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Non-perturbative approach in truncated Fock space including antifermion

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● Outline

- Fock space and its truncation
- Wick-Cutkosky model.
- Yukawa Model
 - Two-body truncation $f\mu$.
 - Three-body truncation $f\mu\mu$.
 - Three-body truncation $f\mu\mu + ff\bar{f}$.
 - Anomalous magnetic moment.
- Conclusion.

● Idea

The state vector is represented as the (exact) Fock decomposition:

$$|p\rangle = \sum_{n=1}^{\infty} \int \psi_n(k_1, \dots, k_n, p) |n\rangle$$

It contains **infinite** number of the Fock components ψ_n .
Approximation: replace this sum by the finite one (truncation):

$$|p\rangle = \sum_{n=1}^N \int \psi_n(k_1, \dots, k_n, p) |n\rangle$$

An alternative to the lattice calculations?

• Eigenvalue equation:

$$\mathcal{H} |p\rangle = M |p\rangle$$

It results in a system of equations for the Fock components ψ_n .

$$\begin{pmatrix} H_{11}^0 & H_{12}^{int} & \dots & H_{1N}^{int} \\ H_{21}^{int} & H_{22}^0 & \dots & H_{2N}^{int} \\ \dots & \dots & \dots & \dots \\ H_{N1}^{int} & H_{N2}^{int} & \dots & H_{NN}^0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \\ \psi_N \end{pmatrix} = M \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \\ \psi_N \end{pmatrix}$$

The coupling constant α in H^{int} may be large. After truncation, the solution of the system of equations is exact (non-perturbative).

The solution requires renormalization.

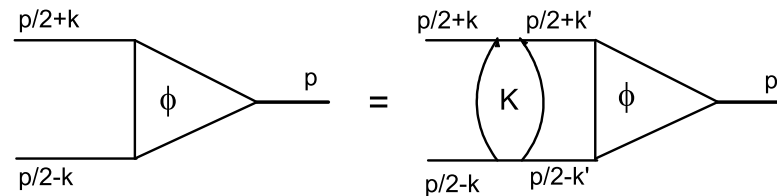
● Can it converge enough fast?

This can be checked in a simple solvable model (Wick-Cutkosky model).

Dae Sung Hwang and V.A. Karmanov,
Many-body Fock sectors in Wick-Cutkosky model,
Nucl. Phys. **B696** (2004) 413.

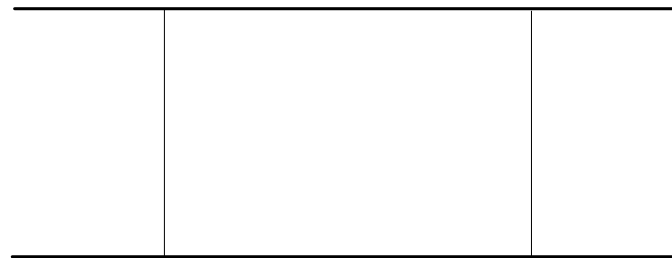
- Spinless particles, massless exchanges.
- The ladder graphs only (however, including all the stretched boxes). No self-energy, no divergences.
- However, **many-body intermediate states, up to infinity, are taken into account.**
- One can compare with truncated calculation.

● Bethe-Salpeter equation



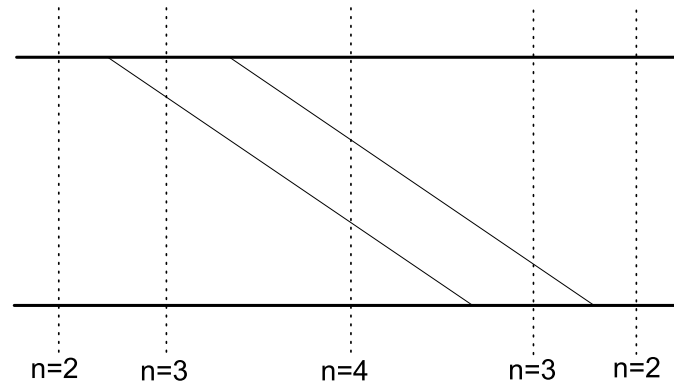
Bethe-Salpeter equation with OBE kernel.

● Time-ordered v.s. Feynman graphs



Feynman ladder graph with two exchanges.

● Time-ordered graph



One of six time-ordered ladder graphs, generated by the Feynman.

In ladder approximation higher Fock sectors contain increasing number of exchanged particles.

• Sum over all intermediate states

Solving Bethe-Salpeter equation, we take into account all the time-ordered graphs, i.e., all the intermediate states.

$$\Phi(x_1, x_2; p) = \langle p | T[\phi(x_1)\phi(x_2)] | 0 \rangle$$

Normalization of the BS amplitude corresponds to

$$\langle p | p \rangle = 1$$

Two-body component is not normalized to 1, but it gives contribution of the two-body sector. In LFD, we can take into account also 3-body sector.

In this way, we can see the convergence (see, how far is the 2+3-body contribution from the full one).

• Normalization integral

$$\langle p|p\rangle = 1 = \int \psi_2^2 \dots + \int \psi_3^2 \dots + \int \psi_4^2 \dots + \dots = N_2 + N_3 + N_4 + \dots$$

Take huge coupling constant: $\alpha = 2\pi$ (when $\alpha_{QED} = \frac{1}{137}$)

Then, 2- and 3-body sectors dominates:

N_2	N_3	$N_{n \geq 4}$	$N_2 + N_3 + N_{n \geq 4}$
9/14=64%	26%	10%	100%

The result is non-perturbative!

Similar hierarchy takes place form e.m. form factors.

• Competition:

Coupling constant – energy

Coupling constant: $\alpha = \frac{g^2}{16\pi m^2}$.

n exchanged particles in intermediate state:

$$M \sim \frac{\alpha^n}{E - \sum_{i=1}^n E_i}$$

If $\alpha > 1$ ($\alpha \rightarrow 2\pi$), then α^n is large.

But $\sum_{i=1}^n E_i$ is also large.

● The approach seems reasonable!

Being developed enough, it might form an alternative to the lattice calculations.

General advantage: knowing state vector, we can calculate any observables.

Particular profit: Minkowski space, wave functions, form factors, etc.

Yukawa model plays role of a testing area.

● Yukawa model and QED

St. Glazek

R. Perry

Yukawa model, 2-body truncation

S.J. Brodsky

S. Chabysheva

V.A. Franke

J.R. Hiller

G. McCartor

S.A. Paston

E.V. Prokhvatilov

Bare masses basis $m_0 \rightarrow m$

J. Vary et al. (ISU) || Harmonic oscillator basis

● Explicitly covariant LFD

V.A. Karmanov, JETP, **44** (1976) 201.

