Physical vacuum of the Thirring and derivative coupling models

Ľubomír Martinovič
Institute of Physics, Slovak Academy of Sciences
Dúbravská cesta 9, 845 11 Bratislava, Slovakia

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ABSTRACT: Simple two-dimensional models with massless and massive fermions are studied aiming at a comparison between vacuum states and correlation functions constructed from known operator solutions in both space-like (SL) and light front (LF) versions of the theory. The models include the Thirring, derivative coupling and the Federbush model. For certain SL Hamiltonians, the physical vacuum as an eigenstate of $H_0 + H_{int}$ is found by means of a Bogoliubov transformation as a specific coherent state qudratic in effective boson operators composed of fermion bilinears. Simplicity of the LF treatment of massive fermions is demonstrated with the LF version of nonperturbative bosonization of Lehmann and Stehr.

INTRODUCTION

Soluble models: a suitable ground for studying structure and relationship between spacelike (SL) and light front (LF) formulations of QFT

Striking differences between the conventional SL and LF theories: mathematical structure as well as some physical aspects (degrees of freedom, Poincare generators ...)

Long time ago: Wightman - detailed and mathematically careful study of simple models in D=1+1 (Cargése 1964)

Here: exactly soluble models - non-perturbative operator solutions of the field equations is known and the exact correlation functions can be calculated and compared – role of the vacuum state and of the operator part ”visible” - how each form of relativistic dynamics works

What is the relationship between the two schemes?

MODELS:

- Massless Thirring model: $-\frac{g}{2}j_\mu j^\mu$
- Derivative coupling model of Schroer: $-gj^\mu\partial_\mu\phi$

- Federbush model: $-g\epsilon_{\mu\nu}j^{\mu}J^{\nu}$

LF and SL ”ingredients” different (status of the vacuum state, description of massive fermion fields in D=1+1)

Physical results (amplitudes) should be the same if both schemes are mathematically sound and physically correct !

Can the two schemes yield equivalent results for the correlation functions?

as the first step: look carefully at the SL versions of these models

useful trick: write the SL Hamiltonian in terms of composite boson operators (bosonize the currents)

interacting terms become bilinear and could be in principle diagonalized by means of a Bogoliubov transformation: the true lowest-energy eigenstate would be a transformed Fock vacuum (an exponential state) and the correlation functions should be calculated as its expectation values

OUTLINE

1. Klaiber's formulation of the massless Thirring model, Hamiltonian treatment, correct physical vacuum

2. SL and LF quantization of the derivative coupling model (DCM) in D=1+1

3. Bosonization of Lehmann and Stehr and its LF version

4. Federbush model

5. Summary, conclusions and next steps

Klaiber's formulation of the massless Thirring model

Thirring model played important role in history of QFT (see Wightman's Cargese lectures)

first solved by Thirring by Bethe Ansatz, K. Johson found and solved a coupled set of equations for the Green's functions (based on conservation of vector and axial vector currents)

operator solution due to B. Klaiber (Boulder 1967), n-point correlation functions constructed

Thirring model may seem obsolete and uninteresting today

all aspects clarified?

not quite true: a series of papers by Faber and Ivanov (discovery of a broken phase claimed based on Nambu – Jona-Lasinio BCS-like Ansatz for the ground state)

similar conclusions done by Fujita et al. using the Bethe Ansatz solution

Question: why is it necessary to make Ansaetze or apply approximative methods when we know the exact solution and can calculate all properties, including vacuum state, directly?

Classical Lagrangian density

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial_\mu} \Psi - \frac{1}{2} g J_\mu J^\mu, \quad J^\mu = \overline{\Psi}\gamma^\mu \Psi. \tag{1}$$

Field equations and current conservation

$$\begin{aligned} i\gamma^\mu \partial_\mu \Psi(x) &= g J^\mu(x) \gamma_\mu \Psi(x), \\ \partial_\mu J^\mu(x) &= 0. \end{aligned} \tag{2}$$

The solution is

$$\Psi(x) = e^{-i(g/\sqrt{\pi})j(x)}\psi(x), \tag{3}$$

where the massless free spinor field satisfies

$$\gamma^\mu \partial_\mu \Psi(x) = 0 \tag{4}$$

and $j(x)$ is the "integrated current"

$$j_\mu(x) = \frac{1}{\sqrt{\pi}}\partial_\mu j(x), \quad J^\mu(x) = j^\mu(x). \tag{5}$$

Free fields essentially define the solution of the interacting model.

