
   - Holographic QCD
3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius
4. Summary

Holographic QCD

- N_c (large) D4 branes fills $\mathcal{M}^{1,3}, x_4(\tau)$, with x_4 compactified in a circle.
  - 4d SU(N_c) YM on the brane
- N_f probe branes D8, $\overline{\text{D8}}$ fills $\mathcal{M}^{1,3}, \{x_5, x_8\}$ and radial $U(\tau)$

$\longrightarrow$ χ symm $SU(N_f) \times SU(N_f)$ dynamically broken.

Holographic QCD

- N_c (large) D4 branes fills $\mathcal{M}^{1,3}, x_4(\tau)$, with x_4 compactified in a circle.
  - 4d SU(N_c) YM on the brane
- N_f probe branes D8, $\overline{\text{D8}}$ fills $\mathcal{M}^{1,3}, \{x_5, x_8\}$ and radial $U(\tau)$ $\longrightarrow$ χ symm $SU(N_f) \times SU(N_f)$ dynamically broken.

Holography: 4d from 5d

- quarks confined
- D8-$\overline{\text{D8}}$ gauge fields independent of $\{x_5, x_8\}$ directions.
- KK modes: vector (v^n_μ) and axial vector mesons (a^n_μ).
  - $\mathcal{A}_\mu(x^\mu, z) = \sum_{n=1}^{\infty} v^{(n)}_\mu(x^\mu)\psi_{2n-1}(z) \; + \; a^{(n)}_\mu(x^\mu)\psi_{2n}(z) + \mathcal{A}_L, \mathcal{A}_R(x)$
    ψ_n: eigenfunctions of e.o.m. (orthonormal).
    λ_n: eigenvalues$\sim m_n^2$ of (axial) vector mesons.

$$S_{D8} \sim \int \mathrm{d}^9 x \, e^{-\phi} \sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} \longrightarrow$$

$$\int \mathrm{d}^4 x \mathcal{L}_{kin+mass} - 2 g_{v^n} \, \mathrm{tr} \left(v^n_\mu \mathcal{V}^\mu \right) + g_{v^l v^m v^n} \left(\partial^\mu v^{l\nu} - \partial^\nu v^{l\mu} \right) \left[v^m_\mu, v^n_\nu \right]$$
$$+ g_{v^l a^m a^n} \left(\partial^\mu v^{l\nu} - \partial^\nu v^{l\mu} \right) \left[a^m_\mu, a^n_\nu \right] + g_{v^l a^m a^n} \left(\partial^\mu a^{n\nu} - \partial^\nu a^{n\mu} \right) \left(\left[v^l_\mu, a^m_\nu \right] - \left[v^l_\nu, a^m_\mu \right] \right)$$

Holography: 4d from 5d

- quarks confined
- D8-$\overline{\text{D8}}$ gauge fields independent of $\{x_5, x_8\}$ directions.
- KK modes: vector (v_μ^n) and axial vector mesons (a_μ^n).
  - $\mathcal{A}_\mu(x^\mu, z) = \sum_{n=1}^{\infty} v_\mu^{(n)}(x^\mu)\psi_{2n-1}(z) \; + \; a_\mu^{(n)}(x^\mu)\psi_{2n}(z) + \mathcal{A}_L, \mathcal{A}_R(x)$
    ψ_n: eigenfunctions of e.o.m. (orthonormal).
    λ_n: eigenvalues$\sim m_n^2$ of (axial) vector mesons.

$$S_{D8} \sim \int \mathrm{d}^9x\, e^{-\phi}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} \longrightarrow$$

$$\int \mathrm{d}^4x \mathcal{L}_{kin+mass} - 2\mathrm{g}_{v^n}\,\mathrm{tr}\left(v_\mu^n \mathcal{V}^\mu\right) + \mathrm{g}_{v^l v^m v^n}\left(\partial^\mu v^{l\nu} - \partial^\nu v^{l\mu}\right)\left[v_\mu^m, v_\nu^n\right]$$

$$+\mathrm{g}_{v^l a^m a^n}\left(\partial^\mu v^{l\nu} - \partial^\nu v^{l\mu}\right)\left[a_\mu^m, a_\nu^n\right] + \mathrm{g}_{v^l a^m a^n}\left(\partial^\mu a^{n\nu} - \partial^\nu a^{n\mu}\right)\left(\left[v_\mu^l, a_\nu^m\right] - \left[v_\nu^l, a_\mu^m\right]\right)$$

Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors
2. D4/D8 Brane Model
   - Holographic QCD
3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius
4. Summary

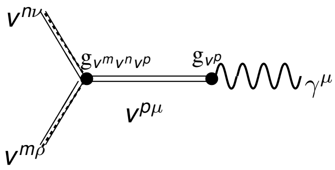


Figure: Vector meson EM scattering.

