Semileptonic and radiative decays of B_c and B^*_c mesons in light-front quark model

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Outline

1. Motivation
2. Semileptonic P→ P decays in Covariant Bethe-Salpeter Model and in LF quark model(LFQM)
3. Radiative M1 transitions
4. Conclusion

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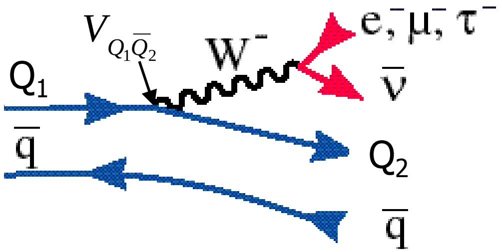
1. Motivation

Semileptonic decays of heavy mesons have provided useful testing ground of SM & QCD dynamics:

- Experiment: easy to access
- Theory : difficult to understand due to the nonperturbative QCD dynamics

(e.g) $P(Q_1\bar{q}) \to P(Q_2\bar{q}) l \nu_l$ semileptonic decays



$$\frac{d\Gamma}{dq^2} = (known) \times |V_{Q_1\bar{Q}_2}|^2 \times \left[a\,|f_+(q^2)|^2 + b\,|f_0(q^2)|^2 \right]$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_1^2 - M_2^2} f_-(q^2)$$

Need accurate values to determine CKM matrix element!

Problem) In LF dynamics, f_-[or f_0] is known to receive zero-mode !
→ Is it possible to construct LF covariant form for f_-[or f_0] ?

- The main goal of this work is

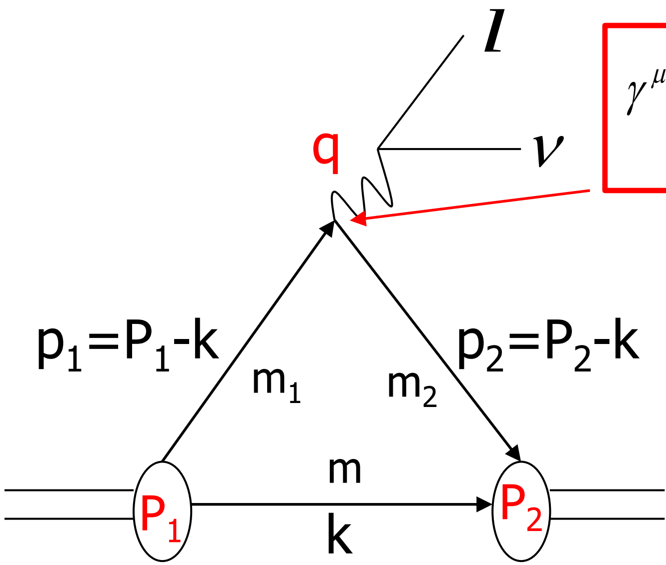
(I) to obtain the LF covariant form factors within our LFQM

(II) to extend the applicability of our LFQM by including bottom-charm sector

⇩

$P \to l\nu$, $P \to Pl\nu$, $B \to Kl^+l^-$, $V \to P\gamma$ decays [Refs: Choi and Ji: PLB 460,461(99), PLB513,330(01); Choi, Ji, and Kisslinger: PRD 65, 074032(02); Choi, PRD 75, 073016(07)]

2. P → P Semileptonic Decays: Covariant BS model



$$\gamma^\mu(1-\gamma_5) \Rightarrow \frac{1}{N_{\Lambda_1}}\gamma^\mu(1-\gamma_5)\frac{1}{N_{\Lambda_2}}$$

Bakker,Choi,Ji(03)

$N_j=p_j^2-m_j^2+i\varepsilon (j=1,2)$

$N=k^2-m^2+i\varepsilon$

$N_{\Lambda j}=p_j^2-\Lambda_j^2+i\varepsilon (j=1,2)$

(Λ_i=momentum cutoffs)

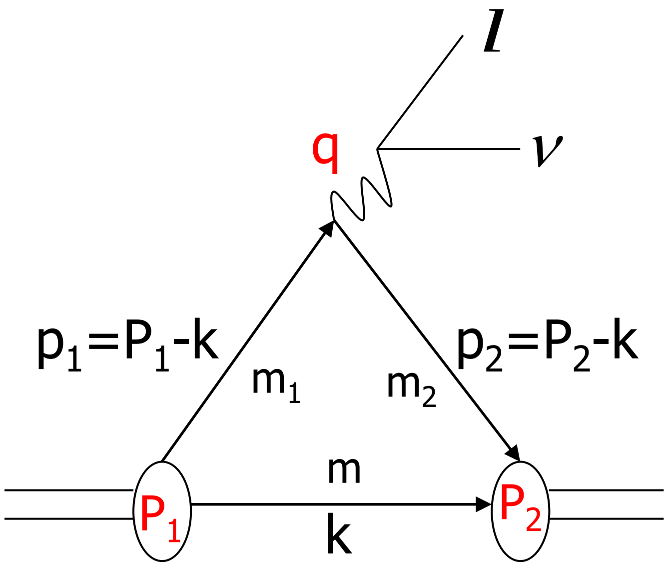
$$\langle J^\mu \rangle \equiv \langle P_2 \,|\, (V-A)^\mu \,|\, P_1 \rangle$$

$$= f_+(q^2)(P_1+P_2)^\mu + f_-(q^2)q^\mu$$

$$= iN\int \frac{d^4k}{(2\pi)^4}\frac{S^\mu}{N_{\Lambda_1}N_1 N N_2 N_{\Lambda_2}}$$

$$S^\mu=\mathrm{Tr}[\gamma_5(\not{p}_1+m_1)\gamma^\mu(\not{p}_2+m_2)\gamma_5(-\not{k}+m)]$$

(1) Manifestly covariant calculation



$$S^{\mu}=Tr[\gamma_5(\not{p}_1+m_1)\gamma^{\mu}(\not{p}_2+m_2)\gamma_5(-\not{k}+m)]$$

$$=a\,P^{\mu}+b\,q^{\mu}+c\,k^{\mu}\quad (P=P_1+P_2)$$

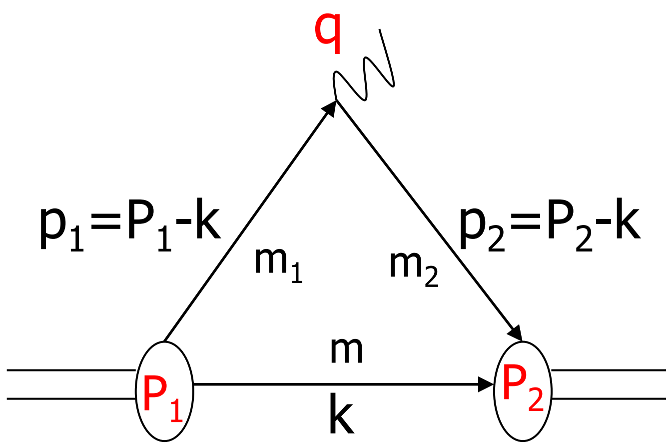
$$\frac{1}{N_{\Lambda_1}\cdots N_{\Lambda_2}}\rightarrow\frac{1}{N}\left(\frac{1}{N_{\Lambda_1}}-\frac{1}{N_1}\right)\left(\frac{1}{N_{\Lambda_2}}-\frac{1}{N_2}\right)$$

$$\frac{1}{N_1 N N_2}=\int_0^1 dx\int_0^{1-x} dy\frac{2}{[N+(N_1-N)x+(N_2-N)y]^3}$$

$$f_+^{Cov}(q^2)=\frac{N}{8\pi^2(\Lambda_1^2-m_1^2)(\Lambda_2^2-m_2^2)}\int_0^1 dx\int_0^{1-x} dy\left\{[3(x+y)-4]\ln\left(\frac{C_{\Lambda_1 m_2}C_{m_1\Lambda_2}}{C_{\Lambda_1\Lambda_2}C_{m_1 m_2}}\right)+\cdots\right\}$$

$$f_-^{Cov}(q^2)=\frac{N}{8\pi^2(\Lambda_1^2-m_1^2)(\Lambda_2^2-m_2^2)}\int_0^1 dx\int_0^{1-x} dy\left\{3(x+y)\ln\left(\frac{C_{\Lambda_1 m_2}C_{m_1\Lambda_2}}{C_{\Lambda_1\Lambda_2}C_{m_1 m_2}}\right)+\cdots\right\}$$

Covariant vs. Light-Front Calculation



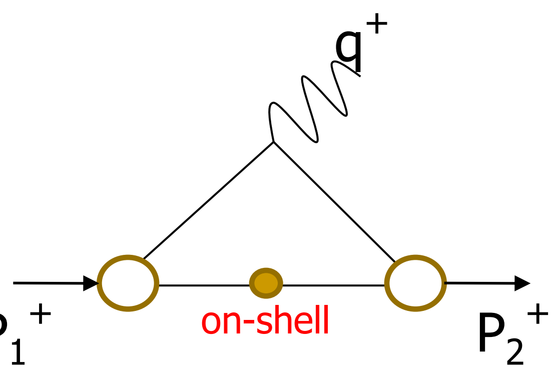
$$\langle J^{\mu}\rangle = f_{+}(q^2)(P_1+P_2)^{\mu} + f_{-}(q^2)q^{\mu}$$

In $q^+=0$ frame:

$$f_{+} = \frac{\langle J^{+}\rangle}{2P_1^{+}}$$

$$f_{-} = f_{+} + \frac{\langle J^{\perp}\rangle\cdot\vec{q}_{\perp}}{\vec{q}_{\perp}^{\,2}}$$

