Color Superconductivity and Confinement in the Chromodieletric Model

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Summary

Introduction

The Chromodielectric model (CDM) provides a reasonable framework to study baryons (such as the nucleon, delta, Roper) at low densities and to study strange quark matter at very high densities. In this regime, two different phases show up: a chiral broken and a chiral symmetric phase, the latter not necessarily absolutely stable. At high densities, the abundance of quarks u, d and s are the same in the chiral symmetric phase and there are no electrons. These two properties are also obtained in a new phase which is expected to occur in QCD at very high densities, known as color flavor locked (CFL) phase.

Introduction

This suggests that strange matter may undergo a transition to the CFL phase, with an energy lowering due to the quark BCS pairing. It is now generally believed that the CFL state (at least for asymptotic densities) is likely to be the ground state for strongly interacting matter. In this work we report on a study in an extended version of the Chromodielectric model (CDM) with the BCS quark pairing implemented. Pairing corrections are carried using the methods developed for nuclear matter [1].

Chromodielectric Model

Lagrangian of the CDM model

$$\mathcal{L} = \overline{\Psi}\left(\imath\partial\!\!\!/ - m\right)\Psi + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma + \partial^\mu\vec{\pi}\cdot\partial_\mu\vec{\pi}\right) - W\left(\sigma,\vec{\pi}\right)$$
$$+\frac{G(\chi)}{f_\pi}\overline{\Psi}\left(\sigma + \imath\vec{\tau}\cdot\vec{\pi}\right)\Psi + G_s(\chi)\overline{\Psi}_s\Psi_s + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - U(\chi),$$

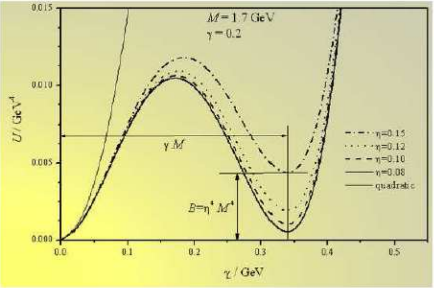
where $G(\chi) = -gf_\pi/\chi$

Chromodielectric Model

$$W\left(\sigma,\vec{\pi}\right)=\frac{m_\sigma^2}{8f_\pi^2}\left(\sigma^2+\pi^2-f_\pi^2\right)^2 \; ,$$

$$U(\chi)=\frac{1}{2}m_\chi^2\chi^2\left[1+\left(\frac{8\eta^4}{\gamma^2}-2\right)\left(\frac{\chi}{\gamma m_\chi}\right)+\left(1-\frac{6\eta^4}{\gamma^2}\right)\left(\frac{\chi}{\gamma m_\chi}\right)^2\right]$$

Chromodielectric Model



Model parameters: $g = 0.023$ GeV, $f_\pi = 0.093$ GeV, $m_\chi = 1.7$ GeV, $\eta = 0.1$, $\gamma = 0.2$.

Summary

Pairing Lagrangian in Leading Order (LO)

Non-linear Lagrangian for quarks interacting with a scalar field (only the confine sector) with a quadratic potential:

$$\mathcal{L} = \overline{\Psi}\,(\partial\!\!\!/ - m)\,\Psi + G(\chi)\overline{\Psi}\Psi - U(\chi) + \frac{1}{2}\partial^{\mu}\chi\partial_{\mu}\chi, \qquad (1)$$

Pairing Lagrangian in Leading Order (LO)

where $G(\chi) = gf_\pi/\chi$ and $U(\chi) = \frac{1}{2}m_\chi^2\chi^2$.

