Light-Cone Model of Transition Form-Factors in Heavy Meson Decay

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J. Pacheco B. C. de Melo

[a]Laboratório de Física Téorica e Computacional, LFTC, UNICSUL (Brazil) and
[b]Instituto de Física Téorica, IFT, UNESP, (Brazil)
Collaborators: S. Pisano, G. Salmé, E. Pace (Italy), B. Loiseau, B. El-Bennich (France), T. Frederico , E. Gambin (Brazil)

July 10, 2009

Light-Front Coordinates

Four-Vector $\Longrightarrow$ $x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$

$x^+ = t + z$ $\qquad x^+ = x^0 + x^3$ $\Longrightarrow$ **Time**

$x^- = t - z$ $\qquad x^- = x^0 - x^3$ $\Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3$$
$$p^- = p^0 - p^3$$
$$p^\perp = (p^1, p^2)$$

Dirac Matrix and Electromagnetic Current

$\gamma^+ = \gamma^0 + \gamma^3 \implies$ **Electr. Current** $J^+ = J^0 + J^3$

$\gamma^- = \gamma^0 - \gamma^3 \implies$ **Electr. Current** $J^- = J^0 - J^3$

$\gamma^\perp = (\gamma^1, \gamma^2) \implies$ **Electr. Current** $J^\perp = (J^1, J^2)$

$p^\mu x_\mu = \frac{p^+x^- + p^-x^+}{2} - \vec{p}_\perp \vec{x}_\perp$

$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$

$p^- \implies$ **Light-Front Energy**

$p^2 = p^+p^- - (\vec{p}_\perp)^2 \implies p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$ **On-shell**

Bosons $\implies S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky

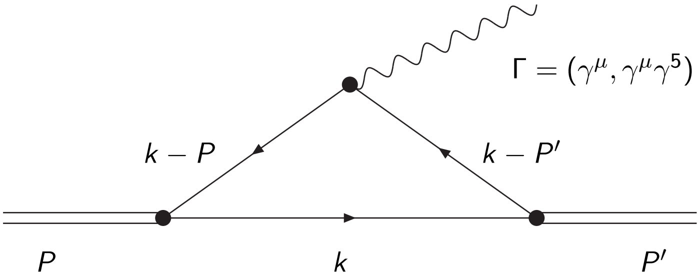
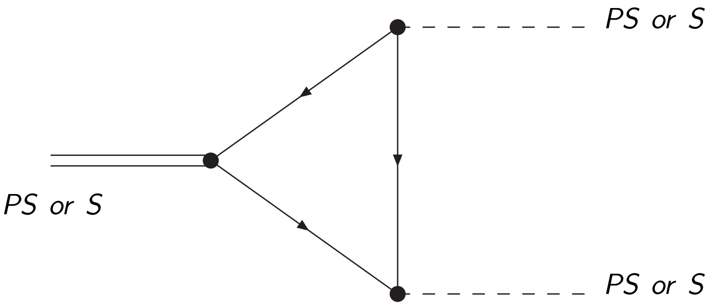


Figure: $P\ (\pi, K, B, D) \longrightarrow P\ (\pi, K, D, B)$



- **Possible Decays**

$$D \longrightarrow \pi\ell\nu \text{ or } K\ell\nu$$
$$B,\ D \longrightarrow \pi\pi \text{ or } K\ K;\quad f_0 \longrightarrow \pi\pi \text{ or } K\ K$$

Bethe-Salpeter Vertex

$$\Lambda_M(k,p) = \frac{\left(k^2 - m_1^2\right) \Gamma_M \left(\left(p-k\right)^2 - m_2^2\right)}{\left(k^2 - \lambda_M^2 + i\varepsilon\right)^n \left(\left(p-k\right)^2 - \lambda_M^2 + i\varepsilon\right)^n}$$

$$\Psi(k,p) = S(k)\ \Lambda(k,p)\ S(k-P)$$

- λ_M: Scale Associate with the Meson Light-Front Valence Wave Function
- **The Factors in the Numerador: Avoids the Cuts due $q\bar{q}$ scattering if $m_1 + m_2 \; < \;$ Meson Mass**
- **In order to confine the quarks: $2 \; \lambda_M \; > \;$ Meson Mass**