Fourier representation

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int dp^1 \{b(p^1)u(p^1)e^{-ip.x} + d^\dagger(p^1)v(p^1)e^{ip.x}\}, \quad p^0 = |p^1|$$

$$\{b(p^1), b^\dagger(q^1)\} = \{d(p^1), d^\dagger(q^1)\} = \delta(p^1 - q^1),$$
$$b(k^1)|0\rangle = d(k^1)|0\rangle = 0. \qquad (6)$$

The spinor $u(p^1)$

$$u^\dagger(p^1) = \big(\theta(-p^1), \theta(p^1)\big), \quad v^\dagger(p^1) = \big(-\theta(-p^1), \theta(p^1)\big). \qquad (7)$$

satisfies

$$(\gamma p)u(p^1) = 0, \quad u(p^1)\overline{u}(p)(p^1) = \frac{1}{2p^0}\gamma.p, \quad u^\dagger(p^1) = \big(\theta(-p^1), \theta(p^1)\big). \qquad (8)$$

Vector current

$$j^0(0,x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}p \int_{-\infty}^{+\infty} \mathrm{d}q \Big\{ f_0(p,q)\Big[b^\dagger(p)b(q)e^{-i(p-q)x} - d^\dagger(p)d(q)e^{i(p-q)x}\Big] +$$

$$+ g_0(p,q)\Big[b^\dagger(p)d^\dagger(q)e^{-i(p+q)x} + d(p)b(q)e^{i(p+q)x}\Big]\Big\},$$

$$j^1(0,x) = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\mathrm{d}p\int\limits_{-\infty}^{+\infty}\mathrm{d}q\Big\{g_0(p,q)\Big[b^\dagger(p)b(q)e^{-i(p-q)x} - d^\dagger(p)d(q)e^{i(p-q)x}\Big] +$$

$$+ f_0(p,q)\Big[b^\dagger(p)d^\dagger(q)e^{-i(p+q)x} + d(p)b(q)e^{i(p+q)x}\Big]\Big\},$$

$$f_0(p,q) = \theta(p)\theta(q) + \theta(-p)\theta(-q), \quad g_0(p,q) = \theta(p)\theta(q) - \theta(-p)\theta(-q). \qquad (9)$$

can be represented in terms of composite fermion operators

$$j^\mu(x) = -\frac{i}{\sqrt{2\pi}}\int\frac{dk^1}{\sqrt{2k^0}}k^\mu\{c(k^1)e^{-i\hat{k}.x} - c^\dagger(k^1)e^{i\hat{k}.x}\}, \qquad (10)$$

where

$$c(k^1) = \frac{i}{\sqrt{k^0}} \int dp^1 \{ \theta\big(p^1 k^1\big) \big[b^\dagger(p^1) b(p^1 + k^1) - d^\dagger(p^1) d(p^1 + k^1) \big] +$$
$$+ \theta\big(p^1(p^1 - k^1)\big) d(k^1 - p^1) b(p^1) \big\}. \qquad (11)$$

Canonical Fock commutation relation follow

$$\big[c(p^1), c^\dagger(q^1)\big] = \delta(p^1 - q^1), \;\; c(k^1)|0\rangle = 0. \qquad (12)$$

Problem: infrared divergence – the two-point correlation function of a massless scalar field in D=1+1 is

$$D^{(+)}(x - y) = \langle 0|\phi(x)\phi(y)|0\rangle = \frac{1}{4\pi} \int \frac{dk^1}{|k^1|} e^{-i\hat{k}.x}. \qquad (13)$$

Large part of the paper devoted to the infrared regularization and to verification of basic properties (Poincare invariance, locality) of the regularized n-point correlation functions of the interacting model. Based on normal ordering of the operator solution:

$$\Psi(x) = e^{(-ig/\pi)j^{(+)}(x)}\psi(x)e^{(-ig/\pi)j^{(-)}(x)}. \tag{14}$$

Comparison with the LF treatment not obvious:

Only massive fields consistently treated in LF theory for D=1+1

$\Rightarrow$ Try to compare solutions of a soluble model with massive fermions in both schemes.

