Dirac-Brueckner mean fields and an effective* density-dependent Dirac-Hartree-Fock interaction in nuclear matter

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*Small parameters not included.

Dirac mean fields

The Dirac equation for a nucleon of momentum k in nuclear matter is

$$(\gamma^\mu k_\mu - M - \Sigma(k))\, u(k,s) = 0$$

The Dirac mean field can be decomposed as

$$\Sigma_t(k) = \Sigma_t^s(k) I - \Sigma_t^0(k)\gamma^0 + \Sigma_t^v(k)\vec{\gamma}\cdot\vec{k} \qquad t = n,\, p$$

In terms of these, we can define the effective mass and momentum,

$$M^*(k) = M + \Sigma^s(k), \quad \vec{k}^* = \vec{k}\,(1 + \Sigma^v(k))$$

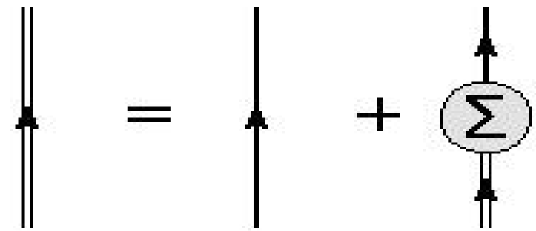
and the single-particle energy of the state, *u(k,s)*,

$$E^*(k) = E + \Sigma^0(k) = \sqrt{k^{*2} + M^{*2}}$$

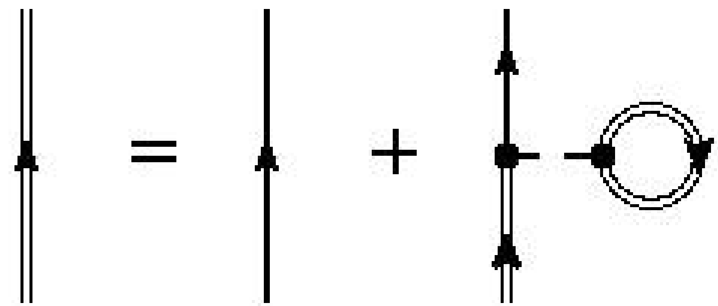
Approximations to the mean field

Dyson equation for the single-particle propagator

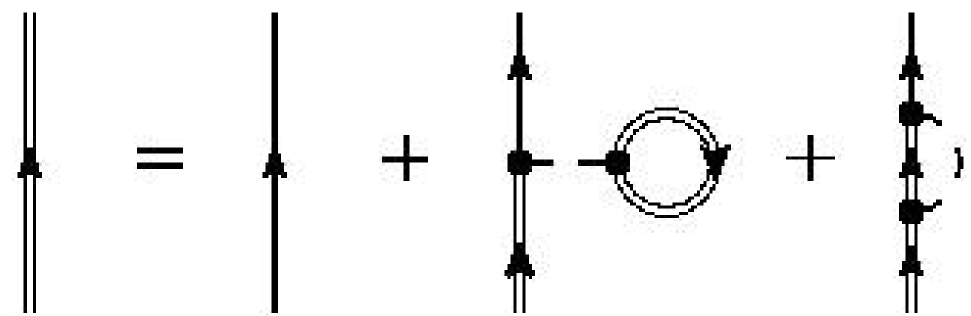
$$G_t(k) = G_0(k) + G_0(k)\Sigma_t(k)G_t(k)$$



Hartree – includes direct meson term with all nucleons



Hartree-Fock – includes direct meson and nucleon exchange terms



Meson exchange - Hartree vs. Hartree-Fock

Low mass mesons included in meson-exchange NN potentials, such as the Bonn potentials

Only the first four enter the Hartree mean field – no pions.

meson	J^π, I	mass (MeV)
σ	0^+, 0	550.0
δ	0^+, 1	983.0
ω	1^-, 0	782.6
ρ	1^-, 1	769.0
η	0^-, 0	548.4
π	0^-, 1	138.03

Without the long-range of pion exchange, the Hartree approximation furnishes a very sharp nuclear surface and a very large surface energy.

Dirac-Hartree effective interactions improve their description of the nuclear surface using either:

1 - Non-linear meson interactions, such as a $\lambda\sigma^4$ term;
2 - Density-dependent coupling coefficients that increase at low density to increase the mean field in the surface.

Hartree-Fock and Brueckner

The Hartree-Fock approximation can describe the nuclear surface in a more fundamental manner than the Hartree approximation,

It still does not provide a good description of the interaction of two nucleons -- it does not take into account properly the hard core of the nucleon-nucleon interaction.

The HF furnishes an overly rigid model of the nucleus -- a compressibility on the order of 500 MeV, (as in the Hartree approximation) -- compared to experimental estimates of 200 – 300 MeV.

Density-dependent Hartree models try to correct for this by adjusting to Brueckner mean fields, which take the hard core into account.