Assuming infinite matter $\overline{\chi} = m_\chi^{-2}G'(\overline{\chi})\langle\overline{\Psi}\Psi\rangle$,

the scalar density $\rho_S = \langle\overline{\Psi}(x)\Psi(x)\rangle$

$$\overline{\chi} = -m_\chi^{-2}\,\frac{gf_\pi}{\overline{\chi}^2}\,\rho_S \tag{2}$$

Pairing Lagrangian in Leading Order (LO)

Assuming the pairing effect to be enough small

$$\chi = \overline{\chi} + \delta\chi$$

the Lagrangian around the mean field $\overline{\chi}$ up to order $\mathcal{O}\left(\delta\chi^2\right)$ is

$$\mathcal{L} = \overline{\Psi}\left(\not{\partial} - m\right)\Psi + G(\overline{\chi})\overline{\Psi}\Psi - \frac{1}{2}m_\chi^2\overline{\chi}^2 + \frac{1}{2}\left(G'(\overline{\chi})\overline{\Psi}\Psi + m_\chi^2\overline{\chi}\right) \times \int d^4x'\mathcal{G}(x,x')\left(G'(\overline{\chi})\overline{\Psi}(x')\Psi(x') - m_\chi^2\overline{\chi}\right). \quad (3)$$

Pairing Lagrangian in Leading Order (LO)

Expanding up to order $(\overline{\Psi}\Psi)^2$ and constructing a pairing Lagrangian keeping terms up to the order $(\overline{\Psi}\Psi - \langle\overline{\Psi}\Psi\rangle)^2$.

The mass of the χ field gains a self-energy contribution given by:

$$\overline{M}^2 = m_\chi^2 - G''(\overline{\chi})\langle\overline{\Psi}\Psi\rangle \ . \quad (4)$$

Pairing Lagrangian in Leading Order (LO)

The expansion for the Lagrangian just to leading order in $(\overline{\Psi}\Psi - \langle\overline{\Psi}\Psi\rangle)$ reads

$$\mathcal{L}_{LO} = \overline{\Psi}\left(\not{\partial} - m\right)\Psi + G(\overline{\chi})\overline{\Psi}\Psi - \frac{1}{2}M^2\overline{\chi}^2 + \frac{1}{2\overline{M}^2}(G'(\overline{\chi}))^2\left((\overline{\Psi}\Psi) - \langle\overline{\Psi}\Psi\rangle\right)^2 , \quad (5)$$

where the term neglected corresponds to NLO in the expansion of the Lagrangian in terms of $(\overline{\Psi}\Psi - \langle\overline{\Psi}\Psi\rangle)$.

Pairing Lagrangian in Leading Order (LO)

Thus pairing of quarks are generated by

$$\mathcal{L}_{\mathrm{I,LO}} = h\overline{\Psi}^{a,i}_{\alpha}\Psi^{a,i}_{\alpha}\overline{\Psi}^{b,j}_{\beta}\Psi^{b,j}_{\beta} \,, \tag{6}$$

the quartic fermionic term in the LO Lagrangian, with the coupling,

$$h = \frac{1}{2}\left(\frac{G'(\overline{\chi})}{\overline{M}}\right)^2 \,. \tag{7}$$

Pairing Lagrangian in Leading Order (LO)

In the complete CDM version

$$\overline{M} = U''(\overline{\chi}) - U'(\overline{\chi}) \frac{\bar{G}''(\overline{\chi})}{\bar{G}'(\overline{\chi})} \ . \tag{8}$$

$$\overline{M} = m_\chi^2 \left(3 + 12 \left(\frac{4\eta^4}{\gamma^2} - 1 \right) \left(\frac{\chi}{\gamma m_\chi} \right) + 10 \left(1 - \frac{6\eta^4}{\gamma^2} \right) \left(\frac{\chi}{\gamma m_\chi} \right)^2 \right) \ . \quad ($$

Summary

Gap structure

The color indices are a, b, flavor i, j and Dirac $\alpha, beta$. Now we introduce the conjugate fields $\Psi_{\mathrm{T}} = B\overline{\Psi}^{\top}$ and $\overline{\Psi}_{\mathrm{T}} = \Psi^{\top}B$ with $B = \gamma^5 C$ and the charge conjugation operator $C = i\gamma^2\gamma^0$. The properties are valid $B^2 = -1$, $B^{\dagger} = B^{\top} = -B = B^{-1}$. The operator $\overline{\Psi}_c \Delta \Psi$ has to obey the following consistence relation

$$\overline{\Psi}_{\mathrm{T}} \Delta \Psi = -\Psi^{\top} \Delta^{\top} \overline{\Psi}_{\mathrm{T}}^{\top} = -\Psi^{\top} B B \Delta^{\top} B B \overline{\Psi}_{\mathrm{T}}^{\top} = -\overline{\Psi}_{\mathrm{T}} B \Delta^{\top} B \Psi \;, \quad (10)$$

expressed as $\Delta = -B\Delta^{\top}B$, because Ψ is a Grassmann variable.