=



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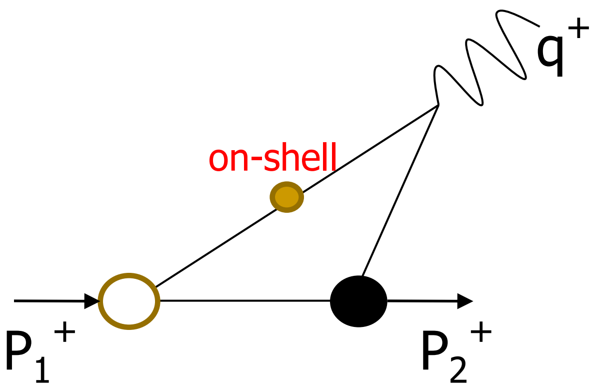


Valence contribution($0<k^+<P_2^+$):

$k^2=m^2$, i.e., $k^-=(k^2_\perp+m^2-i\varepsilon)/k^+$

Nonvalence contribution($P_2^+<k^+<P_1^+$) :

$p_1^2=m_1^2$, i.e., $p_{1on}^-=[m_1^2+k_\perp^2-i\varepsilon]/p_1^+$

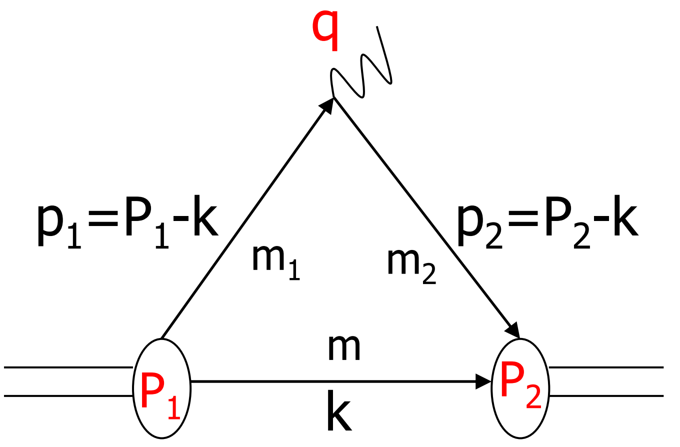
$p_1^2=\Lambda_1^2$, i.e., $p_{1on}^-=[\Lambda_1^2+k_\perp^2-i\varepsilon]/p_1^+$

(2) LF Calculation for the trace term S^μ

$$\not{p}+m=(\not{p}_{on}+m) + (1/2)\,\gamma^+(p^- - p^-_{on})$$

(propagating) (instantaneous)

$$S^\mu = S^\mu_{on} + S^\mu_{inst}$$

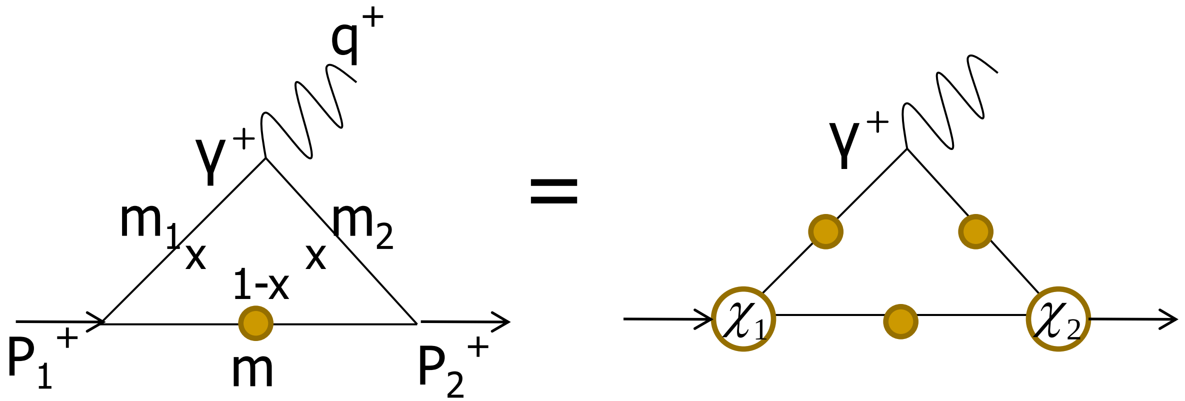


$$S^\mu_{on} = 4\left[p^\mu_{1on}(p_{2on}\cdot k_{on}) - k^\mu_{on}(p_{1on}\cdot p_{2on}) + p^\mu_{2on}(p_{1on}\cdot k_{on}) + m_2 m p^\mu_{1on} + m_1 m p^\mu_{2on} + m_1 m_2 k^\mu_{on}\right]$$

$$\begin{aligned} S^\mu_{inst} = {} & 2(p_1^- - p_{1on}^-)\left[p^\mu_{2on}k^+_{on} - p^+_{2on}k^\mu_{on} + g^{\mu+}(p_{2on}\cdot k_{on} + m_2 m)\right] \\ & + 2(p_2^- - p_{2on}^-)\left[p^\mu_{1on}k^+_{on} - p^+_{1on}k^\mu_{on} + g^{\mu+}(p_{1on}\cdot k_{on} + m_1 m)\right] \\ & + 2(k^- - k^-_{on})\left[p^\mu_{1on}p^+_{2on} + p^+_{1on}p^\mu_{2on} - g^{\mu+}(p_{1on}\cdot p_{2on} - m_1 m_2)\right] \\ & + 2g^{\mu+}k^+_{on}(p_1^- - p_{1on}^-)(p_2^- - p_{2on}^-) \end{aligned}$$

(3) LF Valence contribution($0<k^+<P_2^+$): $k^-=k^-_{on}$

(i) Plus current($\mu = +$)



$$S^+_{on} = \frac{\vec{k}_\perp \cdot \vec{k}'_\perp + A_1 A_2}{(1-x)}$$

$$k'_\perp = k_\perp + (1-x) q_\perp$$

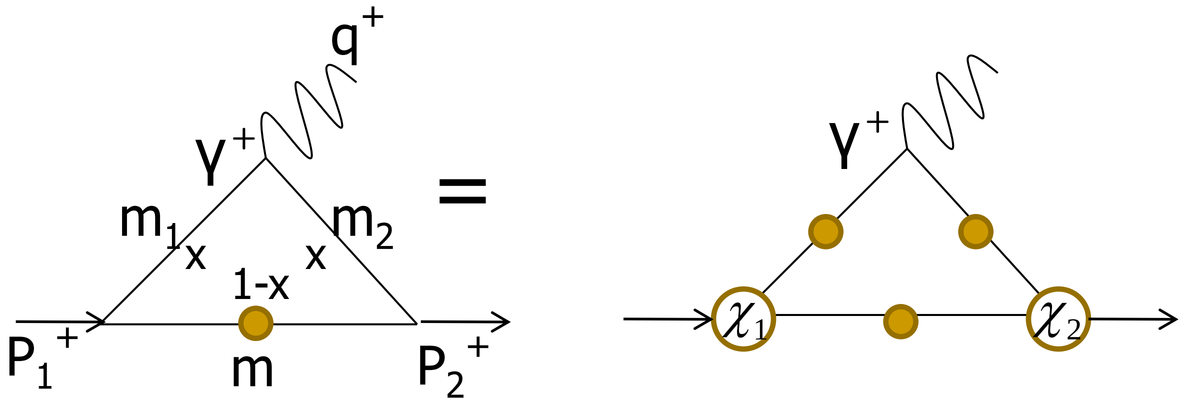
$$A_i = (1-x) m_i + x m$$

$$S^+_{on} = 4\left[p^+_{1on}(p_{2on}\cdot k_{on}) - k^+_{on}(p_{1on}\cdot p_{2on}) + p^+_{2on}(p_{1on}\cdot k_{on}) + m_2 m p^+_{1on} + m_1 m p^+_{2on} + m_1 m_2 k^+_{on}\right]$$

$$\begin{aligned} S^+_{inst} = {} & 2(p_1^- - p^-_{1on})\left[p^+_{2on}k^+_{on} - p^+_{2on}k^+_{on} + g^{++}(p_{2on}\cdot k_{on} + m_2 m)\right] \\ & + 2(p_2^- - p^-_{2on})\left[p^+_{1on}k^+_{on} - p^+_{1on}k^+_{on} + g^{++}(p_{1on}\cdot k_{on} + m_1 m)\right] \\ & + 2(k^- - k^-_{on})\left[p^+_{1on}p^+_{2on} + p^+_{1on}p^+_{2on} - g^{++}(p_{1on}\cdot p_{2on} - m_1 m_2)\right] \\ & + 2g^{++}k^+_{on}(p_1^- - p^-_{1on})(p_2^- - p^-_{2on}) \end{aligned}$$

(3) LF Valence contribution($0<k^+<P_2^+$): $k^-=k^-_{on}$ (continued)

(i) Plus current($\mu = +$)