J. Carbonell, B. Desplanques, V.A. Karmanov, J.-F. Mathiot,
Phys. Reports, **300** (1998) 215.

$$t + z = 0 \quad \rightarrow \quad \omega \cdot x = \omega_0 t - \vec{\omega} \cdot \vec{x}$$

where $\omega = (\omega_0, \vec{\omega})$ such that $\omega^2 = 0$.

The unit vector $\vec{n} = \frac{\vec{\omega}}{|\vec{\omega}|}$ determines the orientation of the light-front plane.

Particular case: $\omega = (1, 0, 0, -1)$
corresponds to the standard approach.

• Special 3D graph techniques

Fermion propagator:

$$\Pi_F(p) = [(\not{p} + m)\delta(p^2 - m^2) - (\not{p} + m_1)\delta(p^2 - m_1^2)] \theta(\omega \cdot p)$$

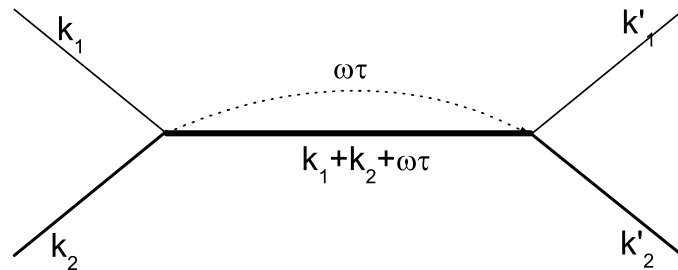
Boson propagator:

$$\Pi_B(p) = [\delta(p^2 - \mu^2) - \delta(p^2 - \mu_1^2)] \theta(\omega \cdot p).$$

Absence of contact (instantaneous) interaction

$$H^{contact} = g^2 \bar{\psi} \psi \phi^2 \quad (\psi \sim \gamma_+)$$

● Example: s -channel exchange



s -channel exchange amplitude

$$\begin{aligned} M &= g^2 \int \delta((k_1 + k_2 + \omega\tau)^2 - m^2) \frac{d\tau}{\tau - i\epsilon} \\ &= \frac{g^2}{m^2 - (k_1 + k_2)^2} = -\frac{g^2}{s - m^2} \end{aligned}$$

● Lagrangian

Physical particles

$$\mathcal{L}^{free} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi + \frac{1}{2} [\partial_\mu\Phi\partial^\mu\Phi - \mu^2\Phi^2]$$

PV particles

$$\mathcal{L}_{PV}^{free} = -i\bar{\Psi}_{PV}\gamma^\mu\partial_\mu\Psi_{PV} + m_1\bar{\Psi}_{PV}\Psi_{PV} - \frac{1}{2} [\partial_\mu\Phi_{PV}\partial^\mu\Phi_{PV} - \mu_1^2\Phi_{PV}^2]$$

Interaction

$$\mathcal{L}^{int} = g_0\bar{\Psi}'\Psi'\Phi' + \delta m\bar{\Psi}'\Psi',$$

where

$$\Psi' = \Psi + \Psi_{PV}, \quad \Phi' = \Phi + \Phi_{PV}.$$

Dressed masses basis m and counter term δm .

• Two-body truncation

The diagram shows an equality between two terms. On the left is a thick horizontal line representing a fermion, labeled Γ_1 below it. This is equal to the sum of two terms. The first term is a thick horizontal line labeled Γ_1 with a small black dot on it, labeled $\delta m^{(2)}$ above and V_1 below. The second term is a thick horizontal line labeled Γ_2 with a curved line (loop) above it, labeled $g_0^{(2)}$ above and V_2 below.

The diagram shows an equality between two terms. On the left is a thick horizontal line representing a boson, labeled Γ_2 below it, with a thin line extending upwards from its right end. This is equal to a thick horizontal line labeled Γ_1 below it, with a thin line extending upwards from its right end, labeled $g_0^{(2)}$ above and V_3 below.

System of equation for physical and Pauli-Villars particles (one PV fermion and one PV boson).

• Two-body equations

$$\begin{aligned}\bar{u}(p_{1i})\Gamma_1^i u(p) &= \bar{u}(p_{1i}) (V_1 + V_2) u(p), \\ \bar{u}(k_{1i})\Gamma_2^{ij} u(p) &= \bar{u}(k_{1i})V_3 u(p),\end{aligned}$$

Spin structure of wf's

$$\begin{aligned}\bar{u}(k_{1i})\Gamma_1^i u(p) &= (m_i^2 - M^2)\psi_1^i \bar{u}(k_{1i})u(p), \\ \bar{u}(k_{1i})\Gamma_2^{ij} u(p) &= \bar{u}(k_{1i}) \left[b_1^{ij} + b_2^{ij} \frac{m\psi}{\omega \cdot p} \right] u(p),\end{aligned}$$

A consequence:

$$V_2 = -g_0^{(2)} \bar{\Sigma}(p) V_3,$$

● Role of PV fermion

Self-energy:

$$\bar{\Sigma}(p) = - \int \frac{d^2 R'_\perp}{(2\pi)^3} \int_0^1 \frac{dx'}{2x'(1-x')} \left[\sum_{i',j'} (-1)^{i'+j'} \frac{\not{p} - \not{k}'_{2j'} + \not{\psi} \tau_{i'j'} + m_{i'}}{2(\omega \cdot p) \tau_{i'j'}} \right],$$

General structure:

$$\bar{\Sigma}(p) = A(p^2) + B(p^2) \frac{\not{p}}{m} + C(p^2) \not{\psi}$$

Without PV fermion $C(p^2)$ diverges.

With the PV fermion $C(p^2)$ converges.

Moreover: $C(p^2) \equiv 0$

● **Solution** (Not yet normalized)

$$\delta m^{(2)} = g_0^2 (A + B)$$

$$\psi_1^0 = a_1, \quad \psi_1^1 = 0,$$

$$b_1^{ij} = 2mg_0a_1, \quad b_2^{ij} = 0.$$

Comments

- The physical and PV two-body components are equal to each other (do not depend on i, j).
- The vertex $F \rightarrow FB$ is constant.
- The ϕ -dependent term is absent.
- **The fermion is structureless (is not dressed).**

● Normalized solution

We could also start with the normalization conditions

$$b_1^{ij} = g,$$

The residue in the vertex is the physical coupling constant.
and

$$b_2^{ij} = 0$$

(automatically satisfied for 2-body truncation)

Since the residue in the physical (on-shell) vertex serves as definition the physical coupling constant, it should not depend on the LF orientation.