- **Three-point Function** $\Rightarrow$ **The General Matrix Element**

$$\langle p' \left| J_q^\mu \left(q^2\right) \right| p \rangle = \frac{N_c}{(2\pi)^4} \int d^4k \; Tr \left[\Lambda_{M'}\left(k, p'\right) S_F\left(k - p'\right) J_q^\mu S(k-p) \Lambda_M\left(k, p\right) S_F\left(k\right) \right]$$

- $PS - q\bar{q}$ **Vertex:** γ^5
- **Fermi Propagator** $\Longrightarrow$ $S_F\left(p\right) = \frac{i}{\not{p} - m_i + i\varepsilon}$

- **The Integration:** Light-Front and Wick Rotation

$$
\begin{aligned}
q^+ &= -q^- = \sqrt{-q^2}\sin\alpha \\
q_x &= \sqrt{-q^2}\cos\alpha, \;\; q_y = 0 \\
q^2 &= q^+q^- - (q_\perp)^2 \; .
\end{aligned}
$$

Breit Frame $(\alpha = 0) \implies q^+ \to 0 \,, q^- = 0 \,; \; \vec{q} \neq 0$

$J^+_\pi = J^0 + J^3 \implies$ No Pair Term Contribuition

$J^-_\pi = J^0 - J^3 \implies$ Pair Term Contribuition

J.P.B.C. de Melo, T. Frederico and H.L. Naus
Phy.Rev. **C59** (1999) 2278

J.P.B.C. de Melo, T. Frederico, E. Pace and G. Salmè
Nucl. Phys. **A 707** (2002) 399

Eletromagnetic Form Factor and Pseudoscalar Constant

$$\left\langle p' \left| J_q^{\mu}\left(q^2\right) \right| p \right\rangle = \left(p + p'\right)^{\mu} F_{PS}^{em}(q^2).$$

- **Weak Decay Constant of the Pseudoscalar Mesons**

$$\left\langle 0 \left| A^{\mu}(0) \right| p \right\rangle = \imath\sqrt{2} f_{ps} p^{\mu}$$

$\Longrightarrow A^{\mu} = \bar{q}(x)\gamma^{\mu}\gamma^{5}\frac{\tau}{2}q(x)$

- **The Final expression for Decay Constant**

$$\imath p^{\mu} f_{\pi} = \frac{m}{f_{\pi}} N_c \int \frac{dk^4}{(2\pi)^4} Tr\left[\gamma^{\mu}\gamma^{5} S(k)\gamma^{5} S(k-p)\right] \Lambda_M(k,p)$$

- **Plus component E.M. Current:** $\gamma^{+} = \gamma^{0} + \gamma^{3}$

$$k^- \quad \Longrightarrow \quad k^- e^{\imath\theta} \; ; \qquad 0^0 \leq \theta \leq 90^0$$

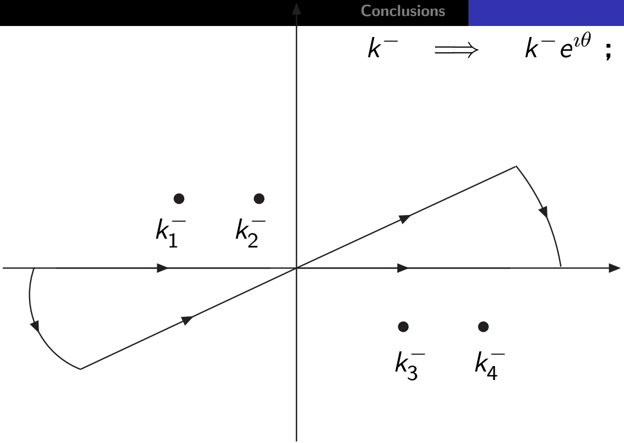


Figure: Wick rotation in the minus momentum component.