Gap structure

We classify the possible invariants that satisfy the consistency relation $\Delta = -B\Delta^{\top}B$. The color and flavor (u,d,s) irreducible tensors belong to the representations of SU(3): $3 \otimes 3 = \overline{3} \oplus 6$. The $\overline{3}$ representation corresponds to antisymmetric tensors given by totally antisymmetric tensor ε_{kij} for flavor and ε_{cab} for color. The 6-symmetric irreducible representation can be associated with the identity and Gellmann matrices for the generators of SU(3) corresponding to λ_1, λ_3, λ_4, λ_6, λ_8 and $\sqrt{\frac{2}{3}}$I, with the trace of products being zero for different matrices and 2 otherwise.

Gap structure

In table I, the non-vanishing structures for the contact interaction Lagrangian of Eq.(6) are presented. The structure simplifies because the gap has no momentum dependence and some structures are not allowed.

Δ	$\bar{3}_c \otimes \bar{3}_f$	$6_c \otimes 6_f$	$\bar{3}_c \otimes 6_f$	$6_c \otimes \bar{3}_f$
I	$\times$	$\times$	-	-
γ^μ	$\times$	$\times$	-	-
γ^5	$\times$	$\times$	-	-
$\gamma^\mu\gamma^5$	-	-	$\times$	$\times$
$\sigma^{\mu\nu}$	-	-	$\times$	$\times$

Gap structure

The gap function has the general form allowed by the interaction (6) and, for infinite quark matter it is given by:

$$\Delta^{a,i;b,j} = \varepsilon_{cab}\varepsilon_{kij}\left(\Delta^{\bar{3}\otimes\bar{3}}_{S;c;k} + \Delta^{\bar{3}\otimes\bar{3}}_{0;c;k}\gamma^0 + \Delta^{\bar{3}\otimes\bar{3}}_{PS;c;k}\gamma^5\right)$$
$$+\mathcal{S}_{cab}\mathcal{S}_{kij}\left(\Delta^{6\otimes 6}_{S;c;k} + \Delta^{6\otimes 6}_{0;c;k}\gamma^0 + \Delta^{6\otimes 6}_{PS;c;k}\gamma^5\right)$$
$$+\mathcal{S}_{cab}\varepsilon_{kij}\Delta^{\bar{3}\otimes 6}_{PV;c;k}\gamma^0\gamma^5 + \varepsilon_{cab}\mathcal{S}_{kij}\Delta^{6\otimes\bar{3}}_{PV;c;k}\gamma^0\gamma^5 \,. \qquad (11)$$

where the six symmetric tensors are represented by $\mathcal{S}_{cab}$.

Gap structure

The color-flavor locked phase corresponds to take into account only the gap structure $\overline{3} \otimes \overline{3}$ terms yielding a ground state symmetric under color and flavor rotations. The gap reduces to:

$$\Delta^{a,i;b,j}_{CFL} = \left(\delta_{ai}\delta_{bj} - \delta_{bi}\delta_{aj}\right)\left(\Delta_S + \Delta_0\gamma^0 + \Delta_{PS}\gamma^5\right) , \quad (12)$$

where we have dropped the notation for the group irreducible representation, and we have used $\varepsilon_{cab}\varepsilon_{cij} = \delta_{ai}\delta_{bj} - \delta_{bi}\delta_{aj}$.