We will use this way of renormalization for the three-body (FBB) truncation.

• Three-body truncation

System of equations

$$\begin{array}{c} p \\ \hline \Gamma_1 \end{array} = \begin{array}{c} p \\ \hline \Gamma_1^i \end{array} \begin{array}{c} \delta m^{(3)} \\ \bullet \end{array} + \begin{array}{c} p \\ \hline \Gamma_2 \end{array} \begin{array}{c} \text{arc} \\ \hline g_0^{(3)} \\ \blacktriangle \end{array}$$

$$\begin{array}{c} \text{diag} \\ \hline \Gamma_2 \end{array} = \begin{array}{c} \text{diag} \\ \hline \Gamma_1 \end{array} \begin{array}{c} g_0^{(3)} \\ \blacktriangle \end{array} + \begin{array}{c} \text{diag} \\ \hline \Gamma_2 \end{array} \begin{array}{c} \delta m^{(2)} \\ \bullet \end{array}$$

$$+ \begin{array}{c} \text{diag} \\ \hline \Gamma_3 \end{array} \begin{array}{c} g_0^{(2)} \\ \blacktriangle \end{array}$$

$$\begin{array}{c} \text{diag} \\ \hline \Gamma_3 \end{array} \begin{array}{c} 2 \\ 1 \end{array} = \begin{array}{c} \text{diag} \\ \hline \Gamma_2 \end{array} \begin{array}{c} g_0^{(3)} \\ \blacktriangle \end{array} \begin{array}{c} 2 \\ 1 \end{array} + \begin{array}{c} \text{diag} \\ \hline \Gamma_2 \end{array} \begin{array}{c} g_0^{(3)} \\ \blacktriangle \end{array} \begin{array}{c} 2 \\ 1 \end{array}$$

● Sector-dependent counter terms

R. Perry, K. Wilson

Our practical realization of this scheme.

V.A. Karmanov, J.-F. Mathiot, A.V. Smirnov, Phys. Rev. **D77** (2008) 085028.

Introduce:

- The mass counter terms in two- and three-body sectors: $\delta m^{(2)}, \delta m^{(3)}$.
- Bare (non-renormalized) coupling constant in the two- and three-body sectors: $\bar{g}_0^{(2)}, \bar{g}_0^{(3)}$.

• Determining counter terms

$\delta m^{(2)}, g_0^{(2)}$ are determined in two-body sector:

$$\delta m^{(2)} = \Sigma(p = m), \quad g_0^{(2)} = g$$

They kill infinities in the two-body sector.

The known $\delta m^{(2)}, g_0^{(2)}$ are inserted in the **two-body** states of three-body sectors.

In addition, there are $\delta m^{(3)}, g_0^{(3)}$ in the **three-body** states of three-body sectors.

$\delta m^{(3)}, g_0^{(3)}$ are determined in from the renormalization conditions in three-body sector.

They should kill infinities in the three-body sector.

This does not mean that we have infinite number of counter terms.

We have usual number of counter terms and an iterative scheme, which precises them from sector to sector.

"Iterative scheme" means that, we do not solve the problem for the truncation N , but first for $n = 2, n = 3, \dots$ and, at last step, for $n = N$.

• Renormalization condition

At

$$s = (k_1 + k_2)^2 = \frac{k_\perp^2 + \mu^2}{x} + \frac{k_\perp^2 + m^2}{1-x} = m^2$$

i.e., at

$$k_\perp^2 = -x^2 m^2 - (1-x)\mu^2 < 0 \quad (\text{non-physical value})$$

we should have:

(i) $b_1^{i=0,j=0}(k_\perp, x) = g$ (relation between $g_0^{(3)}$ and g)

and

(ii) $b_2^{i=0,j=0}(k_\perp, x) = 0$ (kills ω -dependence).

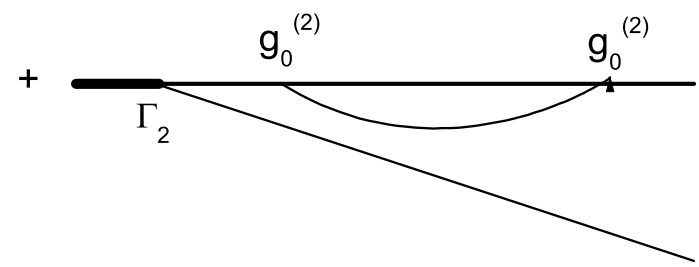
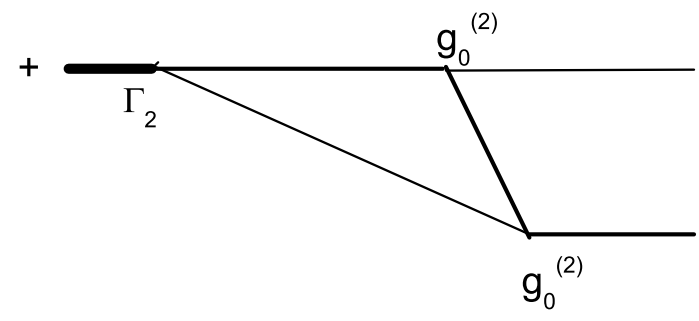
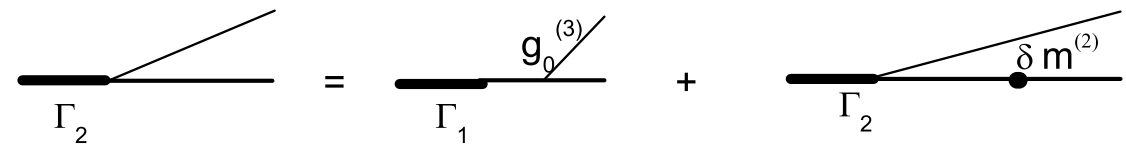
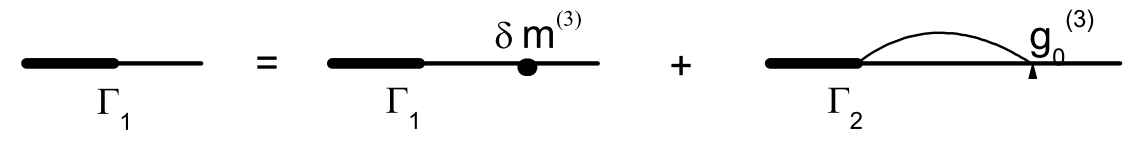
The condition (i) can be always imposed.