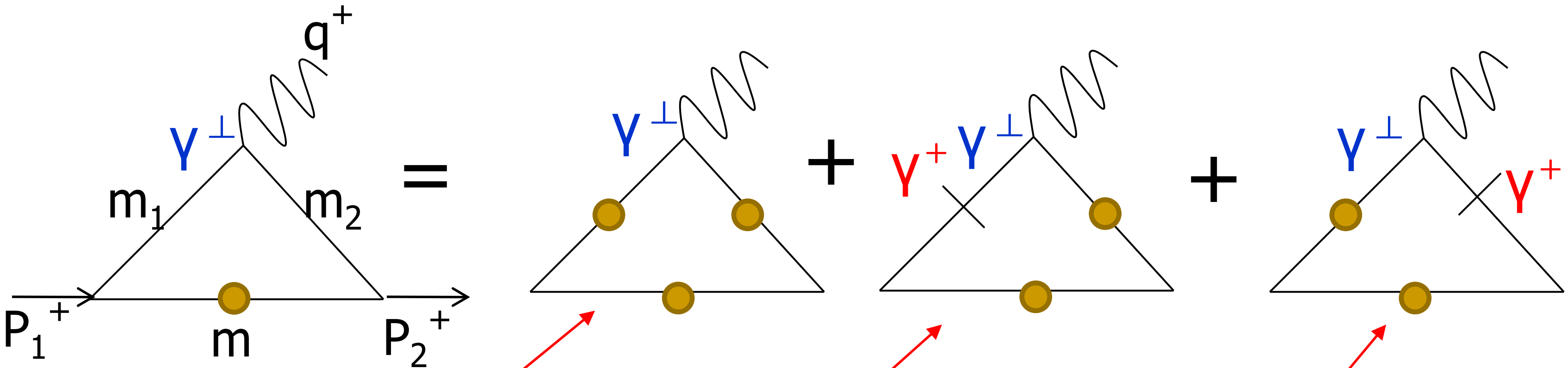
$$f_+^{val} = \frac{N}{8\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2k_\perp \chi_1(x,k_\perp)\chi_2(x,k'_\perp)S_{on}^+$$

$$\chi_1(x,k_\perp) = \frac{1}{x^2(M_1^2 - M_0^2)(M_1^2 - M_{0\Lambda_1}^2)} \qquad M_0^2 = \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m^2}{1-x},\quad M_{0\Lambda_1}^2 = M_0^2(m_1 \to \Lambda_1)$$

$$\chi_2(x,k'_\perp) = \frac{1}{x^2(M_2^2 - M'^2_0)(M_2^2 - M'^2_{0\Lambda_2})} \qquad M'^2_0 = \frac{k'^2_\perp + m_2^2}{x} + \frac{k'^2_\perp + m^2}{1-x},\quad M'^2_{0\Lambda_2} = M'^2_0(m_2 \to \Lambda_2)$$

(3) LF Valence contribution($0<k^+<P_2^+$): $k^-=k^-_{on}$

(ii) Perpendicular current($\mu=\perp$)

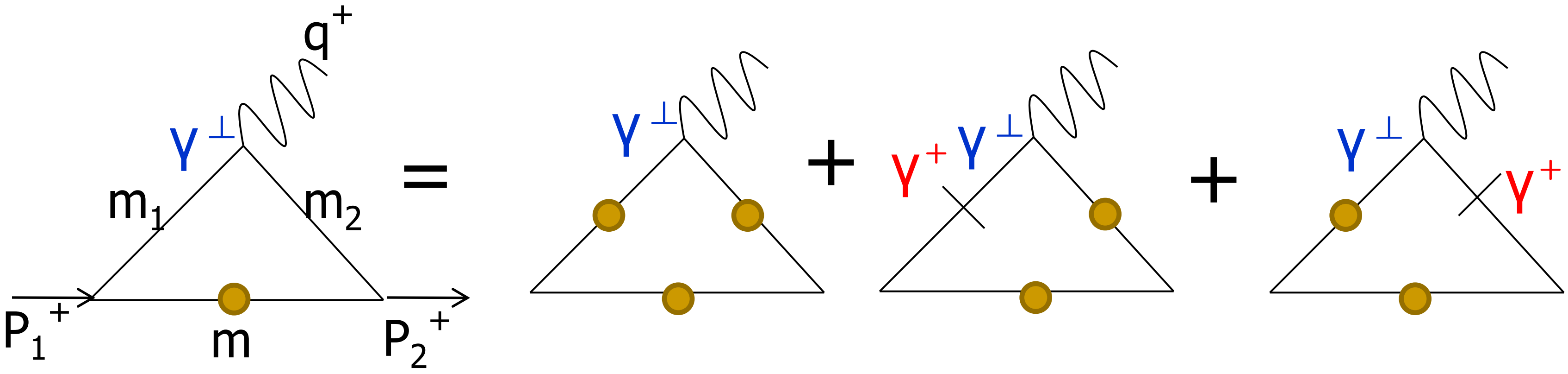


$$S^\mu_{on}=4\left[p^\mu_{1on}(p_{2on}\cdot k_{on})-k^\mu_{on}(p_{1on}\cdot p_{2on})+p^\mu_{2on}(p_{1on}\cdot k_{on})+m_2 m p^\mu_{1on}+m_1 m p^\mu_{2on}+m_1 m_2 k^\mu_{on}\right]$$

$$\begin{aligned}S^\mu_{inst}=&\,2(p_1^- - p_{1on}^-)\left[p^\mu_{2on}k^+_{on}-p^+_{2on}k^\mu_{on}+g^{\mu+}(p_{2on}\cdot k_{on}+m_2 m)\right]\\&+2(p_2^- - p_{2on}^-)\left[p^\mu_{1on}k^+_{on}-p^+_{1on}k^\mu_{on}+g^{\mu+}(p_{1on}\cdot k_{on}+m_1 m)\right]\\&+2(k^- - k^-_{on})\left[p^\mu_{1on}p^+_{2on}+p^+_{1on}p^\mu_{2on}-g^{\mu+}(p_{1on}\cdot p_{2on}-m_1 m_2)\right]\\&+2g^{\mu+}k^+_{on}(p_1^- - p_{1on}^-)(p_2^- - p_{2on}^-)\end{aligned}$$

(3) LF Valence contribution($0<k^+<P_2^+$): $k^- = k^-_{on}$

(ii) Perpendicular current($\mu = \perp$)



$$f_-^{val} = f_+^{val} + \frac{\langle J^\perp \rangle_{val} \cdot \vec{q}_\perp}{\vec{q}_\perp^{\,2}} = \frac{N}{8\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2 k_\perp \chi_1(x, k_\perp) \chi_2(x, k'_\perp)$$

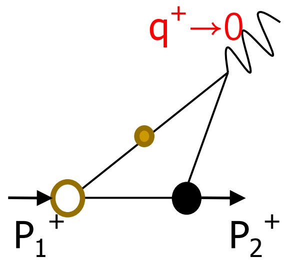
$$\times \left\{ \begin{array}{l} -(1-x)M_1^2 + (m_2 - m)A_1 - m(m_1 - m) \\ + \dfrac{\vec{k}_\perp \cdot \vec{q}_\perp}{q^2} [M_1^2 + M_2^2 - 2(m_1 - m)(m_2 - m)] \end{array} \right\}$$

(4) Zero-mode contribution($P_2^+<k^+<P_1^+$) ($p_1^-=p_{1on}^-$) to J^μ

$$\langle J^\mu \rangle_{Z.M} \propto \lim_{x\to 0} \int_0^{q^+\to 0} dx\, S^\mu(p_1^- = p_{1on}^-) \neq 0 \ \ iff \ \ S^\mu \sim p_1^-(=p_2^- = -k^-)$$

~1/x

$$= 0 \ \ if \ \ S^\mu \sim p_1^- p_i^+ (i=1,2)$$



$$S_{on}^\mu = 4\left[p_{1on}^\mu (p_{2on}\cdot k_{on}) - k_{on}^\mu (p_{1on}\cdot p_{2on}) + p_{2on}^\mu (p_{1on}\cdot k_{on}) + m_2 m p_{1on}^\mu + m_1 m p_{2on}^\mu + m_1 m_2 k_{on}^\mu\right]$$

$$S_{inst}^\mu = 2(p_1^- - p_{1on}^-)\left[p_{2on}^\mu k_{on}^+ - p_{2on}^+ k_{on}^\mu + g^{\mu +}(p_{2on}\cdot k_{on} + m_2 m)\right]$$
$$+ 2(p_2^- - p_{2on}^-)\left[p_{1on}^\mu k_{on}^+ - p_{1on}^+ k_{on}^\mu + g^{\mu +}(p_{1on}\cdot k_{on} + m_1 m)\right]$$
$$+ 2(k^- - k_{on}^-)\left[p_{1on}^\mu p_{2on}^+ + p_{1on}^+ p_{2on}^\mu - g^{\mu +}(p_{1on}\cdot p_{2on} - m_1 m_2)\right]$$
$$+ 2 g^{\mu +} k_{on}^+ (p_1^- - p_{1on}^-)(p_2^- - p_{2on}^-)$$

(1) μ=+ case

Every p_1^- is combined with $p_1^+ = p_2^+$ ⇒ No zero-mode for J^+ !

(4) Zero-mode contribution($P_2^+<k^+<P_1^+$) ($p_1^- = p_{1on}^-$) to J^μ

$$\langle J^\mu \rangle_{Z.M} \propto \lim_{x\to 0} \int_0^{q^+\to 0} dx\, S^\mu(p_1^- = p_{1on}^-) \neq 0 \text{ iff } S^\mu \sim p_1^-(= p_2^- = -k^-)$$
$$= 0 \text{ if } S^\mu \sim p_1^- p_i^+ (i=1,2)$$

$$S_{on}^\mu = 4\left[p_{1on}^\mu(p_{2on}\cdot k_{on}) - k_{on}^\mu(p_{1on}\cdot p_{2on}) + p_{2on}^\mu(p_{1on}\cdot k_{on}) + m_2 m p_{1on}^\mu + m_1 m p_{2on}^\mu + m_1 m_2 k_{on}^\mu\right]$$

$$S_{inst}^\mu = 2(p_1^- - p_{1on}^-)\left[p_{2on}^\mu k_{on}^+ - p_{2on}^+ k_{on}^\mu + g^{\mu+}(p_{2on}\cdot k_{on} + m_2 m)\right]$$
$$+2(p_2^- - p_{2on}^-)\left[p_{1on}^\mu k_{on}^+ - p_{1on}^+ k_{on}^\mu + g^{\mu+}(p_{1on}\cdot k_{on} + m_1 m)\right]$$
$$+2(k^- - k_{on}^-)\left[p_{1on}^\mu p_{2on}^+ + p_{1on}^+ p_{2on}^\mu - g^{\mu+}(p_{1on}\cdot p_{2on} - m_1 m_2)\right]$$
$$+2g^{\mu+}k_{on}^+(p_1^- - p_{1on}^-)(p_2^- - p_{2on}^-)$$

(2) $\mu=\perp$ case

$$S_{Z.M}^\perp(p_1^- = p_{1on}^-) = \lim_{x\to 0} S^\perp(p_1^- = p_{1on}^-) = 2p_1^-(p_{1\perp} + p_{2\perp})$$

$$\left[f_-(q^2)\right]_{Z.M} = \frac{\langle J^\perp\rangle_{Z.M}\cdot\vec{q}_\perp}{\vec{q}_\perp^{\,2}} = i\int_{Z.M}\frac{d^4k}{(2\pi)^4}\frac{2p_1^-\left(1-2\frac{\vec{k}_\perp\cdot\vec{q}_\perp}{q^2}\right)}{N_{\Lambda_1}N_1 N N_2 N_{\Lambda_2}} = \frac{1}{8\pi}\int_0^1 dz(1-2z)\ln\left(\frac{B_{\Lambda_1 m_2}B_{m_1\Lambda_2}}{B_{\Lambda_1\Lambda_2}B_{m_1 m_2}}\right)$$

$$B_{m_1 m_2} = (1-z)m_1^2 + z m_2^2 - z(1-z)q^2$$
$$\vdots$$

(5) Effective inclusion of zero-mode contribution in the valence region

(i) On-shell amp. <J$^+$> is independent of the orientation of LF plane $\omega \cdot x=0$

(ii) Off-shell amp. <J$^\perp$> depends on the orientation of LF plane $\omega \cdot x=0$ Carbonell, Desplanques, Karmanov, Mathiot(98)

→ <J$^\perp$> acquires a spurious ω dependence!