Summary

LO pairing dynamics in CFL phase and self-consistent gap calculation

Pairing of quarks will be dynamically generated by $\mathcal{L}_{\mathrm{I,LO}} = h\overline{\Psi}^{a,i}_{\mathrm{T},\alpha}\Psi^{a,i}_{\mathrm{T},\alpha}\overline{\Psi}^{b,j}_{\beta}\Psi^{b,j}_{\beta}$ where we have make use of the properties of the Grassmann variables and $B^2 = -1$. The gap is the expectation value of the operator

$$\Delta^{a,i;b,j}_{\mathrm{CFL}} = 2h\langle\Psi^{a,i}_{\alpha}\overline{\Psi}^{b,j}_{\mathrm{T},\beta}\rangle, \tag{13}$$

and equating it to the gap decomposed in its three terms from Eq.(12). To make the decomposition of the different gap terms the following contraction is useful:

$$\sum_{a,b,i,j}\delta_{ai}\delta_{bj}\left(\delta_{ai}\delta_{bj} - \delta_{bi}\delta_{aj}\right) = \sum_{i,j}\left(\delta_{ii}\delta_{jj} - \delta_{ij}\delta_{ij}\right) = 6. \tag{14}$$

LO pairing dynamics in CFL phase and self-consistent gap calculation

Therefore, one has $\left(\Delta_S + \Delta_0\gamma^0 + \Delta_{PS}\gamma^5\right)_{\alpha\beta} = \frac{1}{12}\langle \Psi^{i,i}_{\alpha}\overline{\Psi}^{j,j}_{T,\beta}\rangle$. Performing the traces with appropriate spinor operators the gap equations are found:

$$\Delta_X = \frac{h}{48}\mathrm{Tr}\left[\Gamma^X\langle \Psi^{i,i}\overline{\Psi}^{j,j}_T\rangle\right] \ , \qquad (15)$$

with X=S, 0 and PS, with corresponding $\Gamma^X = I$, γ^0 and γ^5, respectively.

LO pairing dynamics in CFL phase and self-consistent gap calculation

We now present the formalism proposed by Gorkov [2] in order to arrive at the Dirac-Gorkov equations of motion. We use the extended form of the time–reversed states $\psi_\top$, already defined by $\psi_\top = \Psi_c = B\overline{\Psi}^\top$, such that now we have an ansatz for the effective single–particle Lagrangian

$$S_{\rm eff} = \int dt\, L_{\rm eff} = \int d^4x \Big\{ \overline{\psi}(x)\,[i\partial\!\!\!/ - M + \gamma_0\mu]\,\psi(x) - \overline{\psi}(x)\Sigma\psi(x) + \frac{1}{2}\overline{\psi}(x)\Delta\psi_{\rm T}(x) + \frac{1}{2}\overline{\psi}_{\rm T}(x)\overline{\Delta}\psi(x) \Big\},$$

where μ is the chemical potential to be used as a Lagrange multiplier to fix the average number of particles.

LO pairing dynamics in CFL phase and self-consistent gap calculation

The symmetries of the effective mean–field Lagrangian under transposition and Hermitian conjugation, yield the following properties of the mean fields: $\Delta = -B\,\Delta^{\top}\,B$ and $\overline{\Delta} = -B\,\overline{\Delta}^{\top}\,B$; $\Sigma = \gamma_0\Sigma^{\dagger}\gamma_0$ and $\Delta = \gamma_0\overline{\Delta}^{\dagger}\gamma_0$ where $\overline{\Delta} = \gamma_0\Delta^{\dagger}\gamma_0$.

The Dirac–Gorkov equations are the following coupled equations of motion for the fields ψ and $\psi_{\top}$

$$\begin{pmatrix} (i\partial\!\!\!/ - M + \gamma_0\mu) - \Sigma & \Delta \\ \overline{\Delta} & (i\partial\!\!\!/ + M - \gamma_0\mu) + \Sigma_{\top} \end{pmatrix} \begin{pmatrix} \psi(x) \\ \psi_{\top}(x) \end{pmatrix} = 0\,.$$