To satisfy (ii), we introduce the ω -dependent counter term

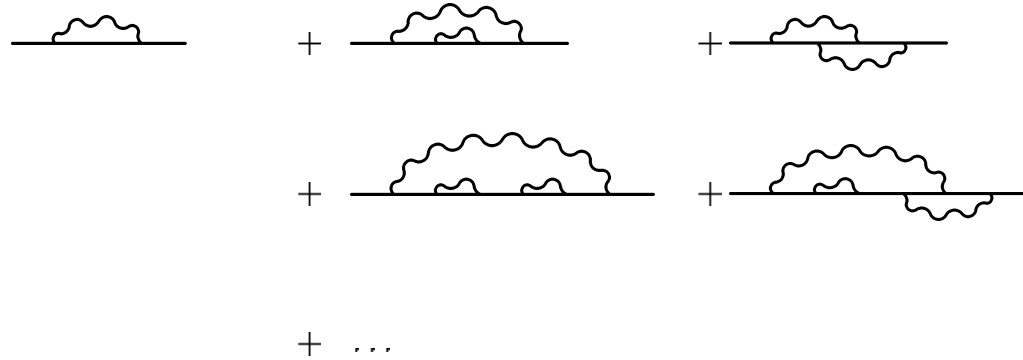
by $g_0^{(3)} \rightarrow g_0^{(3)} + \frac{m\omega}{\omega \cdot p} Z_\omega$

● Reduced system of equations

Excluding Γ_3



• Three-body self-energy



Perturbative expansion, in terms of the pion-nucleon coupling constant g , of the nucleon self-energy.

• Spin structure of 1- and 2-body wf's

(reminder)

$$\bar{u}(k_{1i})\Gamma_1^i u(p) = (m_i^2 - M^2)\psi_1^i \bar{u}(k_{1i})u(p),$$

$$\bar{u}(k_{1i})\Gamma_2^{ij} u(p) = \bar{u}(k_{1i}) \left[b_1^{ij} + b_2^{ij} \frac{m\phi}{\omega \cdot p} \right] u(p),$$

- System of equations involves:

ψ_1^i , $i = 0, 1$ (one-body),

b_1^{ij}, b_2^{ij} , $i, j = 0, 1$ (two-body).

$i = 0, j = 0$ - physical fermion and boson,

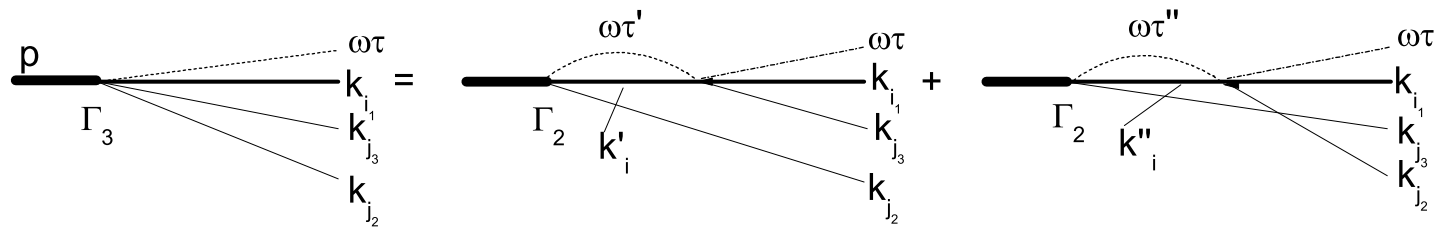
$i = 1, j = 1$ - PV fermion and boson.

(2x2)x2=8 functions; system of 8 equations.

Three-body components are simply expressed through the two-body one and are excluded.

• Three-body wave function

Three-body components are simply expressed through the two-body one and are excluded.



Γ_3 expressed through Γ_2 .

● Spin structure of 3-body wave function

Fermion consists of 1 fermion and 2 spinless bosons.

How many relativistic spin components?

– Answer: **4 spin components**. (One can expect 2, due to parity conservation.)

Nucleon consists of 3 quarks.

How many relativistic spin components?

– Answer: **16 spin components**. (One can expect 8, due to parity conservation.)

For the first glance, it seems:

$$\left(2\frac{1}{2} + 1\right)\left(2\frac{1}{2} + 1\right)\left(2\frac{1}{2} + 1\right) = 8$$

Parity non-conservation does not reduce the number of spin components of the LF wf (for $N \geq 3$).

● Explicit construction

General decomposition:

$$\bar{u}(k_1)\Gamma_3(1, 2, 3)u(p) = \bar{u}(k_1)\left(g_1 S_1 + g_2 S_2 + g_3 S_3 + g_4 S_4\right)u(p)$$

Basis:

$$S_1 = 1, \quad S_2 = m \frac{\not{\omega}}{\omega \cdot p} \quad (\not{\omega} = \omega_\mu \gamma^\mu \rightarrow \gamma_+),$$
$$S_3 = S_1 \gamma_5 C_{ps}, \quad S_4 = S_2 \gamma_5 C_{ps}$$

$$C_{ps} = e^{\mu\nu\rho\gamma} k_{1\mu} k_{2\nu} k_{3\rho} p_\gamma \quad (\neq 0)$$

$$k_1 + k_2 + k_3 \neq p, \quad \text{since} \quad k_{1-} + k_{2-} + k_{3-} \neq p_-$$

$$\text{In our formalism:} \quad k_1 + k_2 + k_3 = p + \omega\tau$$

Remark

Similar formalism can be applied to the 3q nucleon LF wave function.

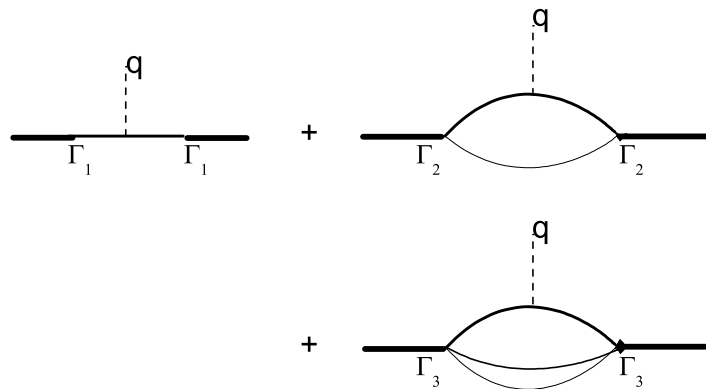
It contains 16 components!