Zero-mode associated with p_1^- ↔ spurious ω dependence in covariant LF dynamics

Jaus(99)

$$S^{\perp}_{Z.M} = 2p_1^-(p_{1\perp} + p_{2\perp}) = 4p_1^- p_{1\perp} - 2p_1^- q_\perp$$

Decompose p_1 in terms of ($P = P_1 + P_2$, q, ω) $\omega = (\omega^+, \omega^-, \vec{\omega}_\perp) = (0,2,0_\perp)$ (Jaus 99)

$$p_1^\mu \equiv P^\mu A_1^{(1)} + q^\mu A_2^{(1)} + \frac{\omega^\mu}{\omega \cdot P} C_1^{(1)}$$

depends on p_1^-

$$C_1^{(1)} = Z_2 - N(= k^2 - m^2)$$

ω free terms

$$\text{where } Z_2 = x(M_1^2 - M_0^2) + m_1^2 - m^2 + (1-2x)M_1^2 - [q^2 + q\cdot P]\frac{k_\perp \cdot q_\perp}{q^2}$$

$$i\int_{Z.M} \frac{d^4k}{(2\pi)^4} \frac{N}{N_{\Lambda_1} N_1 N N_2 N_{\Lambda_2}} = i\int_{val} \frac{d^4k}{(2\pi)^4} \frac{Z_2}{N_{\Lambda_1} N_1 N N_2 N_{\Lambda_2}}$$

$$||$$

$$i\int_{Z.M} \frac{d^4k}{(2\pi)^4} \frac{-p_1^-}{N_{\Lambda_1} N_1 N N_2 N_{\Lambda_2}} = \frac{1}{16\pi^3}\int_0^1 \frac{dx}{(1-x)} \int d^2k_\perp \chi_1(x,k_\perp)\chi_2(x,k'_\perp)(Z_2)$$

Removing ω-dependence[$C_1^{(1)}=0$ or $N \to Z_2$] ↔ Effectively include Z.M. in the valence region!

(i.e. $p_1^- \to -Z_2$)

(5) Effective inclusion of zero-mode contribution in the valence region (continued)

$$S^{\perp}_{Z.M} = 2p_1^-(p_{1\perp} + p_{2\perp}) = 4p_1^- p_{1\perp} - 2p_1^- q_\perp$$

Tensor decomposition:

$$p_1^\mu p_1^\nu \equiv g^{\mu\nu} A_1^{(2)} + \cdots + \frac{1}{\omega \cdot P}(q^\mu \omega^\nu + \omega^\mu q^\nu) \boxed{C_1^{(2)}}$$ depends on $p_1^- p_{1\perp}$

$$\boxed{C_1^{(2)} = A_2^{(1)} C_1^{(1)} + \frac{q \cdot P}{q^2} A_1^{(2)}} \qquad A_1^{(2)} = -p_{1\perp}^2 - \frac{(p_{1\perp} \cdot q_\perp)^2}{q^2}, \quad A_2^{(1)} = \frac{x}{2} - \frac{p_{1\perp} \cdot q_\perp}{q^2}, \quad C_1^{(1)} = Z_2 - N$$

Setting $C_1^{(2)}$=0 leads to $A_2^{(1)} N \to A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)}$

i.e. $p_1^- p_{1\perp} \to -q_\perp \left[A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)} \right]$

Finally, we get

$$S^{\perp}_{Z.M} = 4p_1^- p_{1\perp} - 2p_1^- q_\perp \to \boxed{-4q_\perp \left[A_2^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_1^{(2)} \right] + 2q_\perp Z_2}$$

$$\langle J^\perp \rangle_{Z.M} = \frac{1}{16\pi^3} \int_0^1 \frac{dx}{(1-x)} \int d^2k_\perp \chi_1(x, k_\perp) \chi_2(x, k'_\perp)(\;\downarrow\;)$$

(Summary) Covariant vs. LF calcualtions in BS model

$$f_+^{LF} = \frac{N}{8\pi^3}\int_0^1 \frac{dx}{(1-x)^2}\int d^2k_\perp \chi_1(x,k_\perp)\chi_2(x,k'_\perp)(k_\perp\cdot k'_\perp + A_1A_2)$$

$$f_-^{LF} = \frac{N}{8\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2k_\perp \chi_1(x,k_\perp)\chi_2(x,k'_\perp)\Big\{-x(1-x)M_1^2 - k_\perp^2 - m_1 m$$

$$+2\frac{q\cdot P}{q^2}\left[k_\perp^2 + 2\frac{(k_\perp\cdot q_\perp)^2}{q^2}\right] + 2\frac{(k_\perp\cdot q_\perp)^2}{q^2}$$

$$+\frac{k_\perp\cdot q_\perp}{q^2}[M_2^2 - (1-x)(q^2+qP) + 2xM_0^2 - (1-2x)M_1^2 - 2(m_1-m)(m_1+m_2)]\Big\}$$

$$f_\pm^{Cov}(q^2) = f_\pm^{LF}(q^2)$$

$$f_+^{Cov}(q^2) = \frac{N}{8\pi^2(\Lambda_1^2-m_1^2)(\Lambda_2^2-m_2^2)}\int_0^1 dx\int_0^{1-x} dy\left\{[3(x+y)-4]\ln\left(\frac{C_{\Lambda_1 m_2}C_{m_1\Lambda_2}}{C_{\Lambda_1\Lambda_2}C_{m_1m_2}}\right)+\cdots\right\}$$

$$f_-^{Cov}(q^2) = \frac{N}{8\pi^2(\Lambda_1^2-m_1^2)(\Lambda_2^2-m_2^2)}\int_0^1 dx\int_0^{1-x} dy\left\{3(x+y)\ln\left(\frac{C_{\Lambda_1 m_2}C_{m_1\Lambda_2}}{C_{\Lambda_1\Lambda_2}C_{m_1m_2}}\right)+\cdots\right\}$$

The form factor $f_+(q^2)$ in LFQM $\langle J^+ \rangle = f_+(q^2)(P_1 + P_2)^+$

$$\langle J^+ \rangle^{LFQM}_{q^+=0} = \int_0^1 \frac{dx}{16\pi^3} \int d^2k_\perp \phi_1(x,k_\perp)\phi_2(x,k'_\perp) \sum_{\lambda_1,\lambda_2,\bar{\lambda}} R^{00\dagger}_{\lambda_2\bar{\lambda}} \frac{\bar{u}_{\lambda_2}(p_2)}{\sqrt{p_2^+}} \gamma^+ \frac{u_{\lambda_1}(p_1)}{\sqrt{p_1^+}} R^{00}_{\lambda_1\bar{\lambda}}$$

where

$$\Psi^{S,S_z}_{\lambda_Q\lambda_{\bar{Q}}}(x,k_\perp) = \phi(x,k_\perp) R^{S,S_z}_{\lambda_Q\lambda_{\bar{Q}}}(x,k_\perp)$$

$$\phi(x,k_\perp) \sim \exp(-k^2/2\beta^2) \qquad R^{00}_{\lambda_1\lambda_2} = -\frac{\bar{u}_{\lambda_1}(p_1)\gamma_5 \upsilon_{\lambda_2}(p_2)}{\sqrt{2}\sqrt{M_0^2 - (m_1 - m_2)^2}} \qquad \sum_{\lambda_1\lambda_2} R^{00\dagger}_{\lambda_1\lambda_2} R^{00}_{\lambda_1\lambda_2} = 1$$

$$f_+^{LFQM} = \int_0^1 \frac{dx}{16\pi^3} \int d^2k_\perp \frac{\phi_1(x,k_\perp)}{\sqrt{A_1^2 + k_\perp^2}} \frac{\phi_2(x,k'_\perp)}{\sqrt{A_2^2 + k'^2_\perp}} (k_\perp \cdot k'_\perp + A_1 A_2)$$

$$\sqrt{2N}\frac{\chi_1(x,k_\perp)}{1-x} = \frac{\phi_1(x,k_\perp)}{\sqrt{A_1^2 + k_\perp^2}} \qquad \Updownarrow \qquad \sqrt{2N}\frac{\chi_2(x,k'_\perp)}{1-x} = \frac{\phi_2(x,k'_\perp)}{\sqrt{A_1^2 + k'^2_\perp}}$$

$$f_+^{LFCov} = \frac{N}{8\pi^3} \int_0^1 \frac{dx}{(1-x)^2} \int d^2k_\perp \chi_1(x,k_\perp)\chi_2(x,k'_\perp)(k_\perp \cdot k'_\perp + A_1 A_2)$$ (in BS model)

3. Light-Front Quark Model

PRD59, 074015(99); PLB460, 461(99) by Choi and Ji

Key idea of our LFQM: Using the variational principle to the QCD-motivated effective Hamiltonian, we fix the model parameters!

$$H_{Q\bar{Q}} = \sqrt{m_Q^2 + k^2} + \sqrt{m_{\bar{Q}}^2 + k^2} + V_{Q\bar{Q}}$$

where

$$V_{Q\bar{Q}} = V_0(r) + V_{hyp}(r)$$

$$= a + br - \frac{4\kappa}{3r} + \frac{2 S_Q \cdot S_{\bar{Q}}}{3 m_Q m_{\bar{Q}}} \nabla^2 V_{Coul}$$

Linear → br; HO → (br^2)

and

$$\Psi^{S,S_z}_{\lambda_Q \lambda_{\bar{Q}}}(x, k_\perp) = \phi(x, k_\perp) \Re^{S,S_z}_{\lambda_Q \lambda_{\bar{Q}}}(x, k_\perp)$$

$$\phi(x, k_\perp) \sim \exp(-k^2 / 2\beta^2)$$

Relativistic spin-orbit w.f.