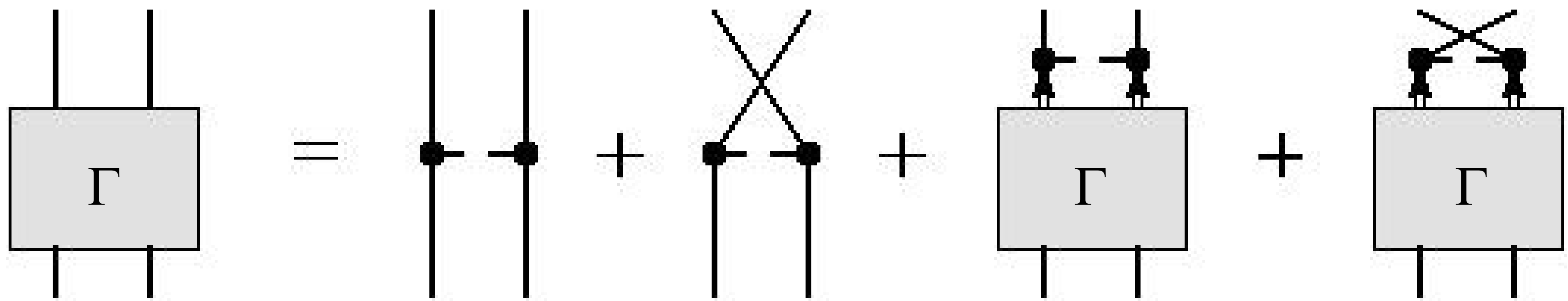
Our objective: to adjust Brueckner mean fields with a density-dependent Hartree-Fock interaction - one that includes the pion explicitly.

The Bethe-Salpeter equation

The Brueckner G-matrix Γ satisfies a Bethe-Salpeter equation.

We calculate it in the ladder approximation, including the anti-symmetrized bare nucleon-nucleon interaction to all orders in the nn,np,pn and pp channels.

In free space, the G-matrix reduces to the two-nucleon scattering T-matrix.



$$\Gamma\left(q',q|P\right) = V\left(q',q\right)+\frac{i}{2}\int\frac{d^4q''}{(2\pi)^4}V\left(q',q''\right)$$

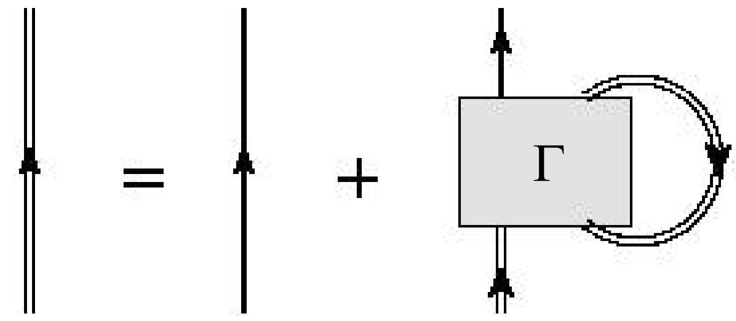
nn or pp

$$\times G_1\left(\frac{P}{2}+q''\right)G_2\left(\frac{P}{2}-q''\right)\Gamma\left(q'',q|P\right)$$

The Brueckner mean field

$$G_t(k) = G_0(k) + G_0(k)\Sigma_t(k)G_t(k)$$

Dyson equation for the single-particle propagator



$$\Sigma_t(k) = -i\int \frac{d^4p}{(2\pi)^4} \sum_{t'=n,p} Tr\left[\Gamma_{tt'}(k,p;k,p)G_{t'}(p)\right]$$

Quasi-particle approximation $\Gamma \rightarrow$ Re[Γ] , $\Sigma \rightarrow$ Re[Σ] :

$$\Sigma^s_t(k) = \sum_{t'=n,p} 2\int \frac{d^3p}{(2\pi)^3}\Gamma^S_{tt'}\left(k-p,k-p|k+p\right)\frac{M^*_{t'}(p)}{E^*_{t'}(p)}\theta\left(k_{Ft'}-|\vec{p}|\right)$$

$$\Sigma^0_t(k) = -\sum_{t'=n,p} 2\int \frac{d^3p}{(2\pi)^3}\Gamma^V_{tt'}\left(k-p,k-p|k+p\right)\theta\left(k_{Ft'}-|\vec{p}|\right) \qquad t=n,p$$

$$\Sigma^v_t(k) = -\sum_{t'=n,p} \frac{2}{k^2}\int \frac{d^3p}{(2\pi)^3}\Gamma^V_{tt'}\left(k-p,k-p|k+p\right)\frac{\vec{k}\cdot\vec{p}^*}{E^*_{t'}(p)}\theta\left(k_{Ft'}-|\vec{p}|\right)$$

The reduced Bethe-Salpeter equation

We simplify the Bethe-Salpeter equation further by
1) projecting onto a set of positive-energy single-particle states
2) and reducing the integral to a three-dimensional one.

$$\begin{aligned}\gamma\left(q', q|P\right) &= v\left(q', q\right) + \frac{1}{2}\int \frac{d^3q''}{(2\pi)^3} v\left(q', q''\right) \\ &\quad \times Q\left(q''|P\right) g\left(q''|P\right) \gamma\left(q'', q|P\right)\end{aligned}$$

nn or pp

where γ and v are the reduced G-matrix and antisymmetrized bare interaction, Q is a Pauli blocking factor and g is a reduced two-particle propagator.