● EM form factors

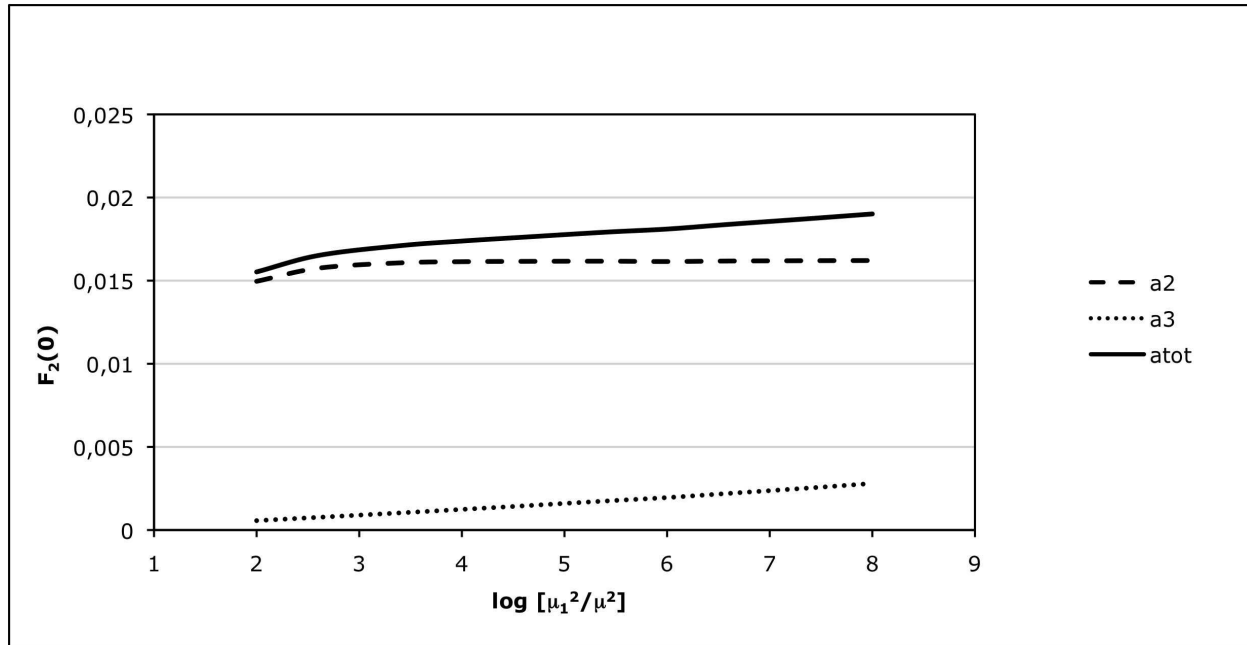
1- and 2-body components are found from equations (model-dependent).

3-body components g_{1-4} are expressed through 2-body components (model-dependent).

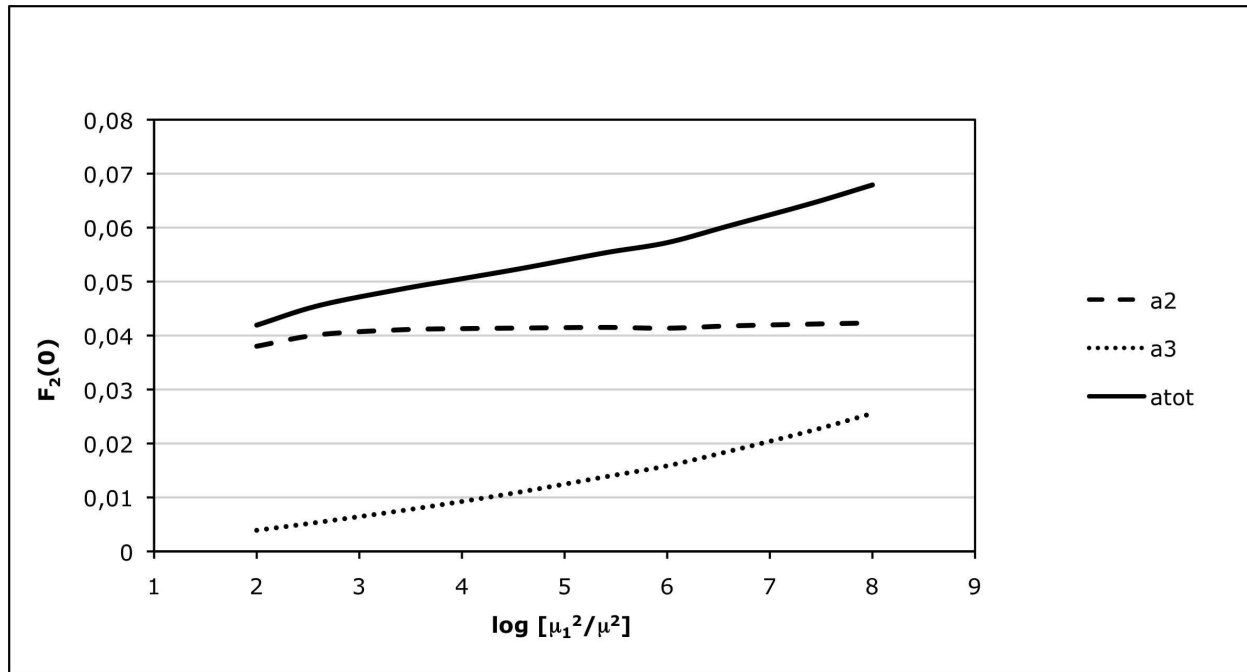
Form-factors are expressed through 1-, 2- and 3-body components (model-independent).



1-, 2- and 3-body contributions in EM form factors



Anomalous magnetic moment A v.s. μ_1 , ($\alpha = 0.2$, $\mu = m = 1$)



Anomalous magnetic moment A v.s. μ_1 ($\alpha = 0.5$, $\mu = m = 1$)

• Z -counter terms

Lagrangian in terms of non-renormalized fields:

$$\mathcal{L} = \bar{\Psi}_0 [i\partial - m_0] \Psi_0 + \frac{1}{2} [\partial_\nu \Phi_0 \partial^\nu \Phi_0 - \mu_0^2 \Phi_0^2] + g_0 \bar{\Psi}_0 \Psi_0 \Phi_0,$$

Renormalized fields and coupling constant

$$\Psi_0 = Z_2^{1/2} \Psi, \quad \Phi_0 = Z_3^{1/2} \Phi, \quad gZ_1 = g_0 Z_2 Z_3^{1/2},$$