Variational Principle

$$\frac{\partial \langle \Psi | [H_0 + V_0] | \Psi \rangle}{\partial \beta} = 0$$

and

$$M_{Q\bar{Q}} = \langle \Psi | [H_0 + V_{Q\bar{Q}}] | \Psi \rangle$$

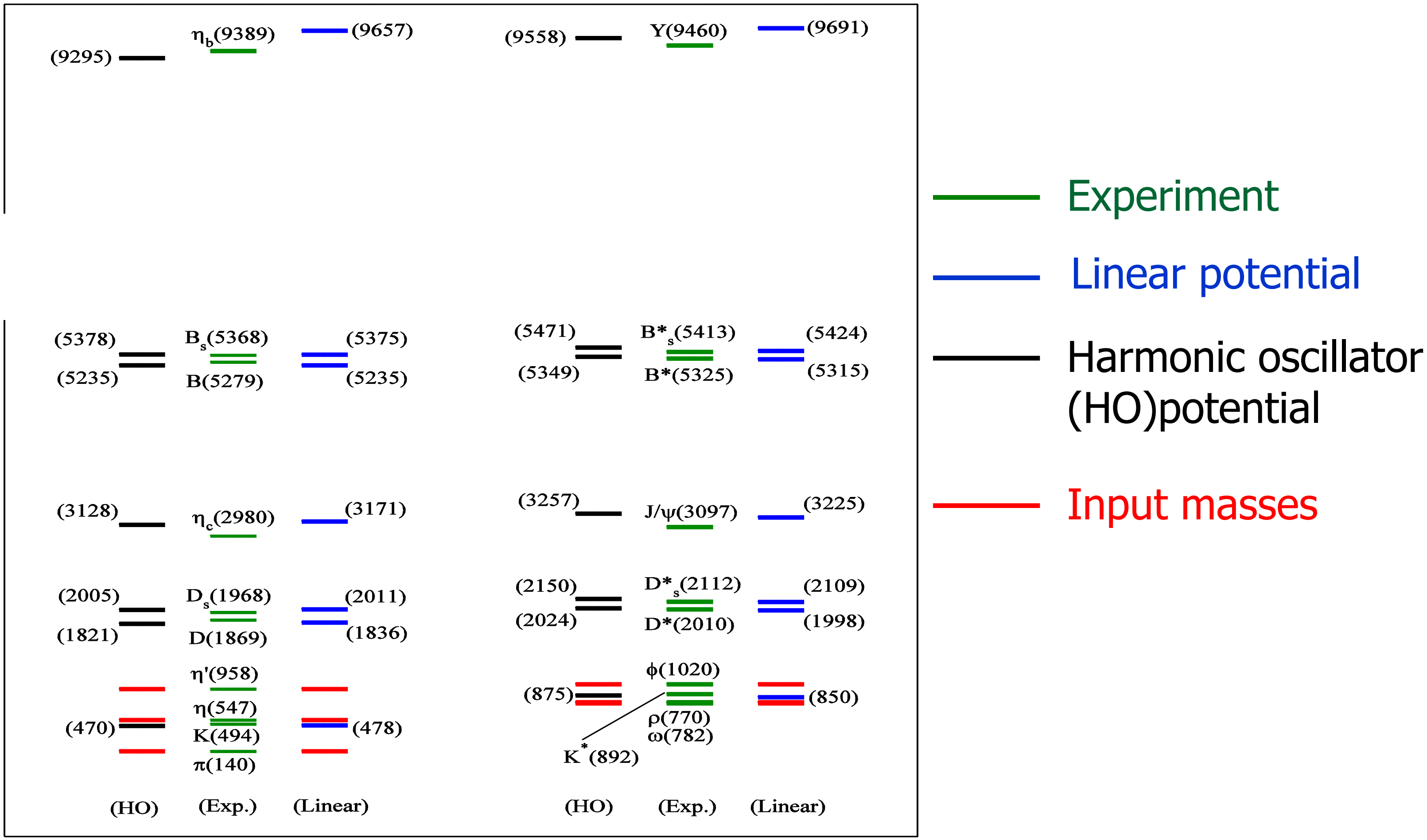
Input parameters for the linear confining potential

: $m_u = m_d = 220$ MeV

$b = 0.18$ GeV^2

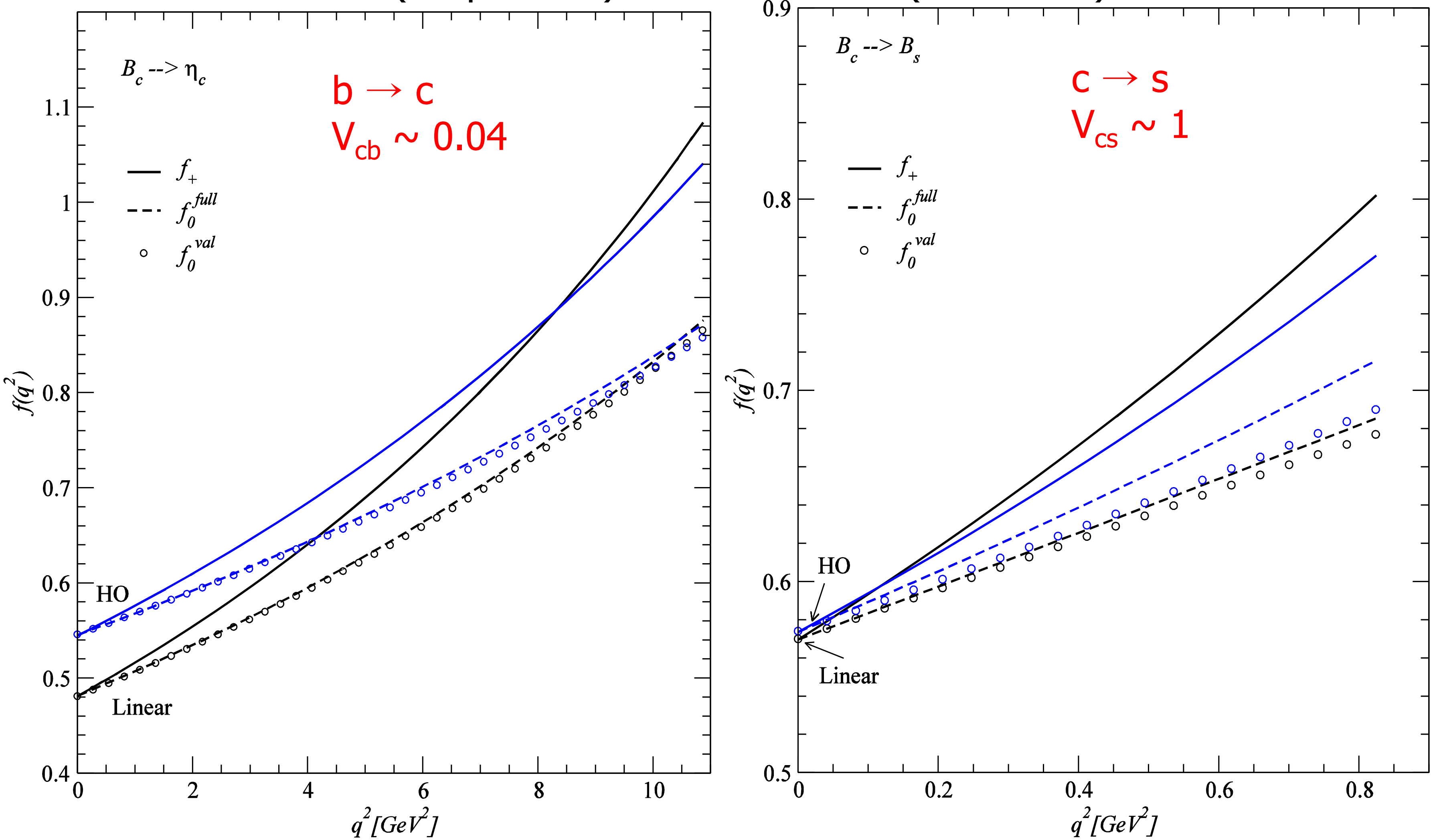
Optimized model parameters(in unit of GeV) and meson mass spectra

Model	m_q	m_s	m_c	m_b	β_{qc}	β_{sc}	β_{cc}	β_{qb}	β_{sb}	β_{cb}	β_{bb}
Linear	0.22	0.45	1.8	5.2	0.4679	0.5016	0.6509	0.5266	0.5712	0.8068	1.1452
HO	0.25	0.48	1.8	5.2	0.4216	0.4686	0.6998	0.4960	0.5740	1.0350	1.8025



analytic continuation

$F(Q^2)$ (in spacelike) $\xrightarrow{\;Q^2 = \vec{q}_\perp^{\,2} \to -q^2\;}$ $F(q^2)$ (in timelike)