This equation is solved by reducing it to coupled equations for the principal parts of 5 (or 6) independent helicity amplitudes, expanding these in partial waves and solving them as matrix equations.

(K. Erkelenz, Phys. Reports, C13 (1974) 191.)

Reconstructing the G-matrix

To calculate the mean field, we need the complete G-matrix, not just its projection. To reconstruct it, we solve for the component functions of a covariant expansion, projected on positive-energy states with distinct masses,

$$\begin{aligned}\Gamma\left(q',q|P\right) &= \Gamma^S\left(q',q|P\right) I^{(1)} I^{(2)} + \Gamma^V\left(q',q|P\right) \gamma_\mu^{(1)} \gamma^{(2)\mu} \\ &\quad + \Gamma^T\left(q',q|P\right) \sigma_{\mu\nu}^{(1)} \sigma^{(2)\mu\nu} + \Gamma^P\left(q',q|P\right) \gamma_5^{(1)} \gamma_5^{(2)} \\ &\quad + \Gamma^A\left(q',q|P\right) \gamma_5^{(1)} \gamma_\mu^{(1)} \gamma_5^{(2)} \gamma^{(2)\mu}\end{aligned}$$

(C.J. Horowitz and B. Serot, Nucl. Phys. A464 (1987) 613.)

Due to the ambiguity between the pseudoscalar and pseudovector pion- and eta-nucleon coupling, we reconstruct the difference between the G-matrix and the bare interaction *V* and use the unprojected expression for the latter.

$$\Gamma\left(q',q|P\right) = V\left(q',q\right) + \Delta\Gamma\left(q',q|P\right)$$

(E. Schiller and H. Müther, Eur. Phys. J. A 11 (2001) 227.)

Brueckner Calculations

We show calculations as a function of the density and the asymmetry α,

$$\alpha = \frac{\rho_n - \rho_p}{\rho}$$

α = 0 nuclear matter

α = 1 neutron matter

We represent the density by an effective Fermi momentum, given by

$$\frac{2}{3\pi^2} k_F^3 = \rho_p + \rho_n = \rho$$

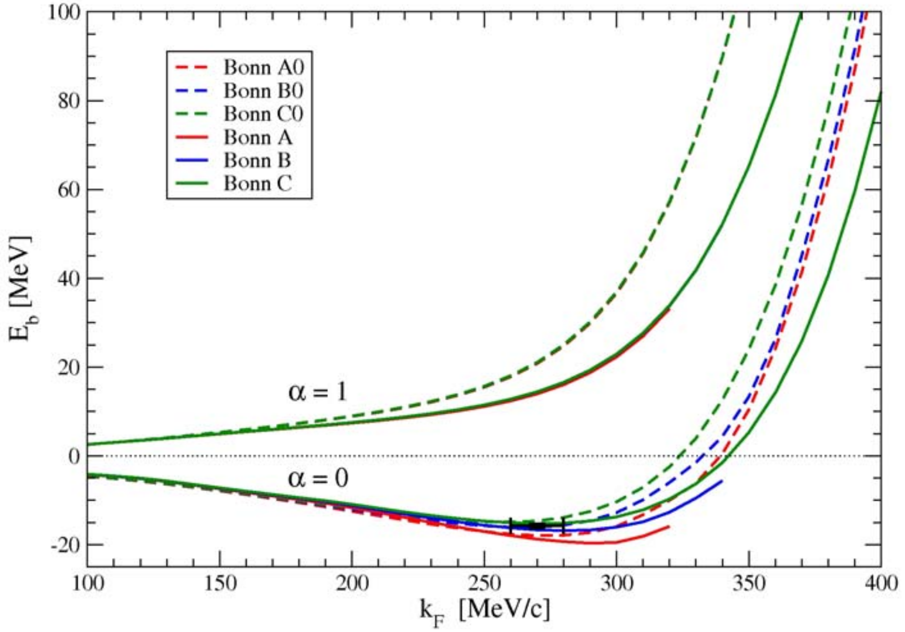
We use the Bonn A, B, C interactions *(B. Brockmann and R. Machleidt, Phys. Rev, C42 (1990) 1965.)*

We take into account the tensor coupling of the ρ meson and use pseudovector coupling for the η and π.

meson	J^π, I	mass (MeV)
σ	0^+, 0	550.0
δ	0^+, 1	983.0
ω	1^-, 0	782.6
ρ	1^-, 1	769.0
η	0^-, 0	548.4
π	0^-, 1	138.03

We use the Thompson form of the reduced nucleon propagator, *g*.