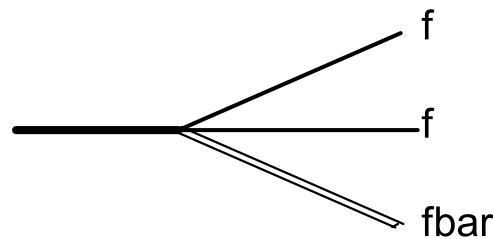
Lagrangian in terms of **renormalized** fields:

$$\begin{aligned} \mathcal{L} &= \bar{\Psi} [i\partial - m] \Psi + \frac{1}{2} [\partial_\nu \Phi \partial^\nu \Phi - \mu^2 \Phi^2] \\ &+ gZ_1 \bar{\Psi} \Psi \Phi + (Z_2 - 1) \bar{\Psi} [i\partial - m] \Psi + \frac{1}{2} (Z_3 - 1) [\partial_\nu \Phi \partial^\nu \Phi - \mu^2 \Phi^2] \\ &+ \delta m \bar{\Psi} \Psi + \frac{1}{2} \delta \mu^2 \Phi^2 \end{aligned}$$

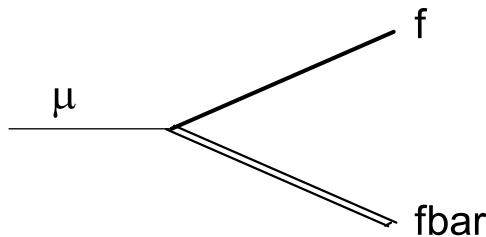
We determine now the counter terms Z 's, δm , $\delta\mu^2$.

● Antifermion

Extending Fock space – adding antifermion.
New Fock component:

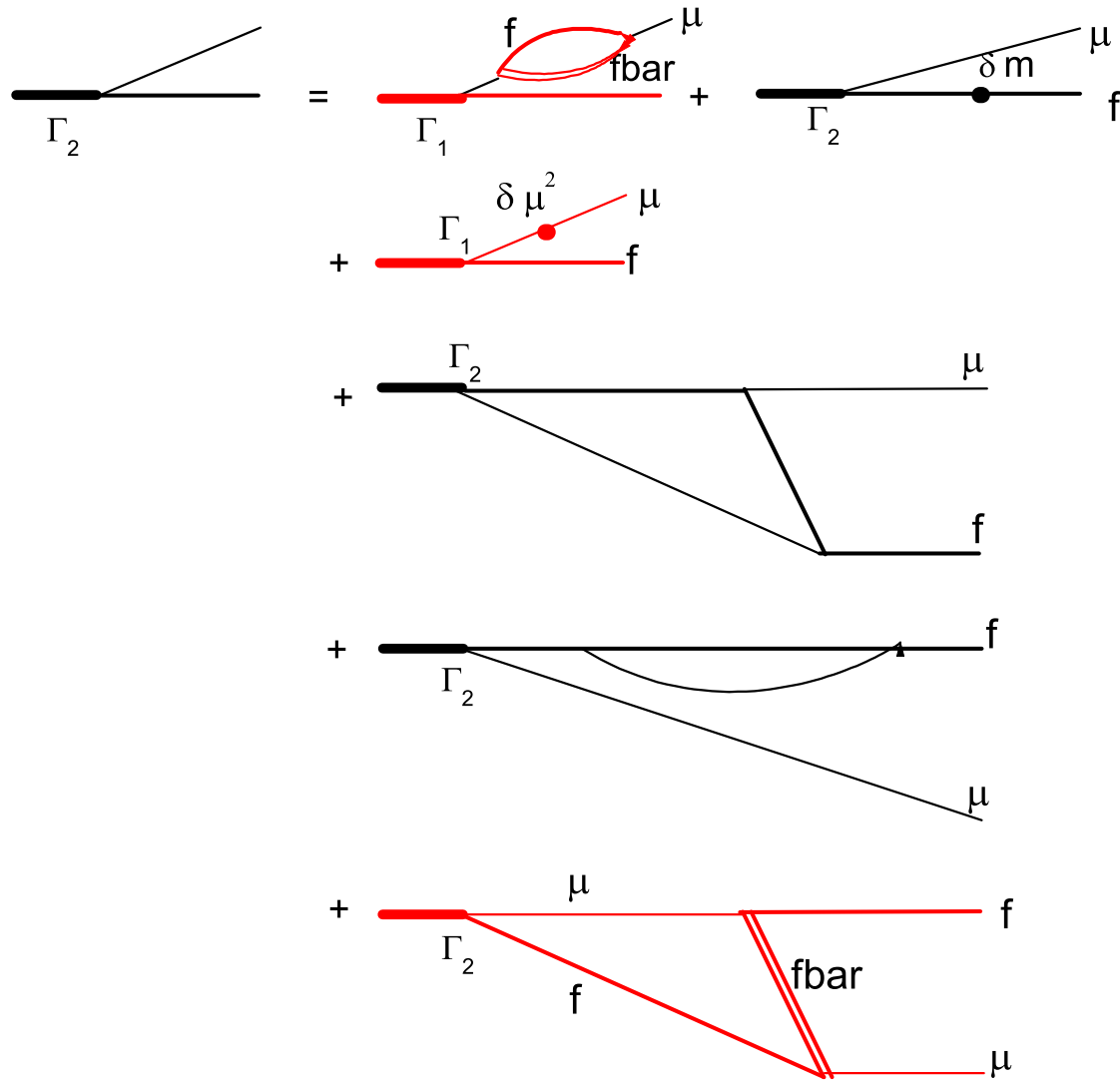


$f - \bar{f}$ pair is always created from $\mu \rightarrow f \bar{f}$:

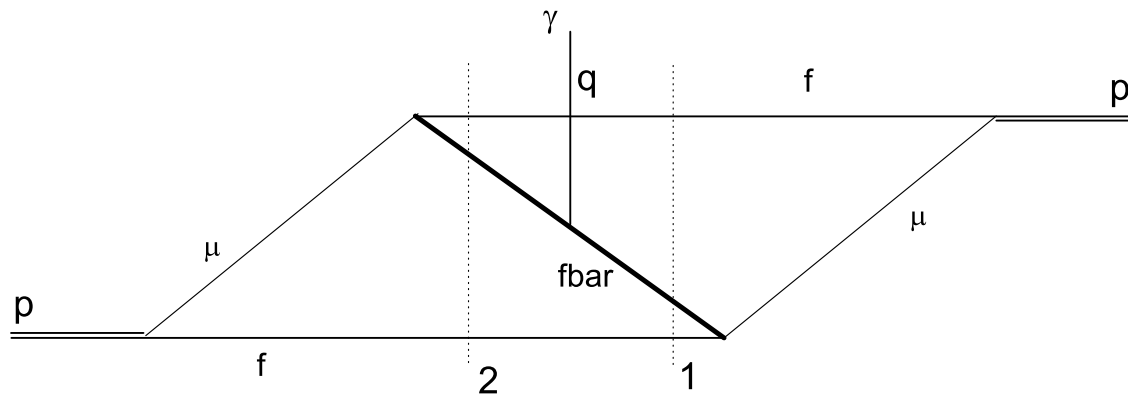
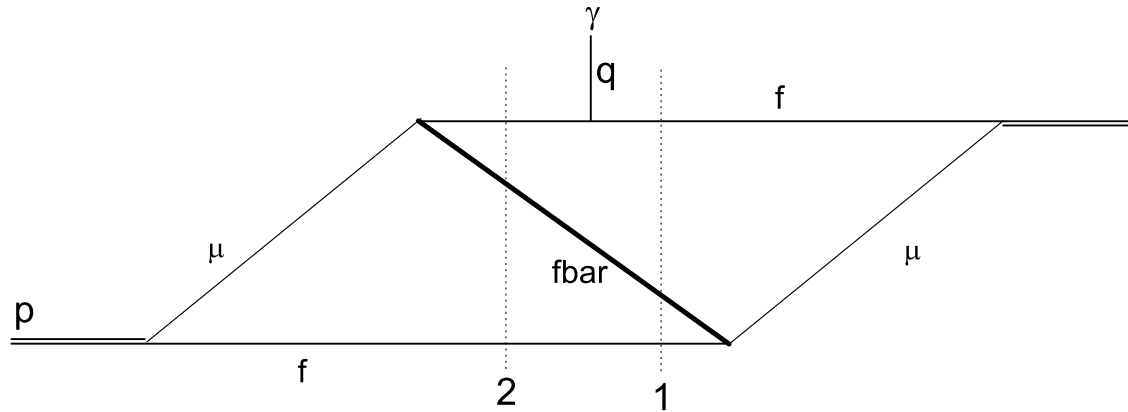


One can exclude $f f \bar{f}$ component and keep \bar{f} in the intermediate states.

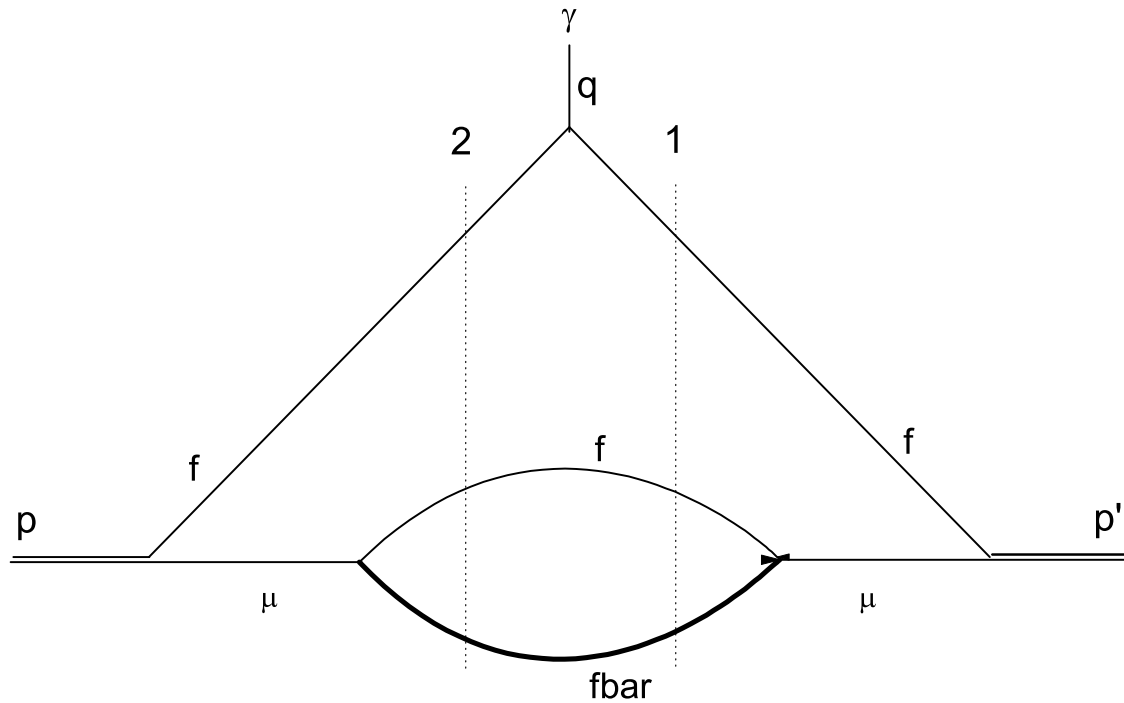
• Reduced system of equations with \bar{f}



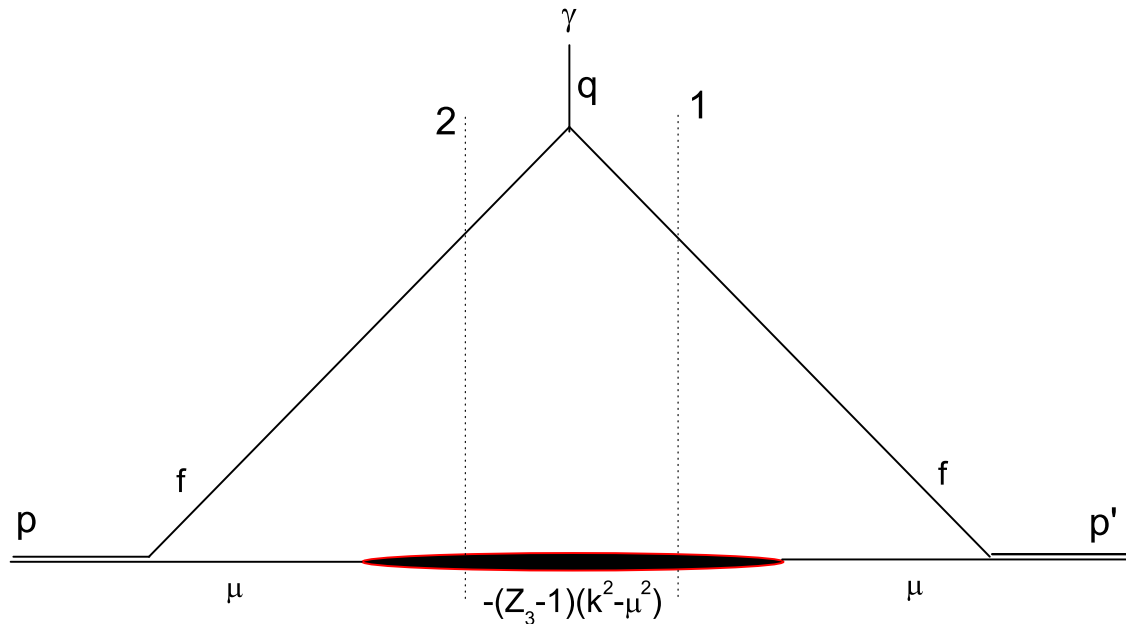
● E.M. vertex: diamond graphs



● E.M. vertex: loop graph



• E.M. vertex: meson counter term graph



$$\text{Meson c.t.} = - (Z_3 - 1)(k^2 - \mu^2) = - \left. \frac{d\Sigma^\mu(k^2)}{dk^2} \right|_{k^2=\mu^2} (k^2 - \mu^2)$$

Σ^μ is the meson self-energy (diverges).