- VMD

$$\langle v^{n\nu}(k)|J^{(\mathcal{V})\mu}(0)|v^{m\rho}(k')\rangle = f^{\mu\nu\rho}_{(k,k')} F_{v^n v^m}(Q^2) \ ,$$

$$F_{v^n v^m}(Q^2) = \sum_{p=1}^{\infty} \frac{g_{v^p} g_{v^n v^m v^p}}{Q^2 + m^2_{v^p}} = \sum_{p=1}^{\infty} \frac{\frac{g_{v^p}}{m^2_{v^p}} g_{v^n v^m v^p}}{\frac{Q^2}{m^2_{v^p}} + 1} = \frac{1}{Q^2} \sum_{p=1}^{\infty} \frac{g_{v^p} g_{v^n v^m v^p}}{1 + \frac{m^2_{v^p}}{Q^2}} \ .$$

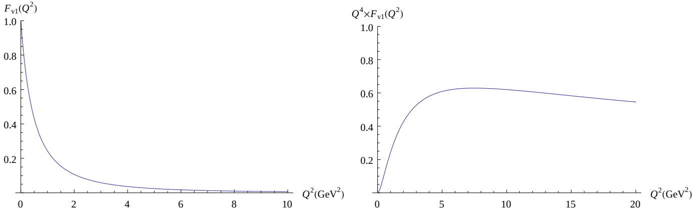
Sum Rules

Table: Masses and coupling constants

n	$\frac{m^2_{v^n}}{M^2_{KK}} = \lambda_{2n-1}$	$\kappa^{-1/2}\frac{g_{v^n}}{M^2_{KK}}$	$\kappa^{1/2} g_{v^n v^1 v^1}$
1	0.66931	2.10936	0.44658
2	2.87432	9.10785	-0.14654
3	6.59118	20.7957	1.8434×10^{-2}
4	11.79669	37.1502	-3.6885×10^{-4}
5	18.48972	58.1701	2.6953×10^{-4}
6	26.67017	83.834	3.0775×10^{-5}
7	36.33796	114.152	1.8572×10^{-5}
8	47.49318	148.103	6.9961×10^{-6}
9	53.6285	188.695	3.5081×10^{-6}

$$F_{v^m v^n}(Q^2 = 0) = \sum_{p=1}^{9} \frac{g_{v^p} g_{v^n v^m v^p}}{m^2_{v^p}} \sim \delta^{mn}$$

v^1 Elastic form factor



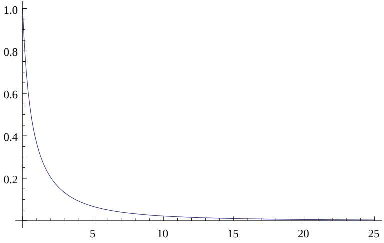
(a) Elastic form factor F_{v^1} vs Q^2 (b) $Q^4 \times F_{v^1}$ vs Q^2

Superconvergence (Grigoryan and Radyushkin):

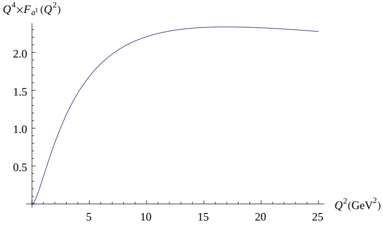
$\sum_{n=1}^{9} g_{v^n} g_{v^n v^1 v^1} \sim -0.000945 (M_{\mathrm{KK}})^2$