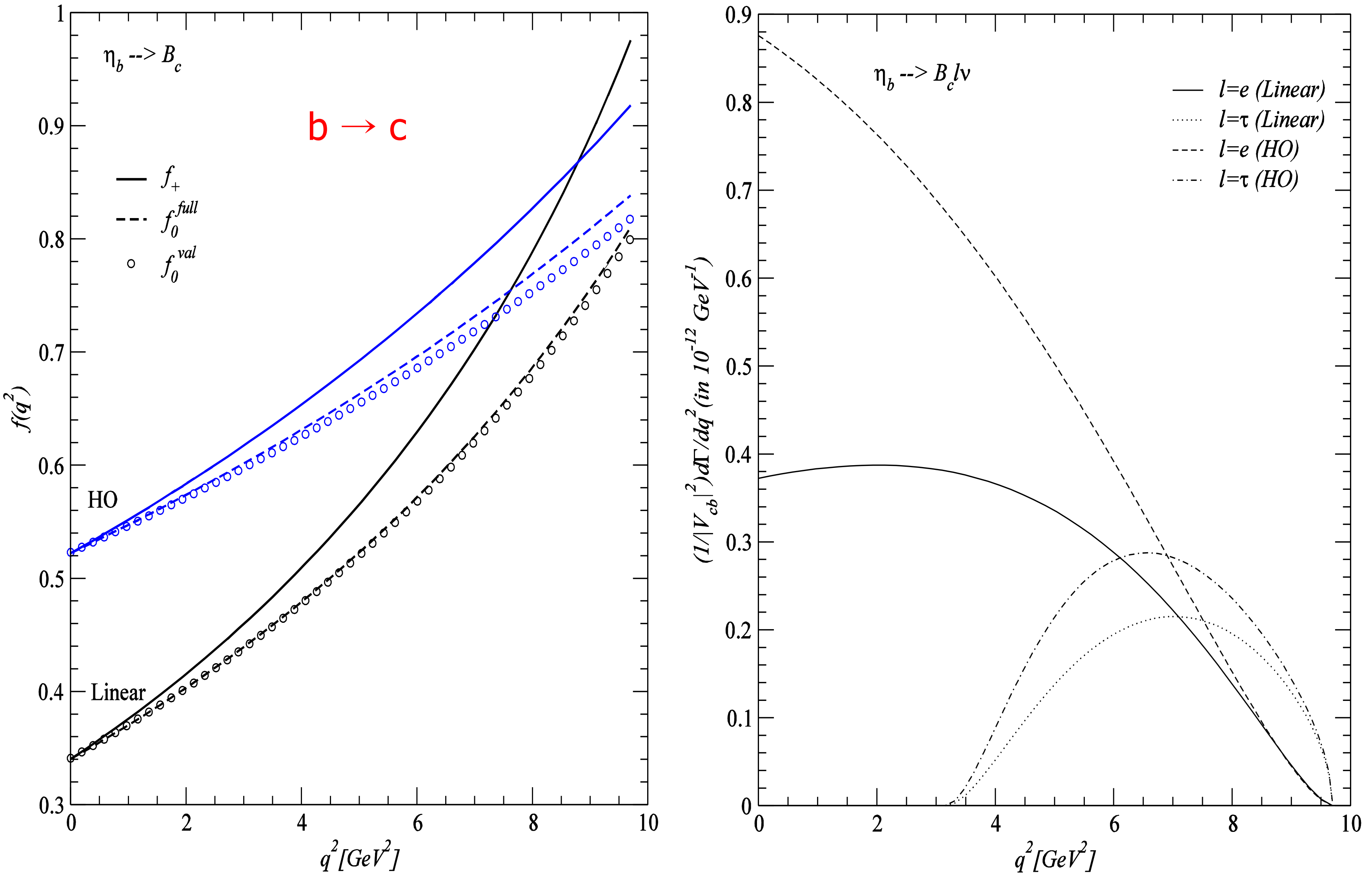
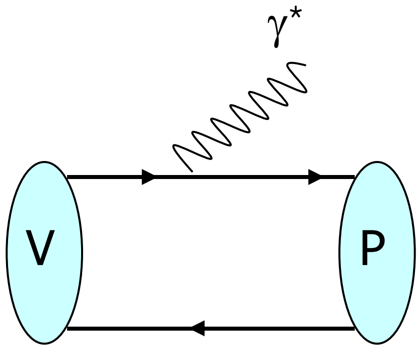

η_b --> B_c
b → c
f_+
f_0^{full}
f_0^{val}
HO
Linear
$f(q^2)$
$q^2 [GeV^2]$
η_b --> $B_c l\nu$
l=e (Linear)
l=τ (Linear)
l=e (HO)
l=τ (HO)
$(1/|V_{cb}|^2) d\Gamma/dq^2$ (in 10^{-12} GeV^{-1})
$q^2 [GeV^2]$

TABLE III: Form factors f_+ and f_0 evaluated at $q^2 = 0$ and $q^2_{\max}$ and decay widths Γ_ℓ(in 10^{-15} GeV) for $B_c \to (D, \eta_c, B, B_s)\ell\nu_\ell$ and $\eta_b \to B_c\ell\nu_\ell(\ell = e, \mu, \tau)$ transitions.

Mode		Linear[HO]	EFG [9, 10]	IKS [5]	NW [15]	HNV [16]	AKN [45]	CD [14]	WSL [17]
$B_c \to D$	$f_{+(0)}(0)$	0.086[0.079]	0.14	0.69	0.1446	-	0.089	-	0.16
	$f_+(q^2_{\max})$	1.129[0.789]	1.20	2.20	1.017	-	-	0.59	1.10
	$f_0(q^2_{\max})$	0.673[0.554]	0.64	-	-	-	-	-	0.59
	$\Gamma_{e(\mu)}$	0.021[0.014]	0.019	0.26	0.020	-	-	0.005(0.03)	0.043
	Γ_τ	0.019[0.012]	-	-	-	-	-	-	-
$B_c \to \eta_c$	$f_{+(0)}(0)$	0.482[0.546]	0.47	0.76	0.5359	0.49	0.622	-	0.61
	$f_+(q^2_{\max})$	1.084[1.035]	1.07	1.07	1.034	1.00	-	0.94	1.10
	$f_0(q^2_{\max})$	0.876[0.872]	0.92	-	-	0.91	-	-	0.86
	$\Gamma_{e(\mu)}$	6.93[7.95]	5.9	14.0	6.8	6.95	8.6	2.1(6.9)	9.81
	Γ_τ	2.31[2.46]	-	3.52	-	2.46	3.3 ± 0.9	-	-
$B_c \to B$	$f_{+(0)}(0)$	0.464[0.428]	0.39	0.58	0.4504	0.39	0.362	-	0.63
	$f_+(q^2_{\max})$	0.729[0.647]	0.96	0.96	0.6816	0.70	-	0.66	0.97
	$f_0(q^2_{\max})$	0.572[0.570]	0.80	-	-	0.71	-	-	0.81
	Γ_e	0.84[0.69]	0.6	2.1	0.638	0.65	-	0.9(1.0)	1.63
	Γ_μ	0.80[0.67]	-	-	-	0.63	-	-	-
$B_c \to B_s$	$f_{+(0)}(0)$	0.570[0.574]	0.50	0.61	0.5917	0.58	0.564	-	0.73
	$f_+(q^2_{\max})$	0.802[0.771]	0.99	0.92	0.8075	0.86	-	0.66	1.03
	$f_0(q^2_{\max})$	0.685[0.716]	0.86	-	-	0.86	-	-	0.87
	Γ_e	15.45[15.20]	12	29	12.35	15.1	15	11.1(12.9)	23.45
	Γ_μ	14.61[14.40]	-	-	-	14.5	-	-	-
$\eta_b \to B_c$	$f_{+(0)}(0)$	0.341[0.523]	-	-	-	-	-	-	-
	$f_+(q^2_{\max})$	0.976[0.918]	-	-	-	-	-	-	-
	$f_0(q^2_{\max})$	0.811[0.839]	-	-	-	-	-	-	-
	$\Gamma_{e(\mu)}$	4.64[7.94]	-	-	-	-	-	-	-
	Γ_τ	1.57[2.11]	-	-	-	-	-	-	-

4. Magnetic dipole(M1) transition $V(1^3S_1) \longrightarrow P(1^1S_0)\,\gamma^*$



Transition form factor:

$$\langle P(P')\,|\,J^{\mu}_{em}\,|V(P,h)\rangle = ie\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\nu}q_{\rho}P_{\sigma}F(q^2)$$

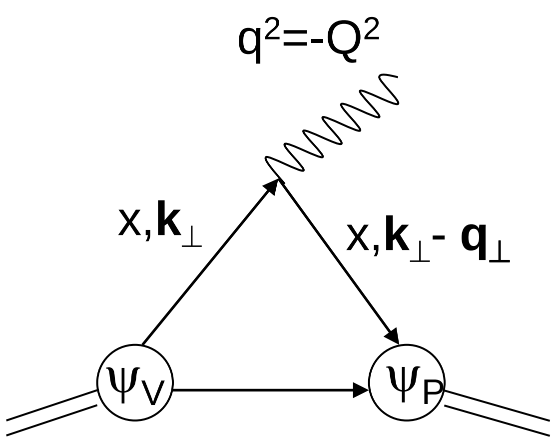
$$0 \le q^2 \le (M_V - M_P)^2$$

Decay width for real photon(γ):

$$\Gamma(V \longrightarrow P\gamma) = (\alpha/3)F(0)^2\,(M^2_V - M^2_P)^3/(2M_V)^3$$

LF in Drell-Yan-West

$q^+(=q^0+q^3)=0$ frame



Hadronic matrix element with '+'-current:

$\langle P'|J^+|P,h=+\rangle$
$=\Sigma_j\, ee_j\int[dx][d^2k_\perp]\; \psi^*_P(x,\mathbf{k}_\perp+(1-x)\mathbf{q}_\perp)\; \psi_V(x,k_\perp)$

analytic continuation

$F(Q^2)$ (in spacelike) $\longrightarrow$ $F(q^2)$ (in timelike)

Decay widths and Br for $V \longrightarrow P\gamma$.

