Taylor-Lagrange renormalization scheme and light-front dynamics

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- Quantised form of the ϕ field :

$$\phi(x) = \int \frac{d^{(D-1)}p}{(2\pi)^{D-1}} \frac{f(\omega_p^2, \vec{p}^2)}{(2\omega_p)} [a_p^+ e^{ipx} + a_p e^{-ipx}]$$

after integration over p^0.

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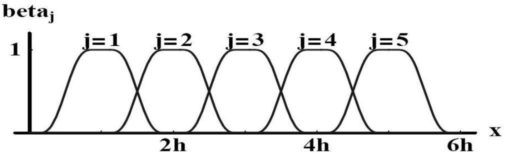

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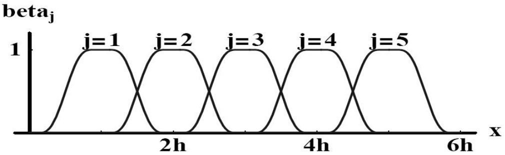

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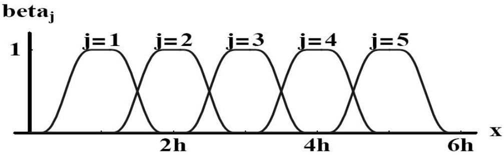

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Crucial properties

1. PU's action is independent of its construction
2. f is a PU $\Rightarrow f^n$ is also a PU

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Regulation of singular distributions

$$A = \int d^D X T(X) f(X) \;=\; \int d^D X T(X) R^k(f(X))$$
$$= \int d^D X \tilde{T}(X) f(X)$$

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- If f is an SRTF, we can generalize :

$$f^{>}(X) = -\frac{X}{k!}\int_1^\infty \frac{dt}{t}(1-t)^k \partial_X^{k+1}[X^k f^{>}(Xt)]$$

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- Example : logarithmic divergence ($k = 1$) :

$$A = \int_0^\infty dXT(X)X\int_1^\infty \frac{dt}{t}\partial_X[f^{>}(Xt)]$$

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Loss of scaling invariance.
$\rightarrow$ "dynamical" limit **depending on X** to escape cut-off type behaviour

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$$\left. \begin{array}{c} \frac{X g'(X)}{g(X)} \text{should be a constant} \\ g \rightarrow 1 \end{array} \right\} \Rightarrow \begin{array}{l} g(X) = X^{(\alpha-1)} \\ X_{max} = H(X_{max}) = (\eta^2)^{\frac{1}{(\alpha-1)}} \end{array}$$

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- Perform the limit **at the end** ie when $\partial_X [XT(X)]$ is self converging

Extension of singular distributions

- Consider a 2D feynman propagator with an arbitrary scale Λ :

$$I = \int \frac{d^2p}{(2\pi)^2} \frac{f^2(p^2)}{p^2 + m^2} = \frac{\Lambda^2}{(2\pi)^2} \int \frac{dX}{X\Lambda^2 + m^2} f(X)$$

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Comparison with P-V

η is a scale $\Rightarrow$ **no infinite limit** to perform.

Extension in the IR

- IR divergent physical amplitude :

$$\begin{aligned} A &= \int_0^\infty dX \, \tilde{T}^<(X) \\ \tilde{T}^<(X) &= (-1)^{k+1} \partial_{(X)}^{k+1} \left[\frac{X^{k+1}}{k!} \int_{\tilde{\eta}X}^1 dt \frac{(1-t)^k}{t^{k+2}} T\left(\frac{X}{t}\right) \right] \end{aligned}$$

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- New expression for photon propagator (LC 2008).
  Departure from ML prescription.

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- **Truncation** at given N.