LO pairing dynamics in CFL phase and self-consistent gap calculation

and one obtains a generalized quark (quasiparticle) propagator

$$S(x) = \begin{pmatrix} G(x) & F(x) \\ \tilde{F}(x) & \tilde{G}(x) \end{pmatrix} = -i \left\langle \begin{pmatrix} \psi(x) \\ \psi_{\mathrm{T}}(x) \end{pmatrix} (\overline{\psi}(x) \, , \, \overline{\psi}_{\mathrm{T}}(x)) \right\rangle \, ,$$

where, by $\langle \cdots \rangle$, we mean the timer-ordered expectation value in the interacting quark matter ground state, $\langle \tilde{0} | T(\cdots) | \tilde{0} \rangle$. The Dirac-Gorkov equations are in fact given in term of the inverse Green-function written in terms of the generalized quark propagator as $\mathcal{G}^{-1} = -iS^{-1}$, such that

$$\mathcal{G}^{-1} \begin{pmatrix} \psi(x) \\ \psi_{\mathrm{T}}(x) \end{pmatrix} = \mathcal{G}^{-1} \, \Psi(x) = 0, \tag{16}$$

LO pairing dynamics in CFL phase and self-consistent gap calculation

We observe that in S , $G(x)$ is the usual quark propagator, while $\tilde{G}(x)$ describes the propagation of quarks in time-reversed states. The off-diagonal terms of $S(x)$ describe the propagation of correlated quarks and are just the relativistic generalization of the anomalous propagators defined by Gorkov [2]. Thus we need to find $\mathcal{G}$ inverting $\mathcal{G}^{-1}$ in order to obtain the self consistent equations for the quark self-energy and quark pairing that can be express in terms of the two–fermion vacuum expectation values as

$$\Sigma = 2h\left\langle \psi(x)\overline{\psi}(x) \right\rangle , \qquad \text{and} \quad \Delta = 2h\left\langle \psi(x)\overline{\psi}_{T}(x) \right\rangle ,$$

LO pairing dynamics in CFL phase and self-consistent gap calculation

where the equation for $\bar{\Delta}$ can be obtained using the Hermiticity condition. The quark mass is $M = m - G(\overline{\chi})$ and one may use the form

$$\overline{\Delta}^{a,i;b,j} = \left(\delta_{ai}\delta_{bj} - \delta_{bi}\delta_{aj}\right)\left(\Delta_S^* + \Delta_0^*\gamma^0 + \Delta_{PS}^*\gamma^5\right). \quad (17)$$

Summary

Self-Consistent Gap equations

To invert $\mathcal{G}^{-1}$ one expands it in a SU(2) isospin matrix basis and expand the color-flavor matrices in their eigenvector basis $\langle ai|s\rangle = v^s_{ai}$ using the respective eigenvalues, λ_s. From the solution of the expansion for the color-flavor matrix $C^{a,i;b,j}$ we find nine eigenvalues, λ_s: 2, -1 and 1, with degeneracies $n^{(s)}$. The eigenvalue 2 appears once; the -1 appears in two isolated times; there are three pairs of -1 and 1. If we write the matrix $\Delta^{(s)}_{\alpha\beta} = 2n^{(s)}h\langle\psi_{\top\alpha}\overline{\psi}_{\beta}\rangle_s$, after a straightforward calculation we arrive at the following gap equation

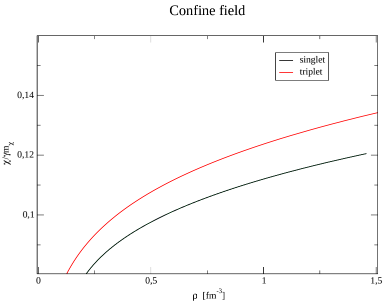
$$\Delta^{(s)}_{\alpha\beta} = 2n^{(s)}hi\sum_{\lambda}\int \frac{d^4k}{(2\pi)^4}\frac{\left(v^{(s)}_{\lambda}\right)_{\alpha}\left(\overline{u}^{(s)}_{\lambda}\right)_{\beta}}{k^0 - \epsilon^{(s)}_{\lambda}(k) + i\delta_{\lambda}}\,. \tag{18}$$

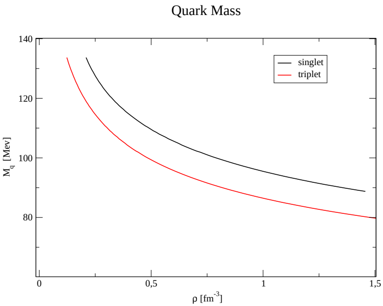
The formalism has been developed, and preliminary results for the solution of the gap equation were obtained.