$\sum_{n=1}^{9} g_{v^n} g_{v^n a^1 a^1} \sim 0.000903 (M_{\mathrm{KK}})^2$

a^1 Elastic form factor



(c) Elastic form factor F_{a^1} versus Q^2



(d) $Q^4 \times F_{a^1}$ versus Q^2

Figure: a^1 elastic form factor

Outline

1 Motivation
- Gauge/String duality vs. the Real QCD
- D4/D8 pion form factors

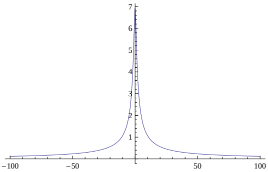
2 D4/D8 Brane Model
- Holographic QCD

3 Work
- Scattering Vector Mesons
- Wave functions
- Quadratic radius

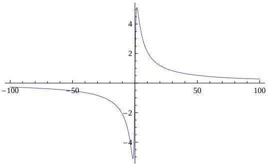
4 Summary

Vanishing of higher 3-vertex coupling

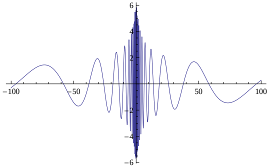
$$g_{v^n} = m_{v^n}^2 \kappa \int dz (1+z^2)^{-1/3} \psi_{2n-1}, \quad g_{v^1 v^1 v^p} = \kappa \int dz (1+z^2)^{-1/3} \psi_1^2 \psi_{2p-1}$$



(a) ψ_1



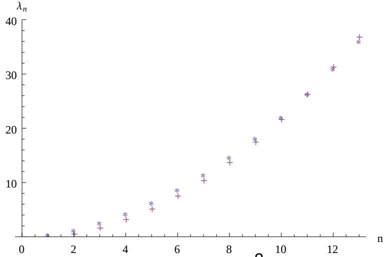
(b) ψ_2



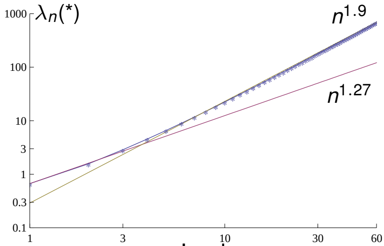
(c) ψ_{60}

- ψ_1^2 selects small z region. In this region $\psi_{2p-1} \sim \cos(z\lambda_{2p-1}^{1/2})$

Regge Trajectory



$\lambda_n(*)$ and $0.25n^2(+)$ vs n



Loglog n

Outline

1. Motivation
   - Gauge/String duality vs. the Real QCD
   - D4/D8 pion form factors
2. D4/D8 Brane Model
   - Holographic QCD
3. Work
   - Scattering Vector Mesons
   - Wave functions
   - Quadratic radius
4. Summary

Using the ρ (vector) meson form factor expression in the low Q^2 limit, in terms of the electric form factor:

$$G_c^\rho(Q^2) = \left(1 - \frac{Q^2}{6m_{v^1}^2} F_{v^1}(Q^2)\right)$$

$$\langle r_\rho^2 \rangle = -6 \frac{\mathrm{d}}{\mathrm{d}Q^2} G_c^\rho(Q^2)|_{Q^2=0}$$

We found

$$\langle r_\rho^2 \rangle = 0.57\mathrm{fm}^2$$

While results from Dyson-Schwinger equations : $\langle r_\rho^2 \rangle = 0.54\mathrm{fm}^2$

Summary

- Besides some weaknesses (mass spectrum, UV behavior), D4-D8 model shows adequate results, concerning vector meson EM scattering.
- Outlook
  - Would our results improve if quarks were massive?
  - What are the general features that makes D4-D8 model interesting?
  - D4-D8 is not really a Top-down model, since M_{KK} and κ are fixed by experimental values. What can be done?

For Further Reading I

- T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113** (2005) 843 [arXiv:hep-th/0412141].
- T. Sakai and S. Sugimoto, Prog. Theor. Phys. **114** (2005) 1083 [arXiv:hep-th/0507073].
- H. R. Grigoryan and A. V. Radyushkin, Phys. Rev. D **76**, 095007 (2007) [arXiv:0706.1543 [hep-ph]].
- H. R. Grigoryan and A. V. Radyushkin, Phys. Lett. B **650**, 421 (2007) [arXiv:hep-ph/0703069].
- S. J. Brodsky and G. F. de Teramond, Phys. Rev. D **77**, 056007 (2008) [arXiv:0707.3859 [hep-ph]].