Test functions and divergences

- creation operators defined with TF $\Rightarrow$

$$\Rightarrow \quad f\left(\frac{p^2}{\Lambda^2}\right) \quad \text{is attached to each boson line of momenta } p$$

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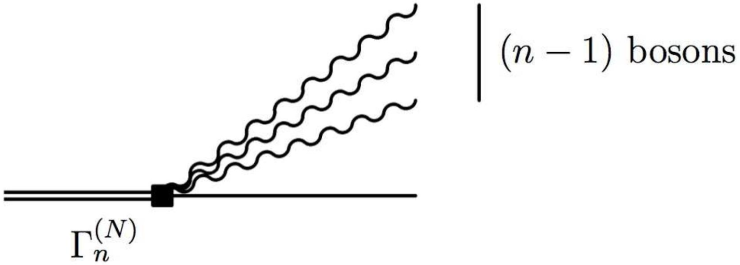

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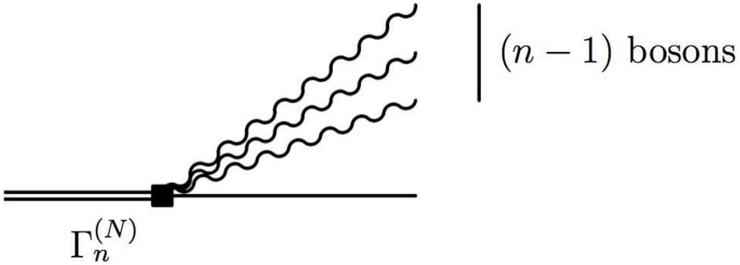
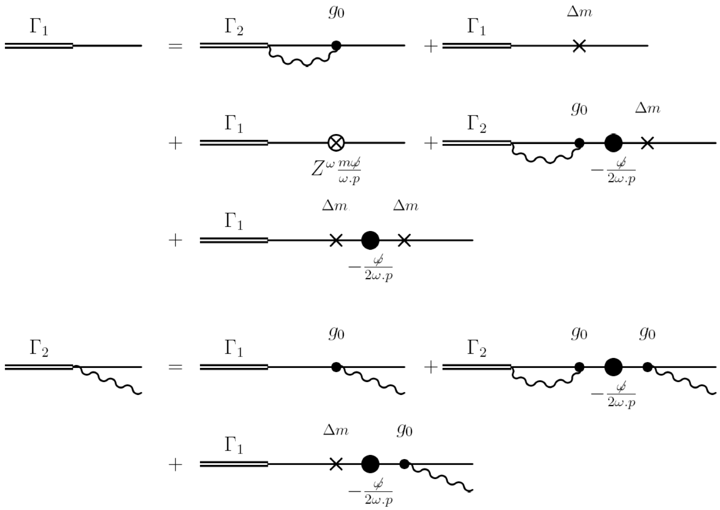


- Finite self energy type diagrams $\Rightarrow$ **mathematically well defined**.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- Solving (1) and (2) we get b_1, b_2 as functions of Δm and g_0 ; and Z_ω as a function of Δm
- Using normalization conditions on b_1, b_2 and the state vectors we get expressions for Δm and Z_ω.

Yukawa $N = 2$, Δm and Z_ω

- Expression for Δm :

$$\Delta m = -\frac{g^2 m}{16\pi} \int d^2\mathbf{k'}_\perp dx' \frac{(2-x')}{\mathbf{k'}_\perp^2 + m^2 x'^2 + \mu^2(1-x')} f(\mathbf{k'}_1^2/\Lambda^2) f(\mathbf{k'}_2^2/\Lambda^2)$$

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- Sufficient to consider :

$$\begin{aligned} I &= \int_0^1 dx' \int_0^\infty d^2\mathbf{k'}_\perp \frac{f(\mathbf{k'}_1^2/\Lambda^2) f(\mathbf{k'}_2^2/\Lambda^2)}{\mathbf{k'}_\perp + m^2 x'^2 + \mu^2(1-x')} \\ &= \pi \log[\eta] - \pi \int_0^1 dx' \log\left[\frac{m^2 x'^2 + \mu^2(1-x')}{m^2}\right] \end{aligned}$$

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- Final result :

$$\Delta m = -\frac{3mg^2}{32\pi^2} \log[\eta] + \frac{mg^2}{16\pi^2} \int_0^1 dx'(2 - x') \log\left[\frac{m^2 x'^2 + \mu^2(1 - x')}{m^2}\right]$$

Compared to the PV result, the finite scale η replaces the PV mass :
Signature of scaling behaviour.

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- Each contribution vanishes $\Rightarrow$ $Z_\omega = 0$ equivalent with P-V results.

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