Brueckner calculations



Bonn A0, B0, C0 – constant mean fields

Bonn A, B, C – momentum-dependent mean fields

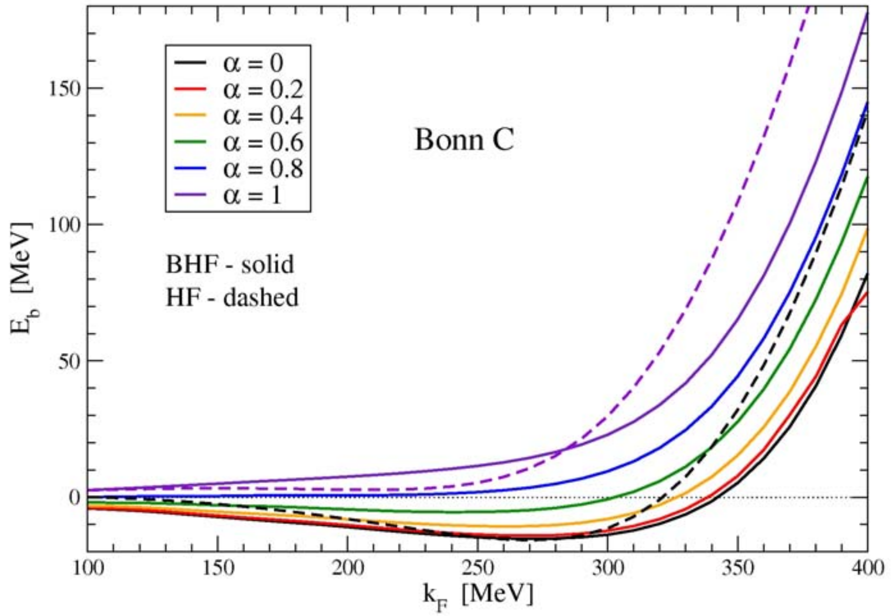
Calculations stop where instability dominates.

Neutron matter – same result for all interactions.

Momentum dependence:
- softens the equation of state of neutron matter;
- increases the Fermi momentum for saturation;
- increases the binding energy per nucleon and incompressibility and decreases the symmetry coefficient at saturation.

	$k_{F,sat}$ [MeV/c]	$E_{b,sat}$ [MeV]	K [MeV]	a_{sym} [MeV]
Bonn A0	273.9	-17.96	331.7	37.1
Bonn B0	264.7	-16.08	254.1	32.9
Bonn C0	257.3	-14.93	215.5	30.1
Bonn A	292.1	-19.67	468.8	36.1
Bonn B	281.9	-16.83	260.3	32.9
Bonn C	270.8	-15.26	252.3	28.4

Brueckner calculations – Bonn C



HF with Bonn C parameters varied (g_σ and g_ω) to saturate at

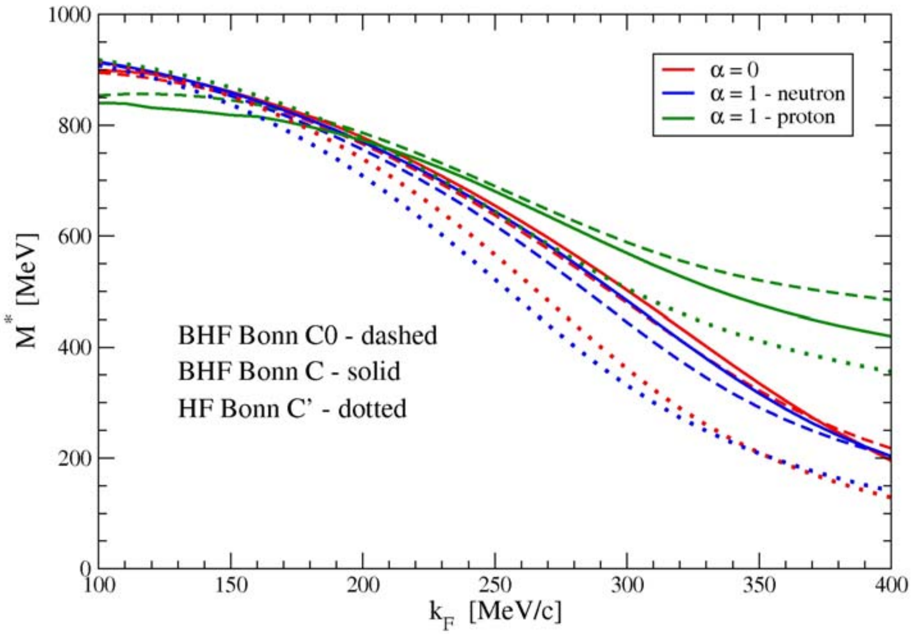
k_F=270 MeV/c

E_b=-15.75 MeV.

M* - effective mass at the Fermi momentum

Neutrons – little difference between M*'s in nuclear and neutron matter

Protons – higher M* than neutrons at high density but lower M* at low density (?)