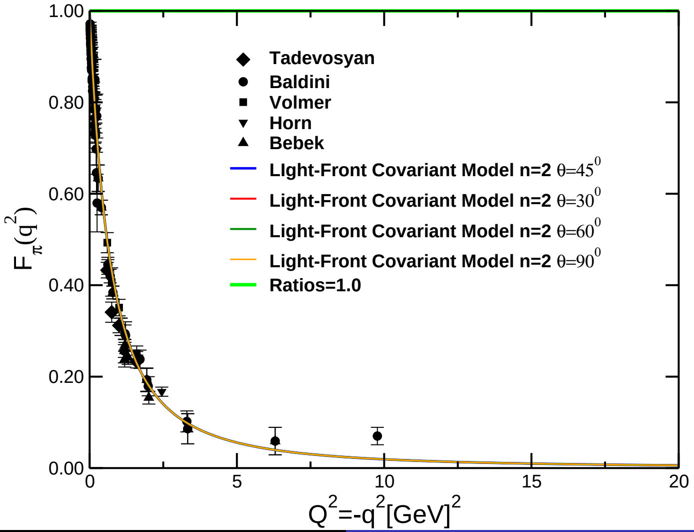
Ref. **PoS (LC2009) 003**, de Melo, J. Gambin, E. and Frederico, T.

Testing Wick Rotation

- **Integration in k^- for n=2**

$$\frac{F_\pi(q^2,\theta)}{F_\pi(q^2,\alpha)} = 1$$

- **Angles** $\Longrightarrow \begin{cases} \theta = 30^0, 45^0, 60^0 \ \textit{and}\ 90^0 \\ \alpha = 30^0, 45^0, 60^0 \ \textit{and}\ 90^0 \\ \theta \neq \alpha \end{cases}$
- $r_\pi \cdot f_\pi = 0.270$


Tadevosyan
Baldini
Volmer
Horn
Bebek
LIght-Front Covariant Model n=2 θ=45⁰
Light-Front Covariant Model n=2 θ=30⁰
Light-Front Covariant Model n=2 θ=60⁰
Light-Front Covariant Model n=2 θ=90⁰
Ratios=1.0
F_π(q²)
Q²=-q²[GeV]²
1.00
0.80
0.60
0.40
0.20
0.00
0
5
10
15
20

•**Motivation:**

Why are Semileptonic Decays Important

•**Pseudoscalar D Semileptonic Decays:**

i) $\Longrightarrow \quad D \Rightarrow K \ \ell \ \nu$

ii) $\Longrightarrow \quad D \Rightarrow \pi \ \ell \ \nu$

•**Rare Semileptonic Decays**

i) $D^0 \Longrightarrow K^- \pi^+ \pi^- e^+ \nu_e$

ii) $D^+ \Longrightarrow \eta e^+ \nu$

Motivation

Goal:: Precision measurements of CKM matrix elements via semileptonic decays

• **Problem:** Form factors parameterizing strong effects require input from theory

• **Nonperturbative New techniques:** Lattice QCD, LQCD

• **Promise more accuracy:** Idea use measurements in 2 charm sector to verify techniques to be used in B sector.

Transition Form Factors: Semileptonic Decays

- **Two Transition Form Factors:** $f_+(q^2)$ **and** $f_-(q^2)$

$$\begin{aligned} \langle (K,\pi)|J^\mu|D\rangle &= (P_1+P_2)^\mu f_+(q^2) + f_-(q^2)(P_2-P_1)^\mu \\ &= f_+(q^2)\left[(P_2+P_1)^\mu - \frac{M_1^2-M_2^2}{q^2}q^\mu\right] \\ &\quad + f_0(q^2)\frac{M_1^2-M_2^2}{q^2}q^\mu, \end{aligned}$$

$$\begin{aligned} f_+(0) &= f_0(0), \\ f_0(q^2) &= f_+(q^2) + \frac{q^2}{M_1^2-M_{ps}^2} f_-(q^2), \end{aligned}$$