● Cancellation of infinities

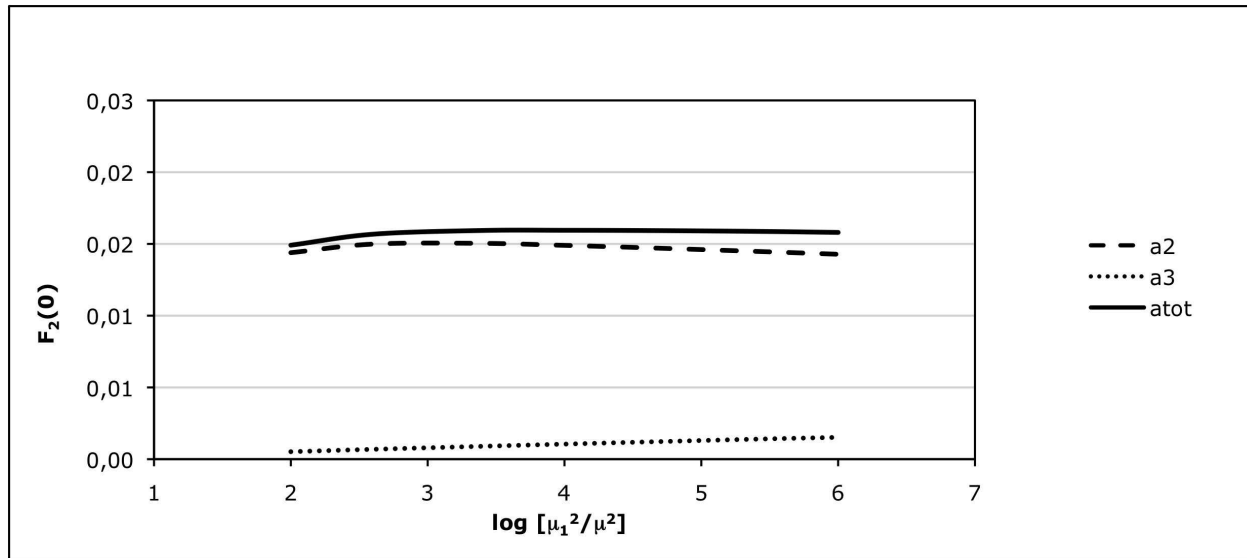
Loop graph asymptotic: $\sim \frac{1}{k_{\perp}^2}$

Counter term graph asymptotic: $\sim -\frac{1}{k_{\perp}^2}$

Loop + counter term = finite

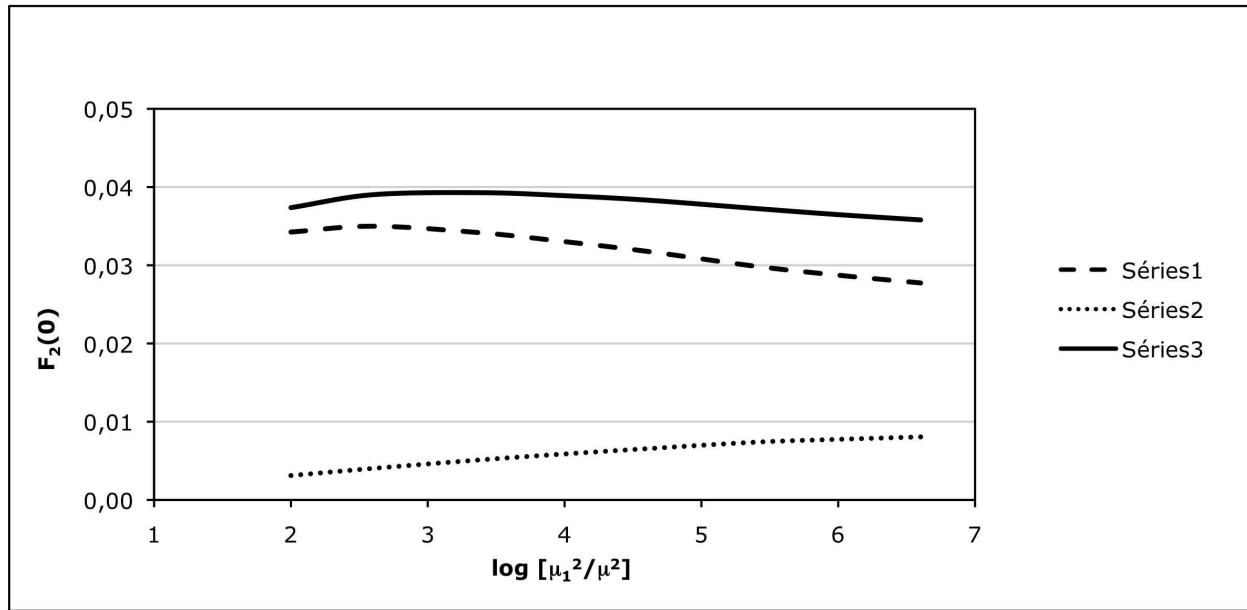
(relative to the limit: P.V. fermion mass $m_1 \rightarrow \infty$)

Limit: P.V. meson mass $\mu_1 \rightarrow \infty$ is taken numerically.



Anomalous magnetic moment A v.s. μ_1 , with g and Z
 ($\alpha = 0.2$, $\mu = m = 1$)

Perturbative value (for $\mu = m$): $A = \frac{\alpha}{4\pi} = 0.016$.



Anomalous magnetic moment A v.s. μ_1 with g and Z
 ($\alpha = 0.5$, $\mu = m = 1$)

Perturbative value (for $\mu = m$): $A = \frac{\alpha}{4\pi} = 0.04$.

● Conclusion

- Non-perturbative approach, based on the truncation of Fock space, is developed.
- Approach is applied to the Yukawa model.
- Fock space is truncated up to three-body states, including state with antifermion ($f f \bar{f}$).
- Anomalous magnetic moment is calculated.
- It is rather stable relative to increase of the meson PV mass.
- The approach seems promising.