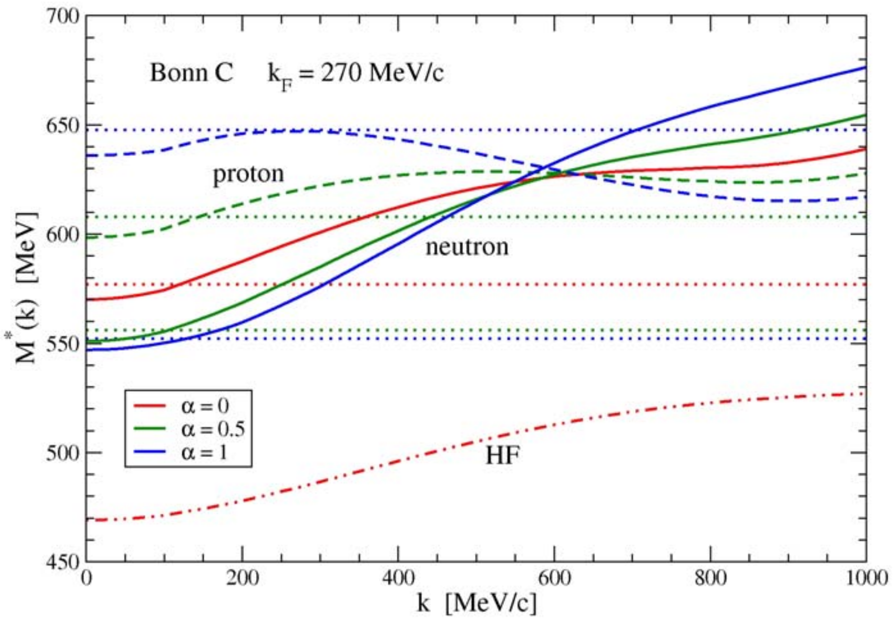


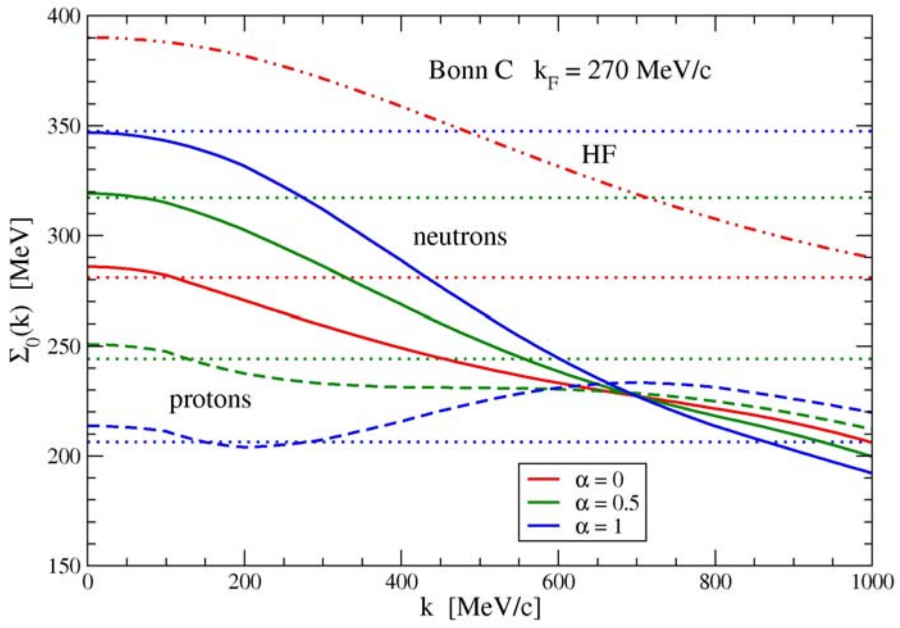
Momentum dependence of the mean fields

Neutron fields - Momentum dependence becomes stronger as asymmetry grows
Proton fields – momentum dependence becomes weaker

Constant fields ‘average’ the momentum dependence of the occupied states.

In symmetric nuclear matter, $\alpha = 0$, the Brueckner and H-F fields have a similar momentum dependence. But the H-F effective mass M* is about 100 MeV smaller and the H-F Σ_0 field 100 MeV larger than the corresponding Brueckner fields.





Brueckner mean fields vs. density-dependent Hartree-Fock fields

What differences would we expect between the ‘ bare’ NN interaction and an effective HF one that describes the Brueckner mean fields?

The mean fields are defined in terms of the Brueckner G-matrix as

$$\Sigma_t(k) = -i \int \frac{d^4p}{(2\pi)^4} \sum_{t'=n,p} Tr\left[\Gamma_{tt'}(k,p;k,p)G_{t'}(p)\right]$$

where the G-matrix is

$$\Gamma\left(q',q|P\right) = V\left(q',q\right) + \Delta\Gamma\left(q',q|P\right)$$

The G-matrix correction to the effective interaction, $\Delta\Gamma$, is density-dependent. As it reduces the overlap of the nucleons, because of the repulsive core of the nucleon-nucleon potential, the effective coupling strengths of the shorter range mesons should be smaller at higher densities. The density-dependent Hartree-Fock interaction should reflect this trend.
We expect the effective HF coupling constants to decrease with density.

A Hartree-Fock fit to the Brueckner mean fields

The average trend of the mean fields is well fit. Binding energies are reproduced within 0.5 MeV.

The momentum dependence of the HF fields is smoother than that of the Brueckner fields.