Frames: The Kinematics of the Model

- $q^+ > 0$ **(Lev-Pace-Salmé Frame)***
- **Elastic Form Factors: Eletromagnetic Processes**

$$P_2^+ = P_1^+ + q^\mu; \quad q^+ \geq 0$$
$$\vec{P}_1 = \vec{P}_2 = 0; \quad (P_1^\mu)^2 = (P_2^\mu)^2 = M_{meson}^2$$

- **Transition Form Factors: Decay Processes**

$$P_1^+ = P_2^+ - q^\mu$$
$$\vec{P}_1 = \vec{P}_2 = 0; \quad q^+ < 0$$

⋆ **de Melo, Frederico,Pace, Pisano, Salmé, Phys.Lett.B671 (2009) ibid. Phys.Rev.D73 (2006) .**

	n	$r_{PS}(fm)$	$f_{PS}(MeV)$	λ_M	m_q	m_Q
Pion (140)	2	0.576	92.4	542	0.220	
$m_u = 220$	3	0.494	92.4	926	0.220	
$m_d = 220$	4	0.456	92.4	1255	0.220	
	10	0.392	92.4	2666.3	0.220	
	12	0.384	92.4	3023.4	0.220	
Exp.(Pion)		0.672	92.42			
Kaon (494)	2	0.474	113	648	0.220	0.508
	3	0.453	113	933	0.220	0.508
	4	0.450	113	1156	0.220	0.508
	10	0.417	113	2478.3	0.220	0.344
	12	0.412	113	2778.8	0.220	0.344
Exp.(Kaon)		0.560	113			

n	m_u	m_s	$< r >/K^+$ (fm)	$< r >/\pi^+$ (fm)
2	0.220	0.344	0.513	0.576
3	0.220	0.344	0.468	0.494
4	0.220	0.344	0.449	0.456
5	0.220	0.344	0.438	0.437
6	0.220	0.344	0.431	0.421
10	0.220	0.344	0.417	0.344

- $D \Longrightarrow \pi\ell\ell$

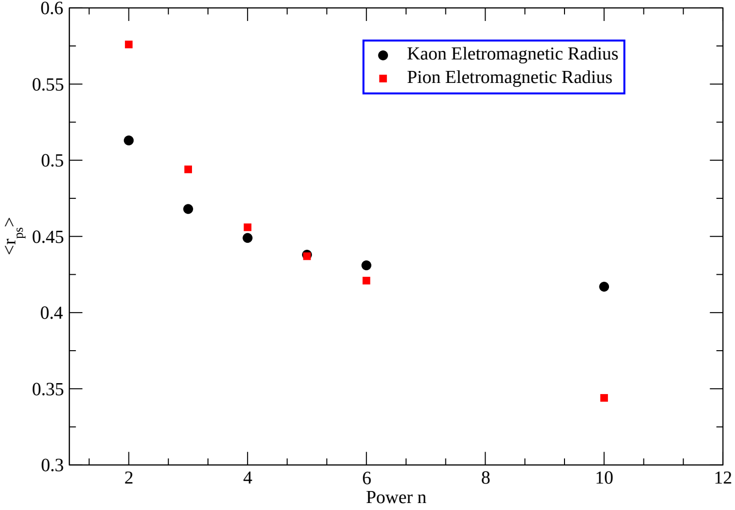
	$f_+(0)$	n	r_{ps}	f_{ps}	λ_M	m_q	m_Q
Pion	**0.9371**	**10**	**0.344**	**0.0924**	**2.665**	**0.220**	**0.220**
Pion (exp.)			**0.672**	**0.0924**			
D^+	**0.9371**	**10**	**0.346**	**0.145**	**2.1385**	**0.220**	**1.458**
Exp. D^+				**0.1445**			
Exp.(CLEO)	**0.665**						