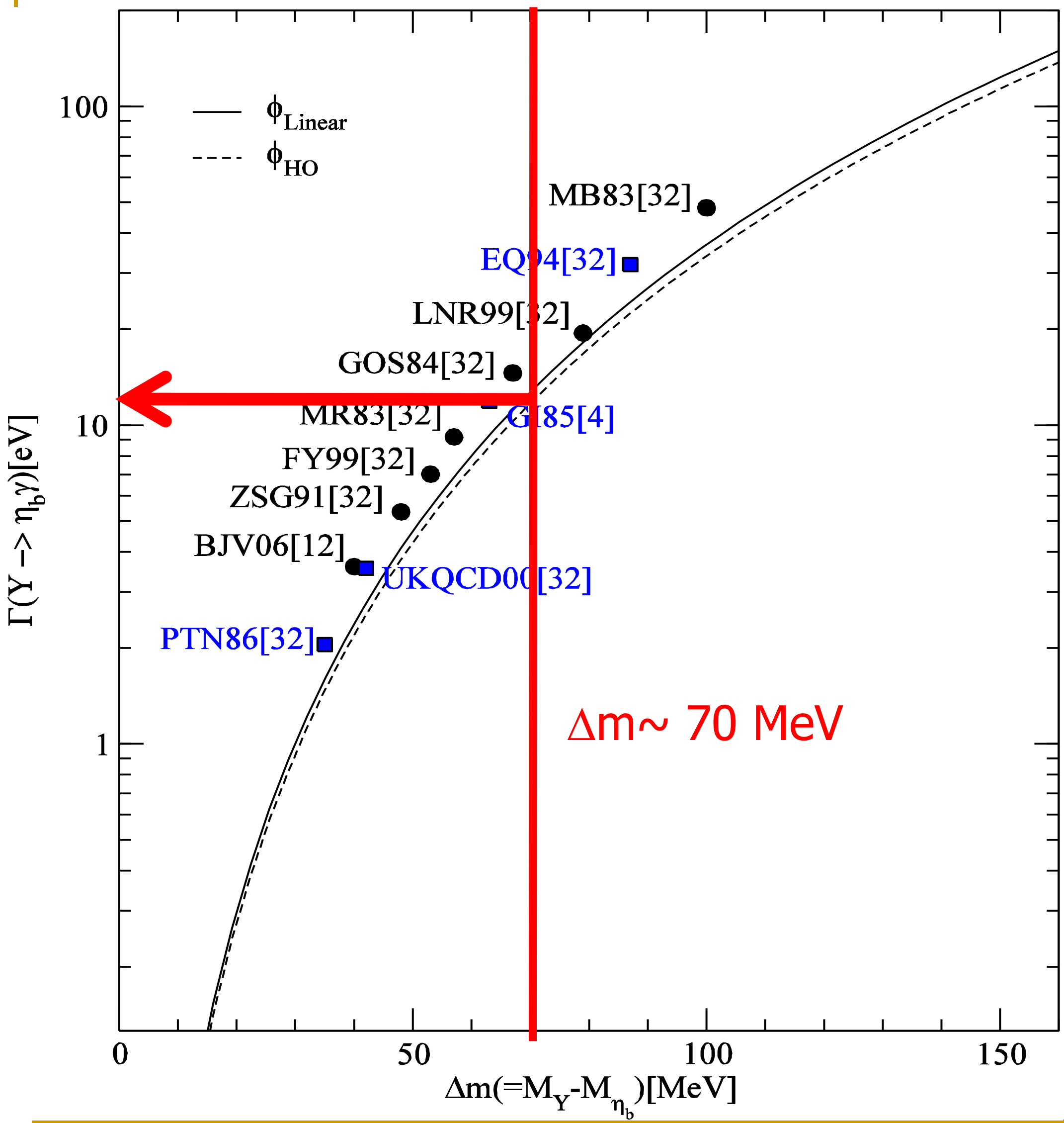
Choi[PRD(07)]

CLEO-c(09)

Decay Mode	$\Gamma_{av.}$[keV]	$Br_{av.}$	Br_{exp}
$J/\psi \rightarrow \eta_c \gamma$	1.67 ± 0.07	$(1.78 \pm 0.12)\%$	$(1.98 \pm 0.39)\%$
$D^{*+} \rightarrow D^+\gamma$	0.93 ± 0.05	$(0.97 \pm 0.37)\%$	$(1.6 \pm 0.4)\%$
$D^{*0} \rightarrow D^0\gamma$	20.5 ± 0.8		$(38.1 \pm 2.9)\%$
$D^{*+}_s \rightarrow D^+_s\gamma$	0.18 ± 0.01		$(94.2 \pm 0.7)\%$
$B^{*+} \rightarrow B^+\gamma$	0.40 ± 0.03		
$B^{*0} \rightarrow B^0\gamma$	0.13 ± 0.01		
$B^{*0}_s \rightarrow B^0_s\gamma$	0.066 ± 0.019		

Dependence of $\Gamma(\Upsilon \longrightarrow \eta_b\gamma)$ on $\Delta m(=M_\Upsilon - M_{\eta b})$

Choi[PRD(07)]

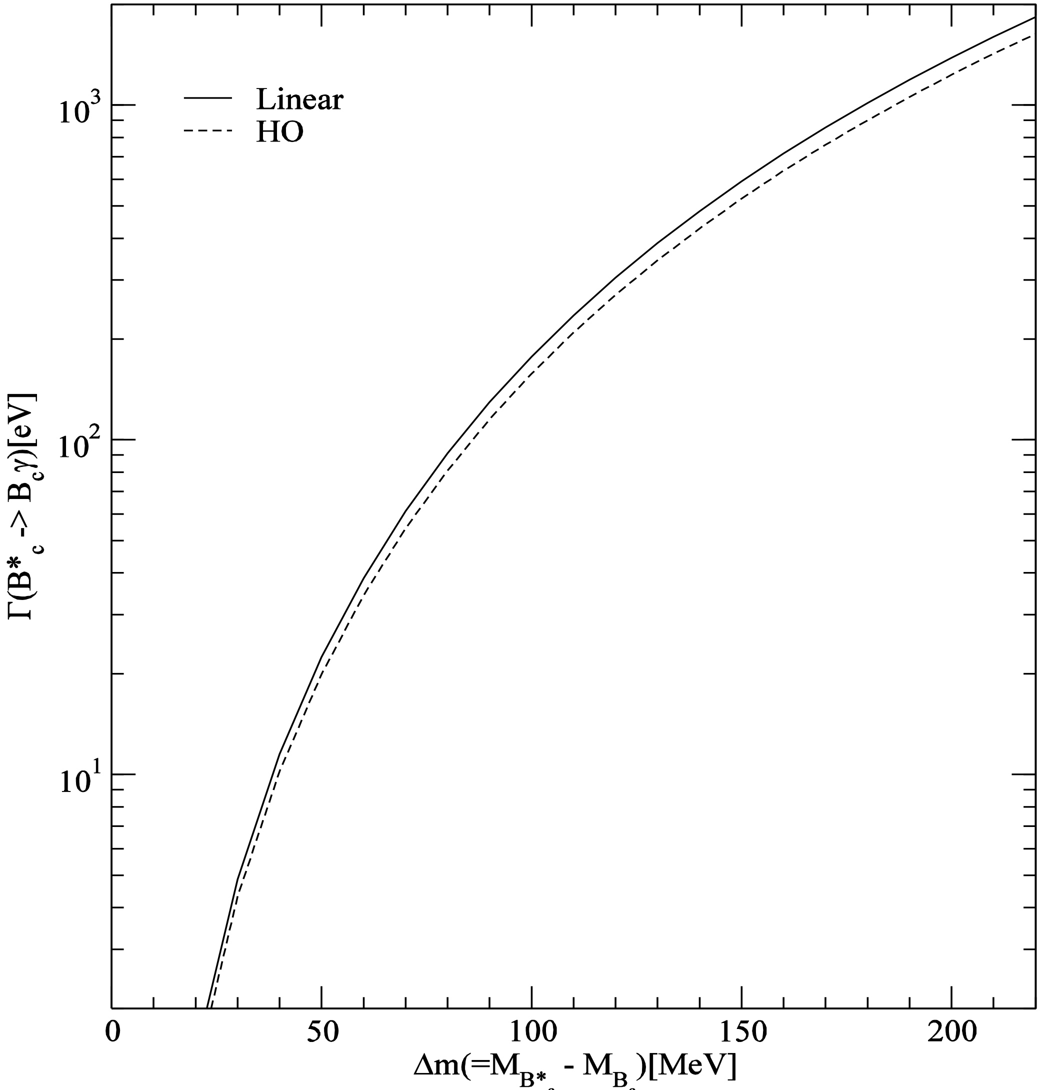


$\Gamma(V \longrightarrow P\gamma)$

$= (\alpha/3)F(0)^2 \ (M^2{}_V - M^2{}_P)^3/(2M_V)^3$

$\sim (\Delta m)^3$

Dependence of $\Gamma(B^*_c \to B_c\gamma)$ on $\Delta m(=M_{B^*c}-M_{B_c})$



$\Gamma(V \longrightarrow P\gamma)$

$= (\alpha/3)F(0)^2\ (M^2_V-M^2_P)^3/(2M_V)^3$

$\sim (\Delta m)^3$

5. Conclusions

(1) In semileptonic P → P decays in manifestly covariant BS model, we found

- Zero-mode contribution to $f_-(q^2)$
- LF covariant form for $f_-(q^2)$ including the zero-mode in the valence region