True ground state of the massless Thirring model:

Hamiltonian

$$H = \int_{-\infty}^{+\infty} \mathrm{d}x \Big[-i\psi^\dagger \alpha^1 \partial_1 \psi - \frac{1}{2} g \big(j^0 j^0 - j^1 j^1 \big) \Big] \tag{15}$$

Fock representation: the free Hamiltonian is ($p \equiv p^1$ often)

$$H_0 = \int_{-\infty}^{+\infty} \mathrm{d}p |p| \Big[b^\dagger(p) b(p) + d^\dagger(p) d(p) \Big]. \tag{16}$$

The interacting Hamiltonian greatly simplifies in terms of composite operators $c(k), c^\dagger(k)$:

$$H_{int} = \frac{g}{\pi} \int\limits_{-\infty}^{+\infty} dk|k| \Big[c^\dagger(k) c^\dagger(-k) + c(k) c(-k) \Big]. \tag{17}$$

Obviously $|0\rangle$ is not an eigenstate of $H = H_0 + H_{int}$. If diagonalization of H by means of some unitary operator $U(\gamma)$ possible, then

$$U(\gamma) H U^{-1}(\gamma)|0\rangle = 0 \tag{18}$$

and $U^{-1}(\gamma)|0\rangle$ will be the physical vacuum state.

Further treatment a la D. C. Mattis and E. H. Lieb, J. Math. Phys. 6, 304 (1965). Luttinger model, massless non-relativistic fermions with four-body interaction:

$$\psi^\dagger(x) = \frac{1}{\sqrt{L}} \sum_k e^{-ikx} (a^\dagger_{1k}, a^\dagger_{2k}),$$

$$H_0 = \sum_k k\big(a^\dagger_{1k} a_{1k} - a^\dagger_{2k} a_{2k}\big),$$

$$H_{int} = \frac{2\lambda}{L} \sum \delta_{k_1+k_2,k_3-k_4} v(k_3 - k_4) a^\dagger_{1k_1} a_{1k_2} a^\dagger 2k_3 a_{2k_4}. \qquad (19)$$

Bogoliubov transformation to diagonalize the Hamiltonian and to find the true ground state.

Main idea: two degrees of freedom $c_1^\dagger, c_2^\dagger$. Choose new operators $C_1^\dagger, C_2^\dagger$

$$C_1^\dagger = uc_1^\dagger - vc_2, \qquad C_1 = u^* c_1 - v^* c_2^\dagger,$$

$$C_2^\dagger = -v c_1^\dagger + u c_2, \qquad C_2^\dagger = -v^* c_1 + u^* c_2^\dagger. \tag{20}$$

Requiring that the new operators satisfy the original commutation relations, we find

$$u = \cosh\gamma, \qquad v = \sinh\gamma. \tag{21}$$

This transformation can be realized in terms of a unitary operator

$$U(\gamma) = e^{\gamma\left[c_1^\dagger c_2^\dagger - c_2 c_1\right]} \tag{22}$$

Indeed,

$$\begin{aligned} U(\gamma) c_1^\dagger U^{-1}(\gamma) &= c_1^\dagger \cosh\gamma - c_2 \sinh\gamma, \\ U(\gamma) c_2^\dagger U^{-1}(\gamma) &= -c_1^\dagger \sinh\gamma + c_2 \cosh\gamma. \end{aligned} \tag{23}$$

Generalization to infinite number of degrees of freedom: work with $\gamma(p)$, transform H_{int} so that the non-diagonal terms vanish. Knowing $U(\gamma(p))$, the lowest-energy eigenstate = the true physical vacuum will be

$$|\Omega\rangle = U^{-1}(\gamma)|0\rangle. \tag{24}$$

Calculate correlation functions using this ground state.