Summary

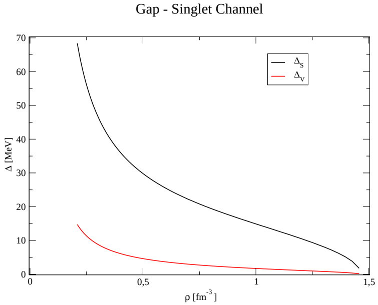
Results

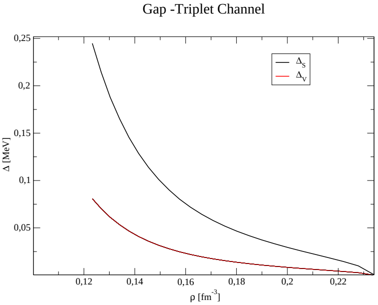
Parameters: g = 0.023 GeV $m_\chi = 1.0$ GeV, $\gamma = 0.2$ and $\eta = 0.1$
They imply in no gap in Phase II.





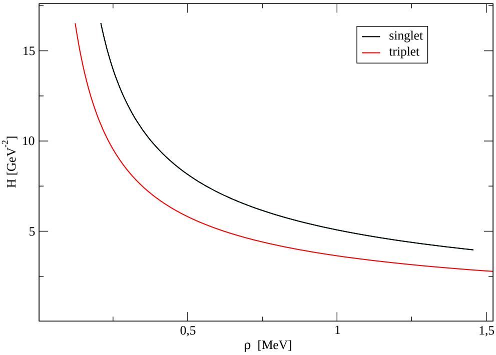
Results


Gap - Singlet Channel
Δ [MeV]
ρ [fm^-3]
0
10
20
30
40
50
60
70
0,5
1
1,5
Δ_S
Δ_V

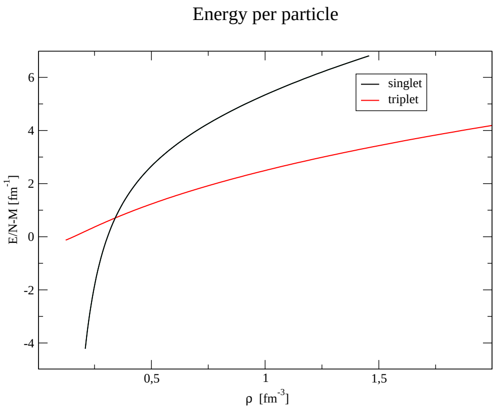

Gap -Triplet Channel
Δ [MeV]
ρ [fm^-3]
0,05
0,1
0,15
0,2
0,25
0,12
0,14
0,16
0,18
0,2
0,22
Δ_S
Δ_V

Results

Effective Coupling Constant



Results


Energy per particle
singlet
triplet
E/N-M [fm^-1]
ρ [fm^-3]
6
4
2
0
-2
-4
0,5
1
1,5

Summary

Conclusions

It is possible to obtain color superconductivity in a self consistent way in the CDM. The confinement affects directly the pairing coupling constant.

$$h = \frac{1}{2}\left(\frac{G'(\overline{\chi})}{\overline{M}}\right)^2 . \qquad (19)$$

$$\overline{M} = m_\chi^2 \left(3 + 12\left(\frac{4\eta^4}{\gamma^2} - 1\right)\left(\frac{\chi}{\gamma m_\chi}\right) + 10\left(1 - \frac{6\eta^4}{\gamma^2}\right)\left(\frac{\chi}{\gamma m_\chi}\right)^2\right) . (2$$

Conclusions

The singlet channel gap is much stronger than in the triplet. The gluon mass m_χ needs to be smaller at high density around $0.5 GeV$ to increase the triplet channel. How to implement Color

Superconductivity on the Light Cone?

Acknowledgements

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We hope to see all of you in Valencia, Spain next year!!!