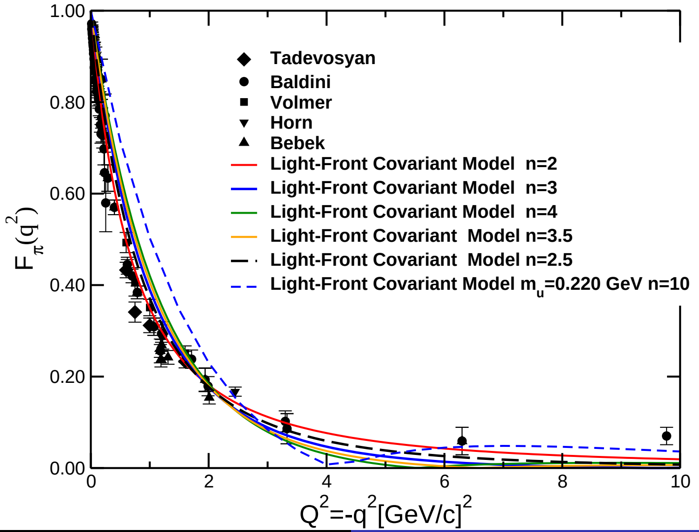
	$f_+(0)$	n	r_{ps}	f_{ps}	λ_M	m_q	m_Q
Kaon	0.04	10	0.153	0.113	7.8711	0.185	0.309
	0.230	10	0.227	0.113	4.6724	0.190	0.314
	0.632	10	0.324	0.113	3.2519	0.200	0.324
	0.697	10	0.338	0.113	3.115	0.202	0.326
	0.729	10	0.344	0.113	3.0564	0.203	0.327
	0.757	10	0.346	0.113	3.002	0.204	0.328
	0.913	10	0.380	0.113	2.7444	0.210	0.334
	1.100	10	0.417	0.113	2.4783	0.220	0.344
		10	0.480	0.113	2.0764	0.260	0.384
		10	0.489	0.113	2.0053	0.280	0.404
		10	0.492	0.113	1.9822	0.290	0.414
		10	0.493	0.113	1.9650	0.300	0.424
Exp.(Kaon)			0.560	0.113			
D^+	0.04	10	0.337	0.145	2.1884	0.185	1.424
	0.230	10	0.337	0.1445	2.814	0.190	1.429
	0.632	10	0.340	0.145	2.1674	0.200	1.439
	0.729	10	0.341	0.145	2.1633	0.203	1.442
	0.697	10	0.342	0.145	2.1647	0.202	1.441
	0.757	10	0.343	0.145	2.162	0.204	1.443
	0.913	10	0.344	0.145	2.1532	0.210	1.449
	1.1006	10	0.346	0.145	2.1385	0.220	1.459
Ex. D^+				0.145			
Exp.(BaBar)	0.727						