$\Delta g_\sigma \sim -0.2 \quad \Delta g_\omega \sim -3.5$

$\Delta g_\delta \sim -3.1 \rightarrow \Delta g_\delta \sim -8.0$

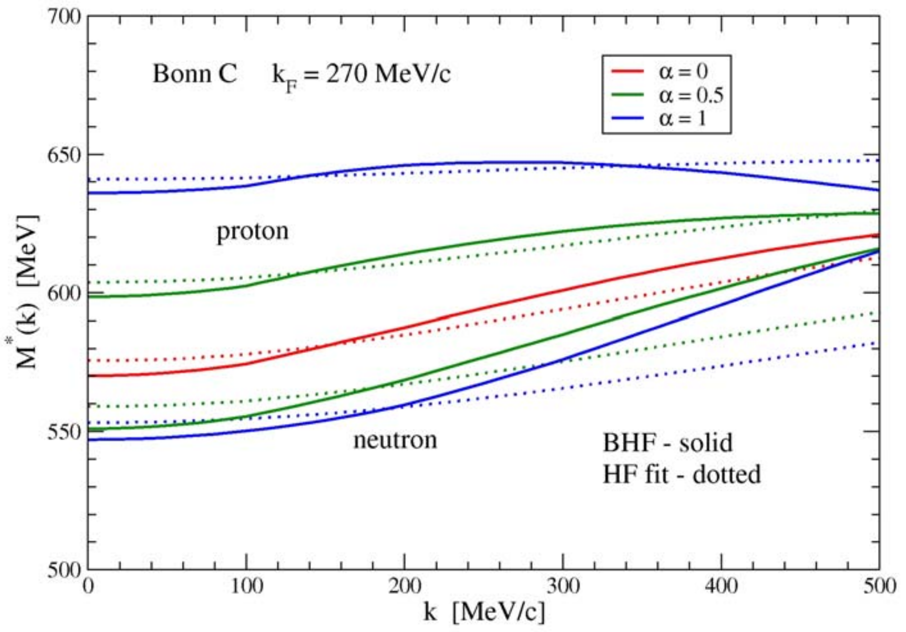
$\Delta g_\rho \sim 0.6 \rightarrow \Delta g_\rho \sim -1.4$

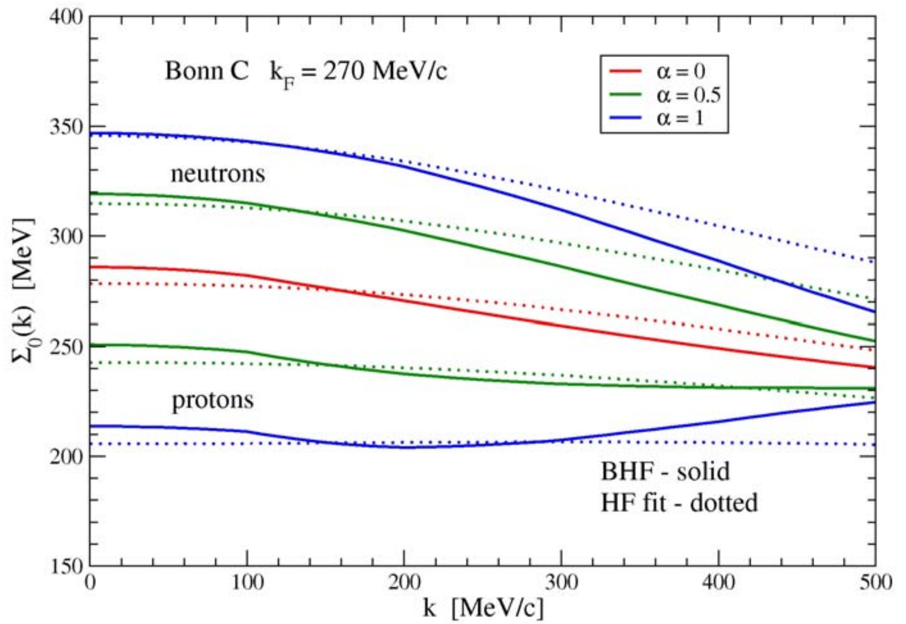
as $\alpha = 0 \rightarrow \alpha = 1$.

Bonn C:

$g_\sigma \sim 10.0 \quad g_\omega \sim 15.9$

$g_\delta \sim 8.0 \quad g_\rho \sim 3.5$

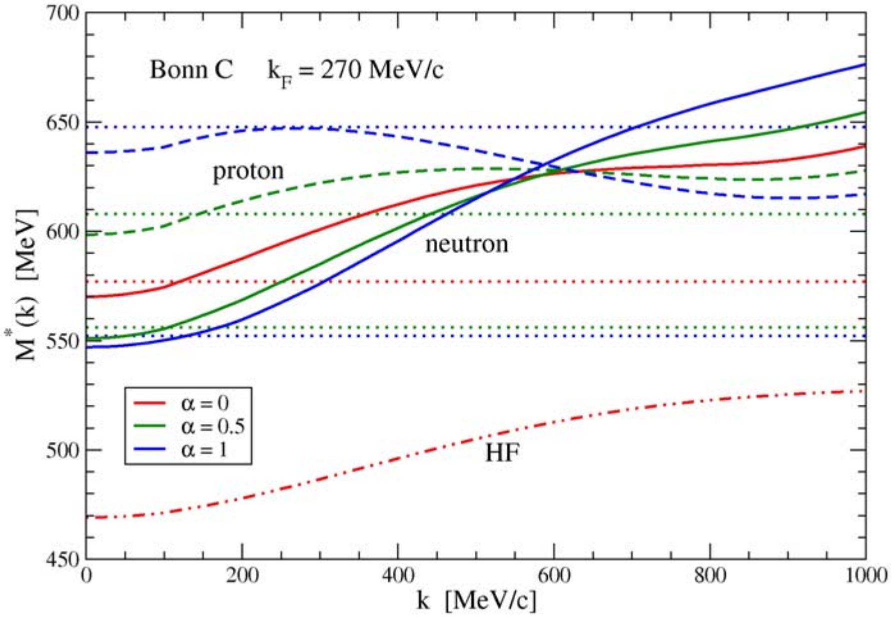




A zero-range contribution to the interaction

The momentum dependence of HF mean fields obtained with the bare interaction is very similar to that of the of the Brueckner mean fields.

This suggests that the effects of the ladder sum could be represented by simply including a zero-range contribution in the interaction,



$$\Delta V = \frac{g_{z\sigma}^2}{m_\sigma^2} 1_1 1_2 - \frac{g_{z\omega}^2}{m_\omega^2} \gamma_1^\mu \gamma_{\mu 2} + \left(\frac{g_{z\delta}^2}{m_\delta^2} 1_1 1_2 - \frac{g_{z\rho}^2}{m_\rho^2} \gamma_1^\mu \gamma_{\mu 2} \right) \vec{\tau}_1 \cdot \vec{\tau}_2$$

(E. Schiller and H. Müther, Eur. Phys. J. A 11 (2001) 227.)

Note that the signs of the terms are opposite of what would be expected of the exchange of zero-range mesons.

Including zero-range terms in the Hartree-Fock fit

The momentum dependence of the Brueckner fields is reproduced better. Binding energies are still reproduced only within 0.5 MeV.

$\Delta g_\sigma \sim 0.0 \quad \Delta g_\omega \sim 0.3$

$\Delta g_\delta \sim -1.0 \rightarrow \Delta g_\delta \sim -2.5$
$\Delta g_\rho \sim 0.9 \rightarrow \Delta g_\rho \sim -0.3$

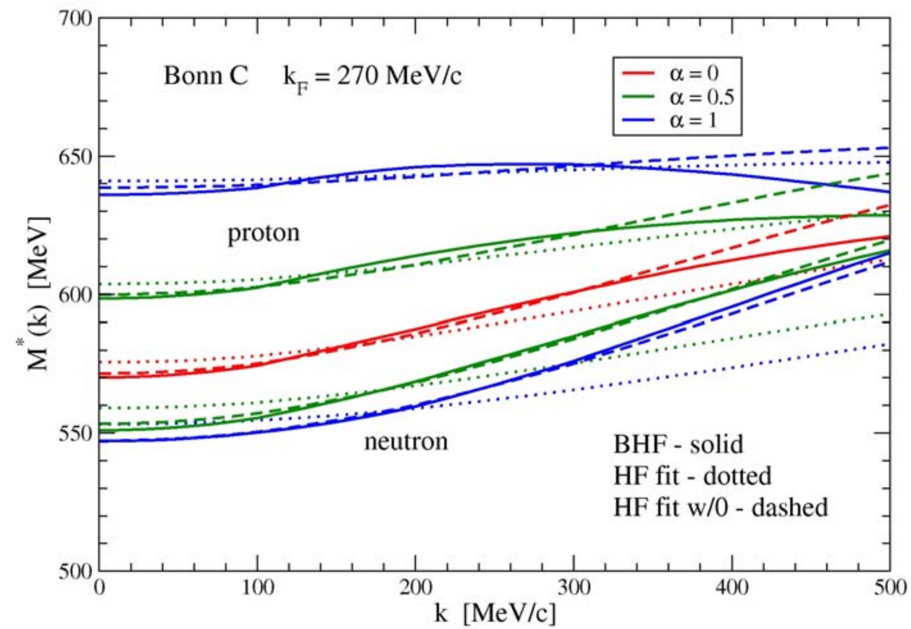
as $\alpha = 0 \rightarrow \alpha = 1$.