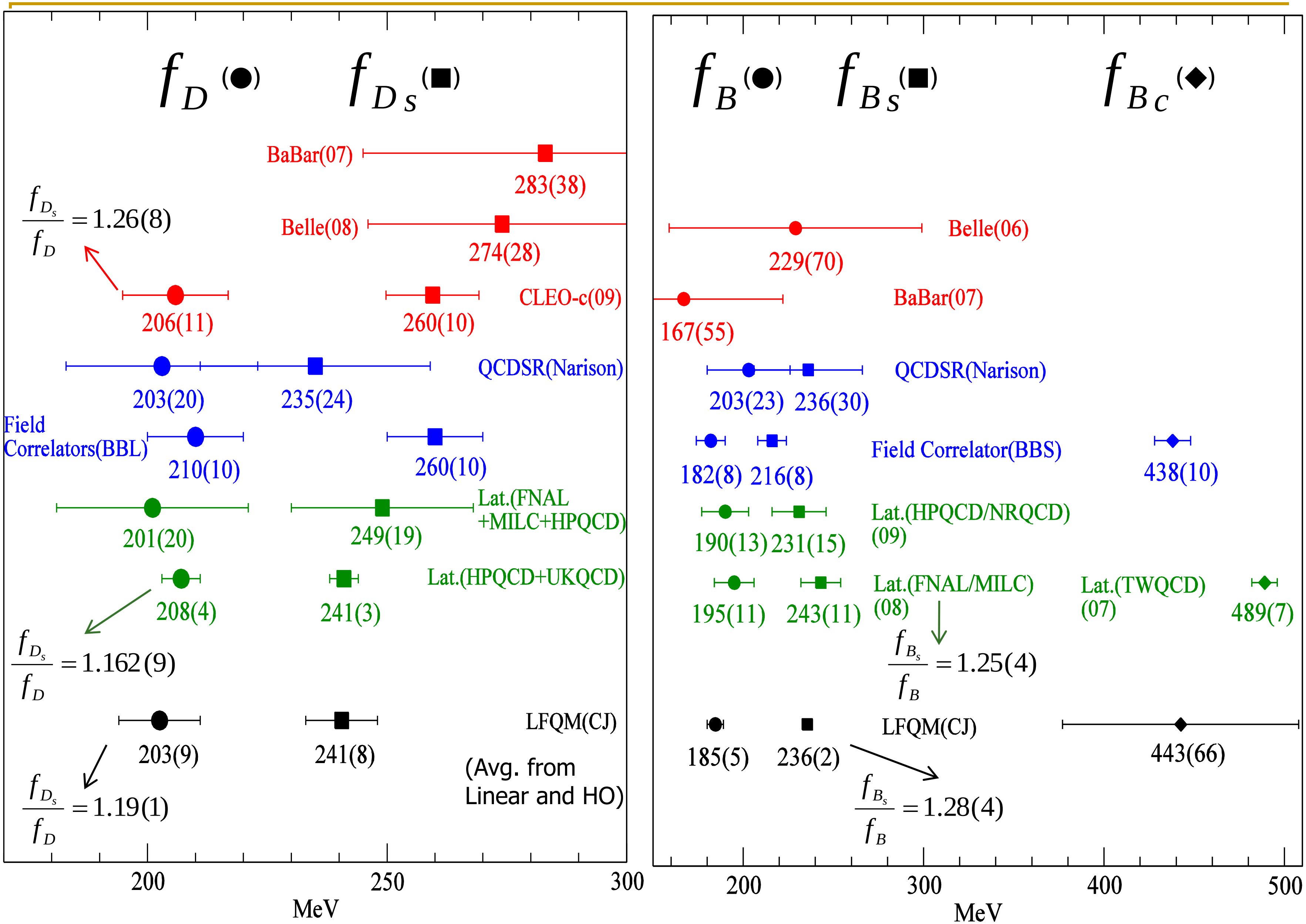
⇓

- Guide a way to construct LF covariant form factors in LFQM

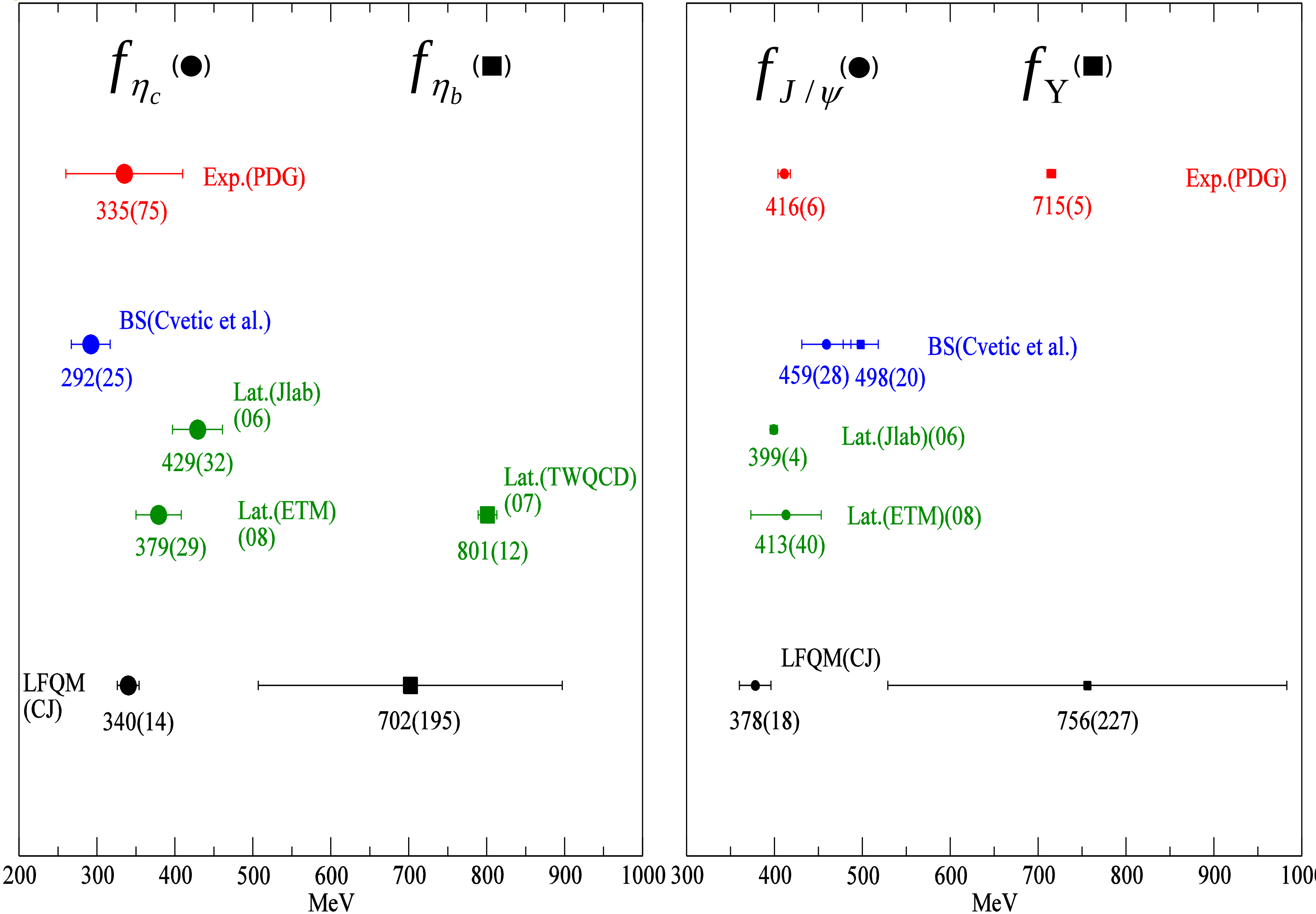
(2) Magnetic dipole(M1) V → Pγ transition form factor is immune to the zero-model!

- Future works for other exclusive processes:

(1) Semileptonic P → V transitions
(2) Rare decays such as B → K*
(3) Nonleptonic decays



f_D (●) f_{D_s} (■)
BaBar(07)
283(38)
$\frac{f_{D_s}}{f_D} = 1.26(8)$
Belle(08)
274(28)
206(11)
260(10)
CLEO-c(09)
QCDSR(Narison)
203(20)
235(24)
Field Correlators(BBL)
210(10)
260(10)
Lat.(FNAL +MILC+HPQCD)
201(20)
249(19)
Lat.(HPQCD+UKQCD)
208(4)
241(3)
$\frac{f_{D_s}}{f_D} = 1.162(9)$
LFQM(CJ)
203(9)
241(8)
(Avg. from Linear and HO)
$\frac{f_{D_s}}{f_D} = 1.19(1)$
200
250
300
MeV
f_B (●) f_{B_s} (■) f_{B_c} (◆)
Belle(06)
229(70)
BaBar(07)
167(55)
QCDSR(Narison)
203(23)
236(30)
Field Correlator(BBS)
182(8)
216(8)
438(10)
Lat.(HPQCD/NRQCD) (09)
190(13)
231(15)
Lat.(FNAL/MILC) (08)
Lat.(TWQCD) (07)
195(11)
243(11)
489(7)
$\frac{f_{B_s}}{f_B} = 1.25(4)$
LFQM(CJ)
185(5)
236(2)
443(66)
$\frac{f_{B_s}}{f_B} = 1.28(4)$
200
300
400
500
MeV



f_{η_c} (●)
f_{η_b} (■)
Exp.(PDG)
335(75)
BS(Cvetic et al.)
292(25)
Lat.(Jlab)
(06)
429(32)
Lat.(TWQCD)
(07)
Lat.(ETM)
(08)
379(29)
801(12)
LFQM
(CJ)
340(14)
702(195)
$f_{J/\psi}$ (●)
f_{Υ} (■)
416(6)
715(5)
Exp.(PDG)
BS(Cvetic et al.)
459(28)
498(20)
399(4)
Lat.(Jlab)(06)
Lat.(ETM)(08)
413(40)
LFQM(CJ)
378(18)
756(227)
200
300
400
500
600
700
800
900
1000
MeV
300
400
500
600
700
800
900
1000
MeV