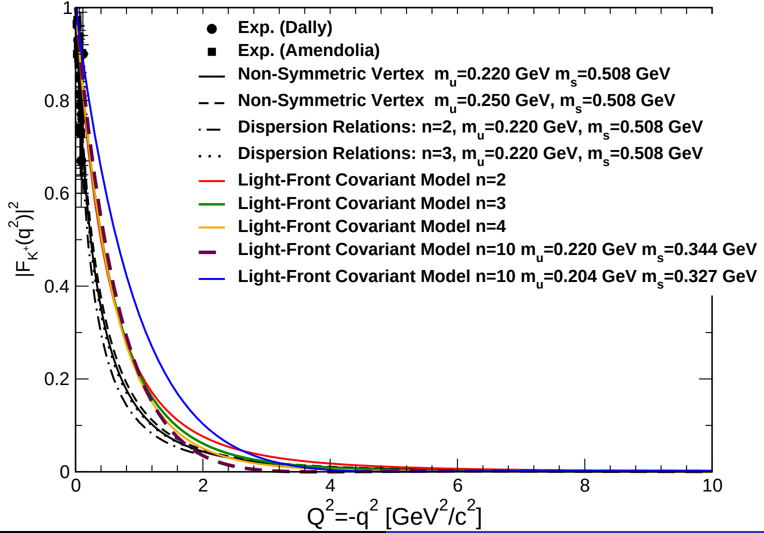
Lattice QCD and Experimental Values

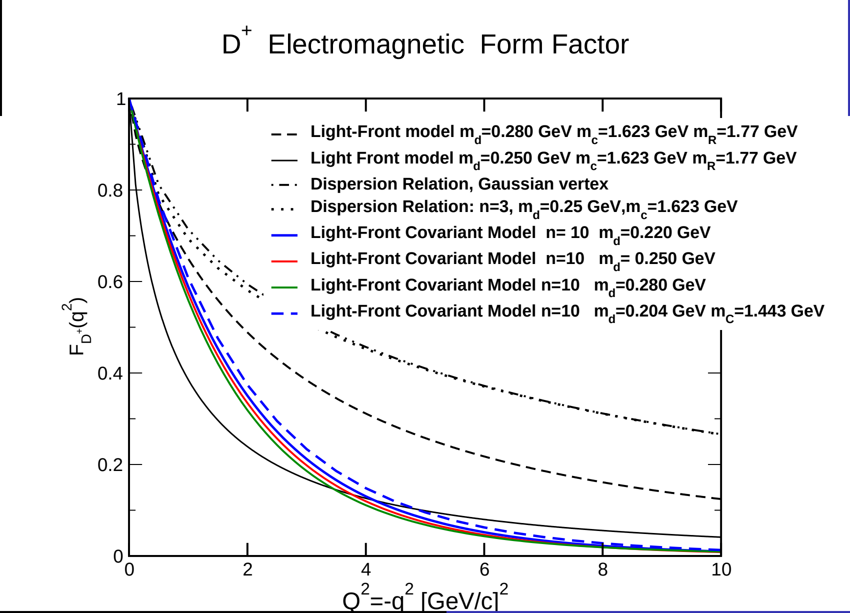
	$f_+^K(0)$	$f_+^\pi(0)$
LQCD [1]	0.664	0.576
LQCD [2]	0.733	0.643
Belle [3]	0.695	0.624
BaBar [4]	0.728	
CLEO-c [5]	0.763	0.628
CLEO (2009) [6]	0.739	0.665

Ref. [1] A. Abada et al., Phy. Rev. D66, 074504 (2002)
[2] C. Aubin et al., Phy. Rev. 94, 011601 (2005)
[3] L. Widham et al., Phys. Rev. Lett. 97, 061804 (2006)
[4] B. Aubert et al., Phys. Rev. D 76, 052005 (2007)
[5] J. Y. Ge at al., Phy. Rev. D 79, 052010 (2009)
[6] D. Besson et al., hep-exp/09062983 (2009)

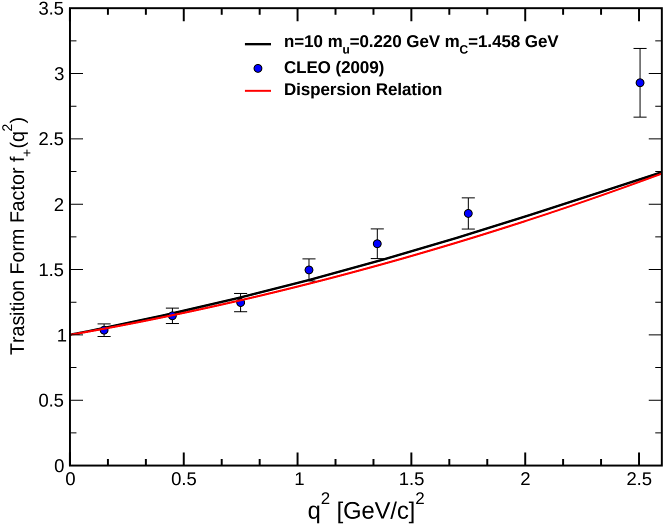

0.6
0.55
0.5
0.45
0.4
0.35
0.3
<r_ps>
2
4
6
8
10
12
Power n
Kaon Eletromagnetic Radius
Pion Eletromagnetic Radius


Tadevosyan
Baldini
Volmer
Horn
Bebek
Light-Front Covariant Model n=2
Light-Front Covariant Model n=3
Light-Front Covariant Model n=4
Light-Front Covariant Model n=3.5
Light-Front Covariant Model n=2.5
Light-Front Covariant Model m_u=0.220 GeV n=10
$F_\pi(q^2)$
$Q^2=-q^2[\mathrm{GeV/c}]^2$
1.00
0.80
0.60
0.40
0.20
0.00
0
2
4
6
8
10