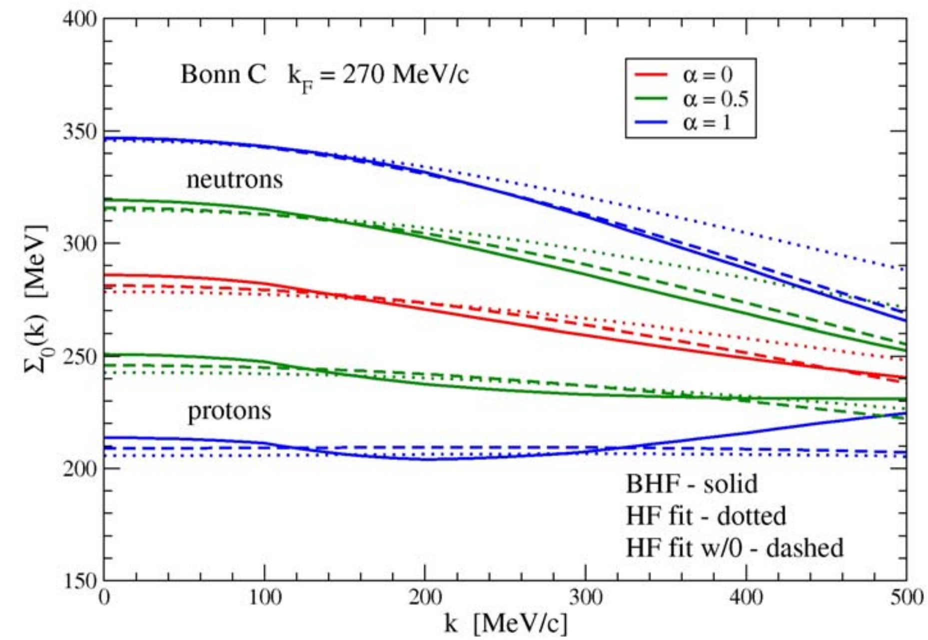
$\Delta g_{z\omega} \sim 7.5 \quad \Delta g_{z\rho} \sim 0.0$
$\Delta g_{z\delta} \sim 0.5$

$\Delta g_{z\sigma} \sim 0.7 \rightarrow \Delta g_{z\sigma} \sim 0.2$

as $\alpha = 0 \rightarrow \alpha = 1$.

Variable meson masses?





How seriously should we take this?

A few things we've left out:

1) Retardation terms → covariance of the interaction;
2) Negative energy states and full Dirac structure;
3) Pairing instability → a density dependent gap;
4) Quasiparticle approximation → Re[Γ] → Γ, Re[Σ] → Σ;
→ spectral function A(k,ω);
5) σ meson dynamics → correlated π's and π – Δ dynamics;
6) Δ's;
7) Other mesons → Rho – Brown scaling;
8) Nucleon structure;
9) RPA/particle-hole and higher-order correlations;
10) ...

The results can still serve as a guide to the form of the interaction we will need to describe nuclei.

Pairing instability

Pairing leads to a density-dependent gap in the single-particle spectrum. A self-consistent treatment leads to a BCS-like equation for the two (±) isolated bound state poles in the gap.

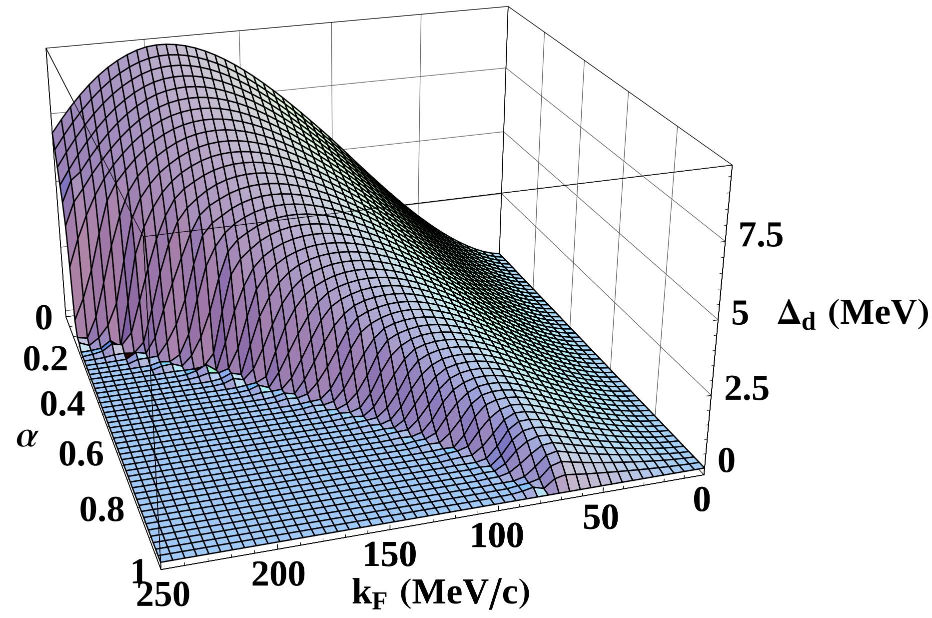
***W.H. Dickhoff, Phys. Lett. B 210 (1988) 15*.**

We do not treat pairing self-consistently.

With constant mean fields, we obtain good convergence with a continuous spectrum – no gap.

With momentum-dependent mean fields, fluctuations in the $^3S_1$-$^3D_1$ channel demand a gap, which we take to have a constant value of 4 MeV.

$^3S_1$-$^3D_1$



B. Funke Haas, BVC and T. Frederico, Nucl. Phys. A788 (2007) 316c.