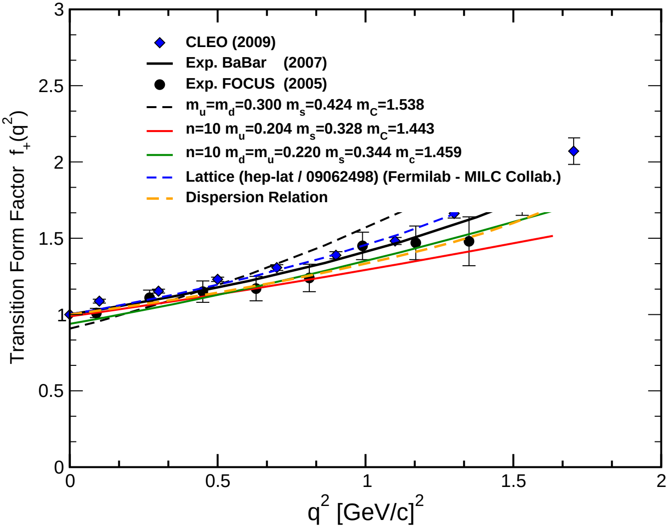

Exp. (Dally)
Exp. (Amendolia)
Non-Symmetric Vertex m_u=0.220 GeV m_s=0.508 GeV
Non-Symmetric Vertex m_u=0.250 GeV, m_s=0.508 GeV
Dispersion Relations: n=2, m_u=0.220 GeV, m_s=0.508 GeV
Dispersion Relations: n=3, m_u=0.220 GeV, m_s=0.508 GeV
Light-Front Covariant Model n=2
Light-Front Covariant Model n=3
Light-Front Covariant Model n=4
Light-Front Covariant Model n=10 m_u=0.220 GeV m_s=0.344 GeV
Light-Front Covariant Model n=10 m_u=0.204 GeV m_s=0.327 GeV
|F_K+(q^2)|^2
Q^2=-q^2 [GeV^2/c^2]
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10


D+ Electromagnetic Form Factor
Light-Front model m_d=0.280 GeV m_c=1.623 GeV m_R=1.77 GeV
Light Front model m_d=0.250 GeV m_c=1.623 GeV m_R=1.77 GeV
Dispersion Relation, Gaussian vertex
Dispersion Relation: n=3, m_d=0.25 GeV,m_c=1.623 GeV
Light-Front Covariant Model n= 10 m_d=0.220 GeV
Light-Front Covariant Model n=10 m_d= 0.250 GeV
Light-Front Covariant Model n=10 m_d=0.280 GeV
Light-Front Covariant Model n=10 m_d=0.204 GeV m_C=1.443 GeV
F_D+(q^2)
Q^2=-q^2 [GeV/c]^2
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10

D -----> π lepton lepton



D–> Kll


CLEO (2009)
Exp. BaBar (2007)
Exp. FOCUS (2005)
m_u=m_d=0.300 m_s=0.424 m_C=1.538
n=10 m_u=0.204 m_s=0.328 m_C=1.443
n=10 m_d=m_u=0.220 m_s=0.344 m_c=1.459
Lattice (hep-lat / 09062498) (Fermilab - MILC Collab.)
Dispersion Relation
Transition Form Factor f_+(q^2)
q^2 [GeV/c]^2
0
0.5
1
1.5
2
2.5
3

Final Remarks

- Analytical Power Law Vertex Model for Pseudoscalar Mesons
- Simple Computation of Form-Factors and Decay Constants
- Wick Rotation: Covariant Calculation $\Longrightarrow q^+ > 0$
- Triangle plus Z Diagram : Poles Separated
- Easy to Test Different Analytical Models

$\Longrightarrow$ Asymptotic Form of Form-Factors

$\Longrightarrow$ Mesons Decays - PS to PS

- Next

Vector Mesons Decays

Rare Semileptonic Decays