DETAILS:

H_0 satisfies

$$\left[H_0, c(k)\right] = -|k|c(k), \quad \left[H_0, c^\dagger(k)\right] = |k|c^\dagger(k). \tag{25}$$

Remark: mathematically correct treatment requires cut-offs or test functions to have well defined quantities, here the approach a little heuristic (but checked in a finite volume)

Define the operator T (free Hamiltonian of massless bosons) with the same commutation property:

$$T = \int_{-\infty}^{+\infty} \mathrm{d}q |q| c^\dagger(q) c(q),$$

$$\big[T, c(k)\big] = -|k| c(k)\big], \quad \big[T, c^\dagger(k)\big] = |k| c^\dagger(k). \tag{26}$$

Consider now the unitary operator U,

$$U = e^{iS}, \ \ S = -\frac{i}{2} \int_{-\infty}^{+\infty} \mathrm{d}p \gamma(p) \big[c^\dagger(p) c^\dagger(-p) - c(p) c(-p)\big]. \tag{27}$$

Form new free and interacting Hamiltonians

$$\hat{H}_0 = H_0 - T, \quad \hat{H}_{int} = H_{int} + T. \tag{28}$$

By construction, due to $\big[S, \hat{H}_0\big] = 0$, $\hat{H}_0$ is invariant with respect to U:

$$\hat{H}_0 \to e^{iS} \hat{H}_0 e^{-iS} = \hat{H}_0 + i\big[S, \hat{H}_0\big] + \ldots = \hat{H}_0. \tag{29}$$

On the other hand, $\hat{H}_{int}$ transforms non-trivially due to

$$\big[S, c(k)\big] = i\gamma(k) c^\dagger(-k), \quad \big[S, c^\dagger(k)\big] = i\gamma(k) c(-k), \;\; \gamma(-k) = \gamma(k). \tag{30}$$

Using the operator identity $e^A B e^{-A} = B + [A, B] + \frac{1}{2}[A, [A, B]] + + \frac{1}{3!}[A, [A, [A, B]]] + \ldots$:

$$e^{iS} c(k) e^{-iS} = c(k) + i\big(i\gamma(k)\big) c^\dagger(-k) + \frac{i^2}{2}\big(i\gamma(k)\big)^2 c(k) + \frac{i^3}{3!}\big(i\gamma(k)\big)^3 c^\dagger(-k) + + \ldots . \tag{31}$$

Thus

$$c(k) \to e^{iS} c(k) e^{-iS} = c(k)\cosh\gamma(k) - c^{\dagger}(-k)\sinh\gamma(k),$$
$$c^{\dagger}(k) \to e^{iS} c^{\dagger}(k) e^{-iS} = c^{\dagger}(k)\cosh\gamma(k) - c(-k)\sinh\gamma(k). \qquad (32)$$

It follows

$$\hat{H}_{int} \to e^{iS}\hat{H}_{int}e^{-iS} =$$
$$\int\limits_{-\infty}^{+\infty} \mathrm{d}k|k| \Big\{ \Big[c^{\dagger}(k)c^{\dagger}(-k) + c(k)c(-k) \Big] \Big[\frac{g}{2\pi} \Big(\cosh^2\gamma(k) + \sinh^2\gamma(k) \Big) -$$
$$- \cosh\gamma(k)\sinh\gamma(k) \Big] -$$
$$-c^{\dagger}(k)c(k) \Big[4\frac{g}{2\pi}\sinh\gamma(k)\cosh\gamma(k) - \Big(\cosh^2\gamma(k) + \sinh^2\gamma(k) \Big) \Big] -$$

$$-\delta(0)\Big[2\sinh\gamma(k)\cosh\gamma(k)+\sinh^2\gamma(k)\Big]\Big\}. \tag{33}$$

The last (divergent) term removed by normal ordering. Diagonal form if

$$\gamma(k)=\gamma_D=\frac{1}{2}\arctan\frac{g}{\pi}. \tag{34}$$

Thus we have achieved

$$e^{iS}\hat{H}_{int}e^{-iS}|0\rangle=0\Rightarrow|\Omega\rangle=e^{-iS}|0\rangle. \tag{35}$$

or

$$|\Omega\rangle=\exp\Big[-\frac{1}{2}\gamma_D\int_{-\infty}^{+\infty}\mathrm{d}p\big[c^\dagger(p)c^\dagger(-p)-c(p)c(-p)\big]\Big]|0\rangle. \tag{36}$$

Simplification due to the operator identity (Kirzhnits)

$$e^{\tau[A+B]} = e^{\alpha(\tau)B} e^{\beta(\tau)C} e^{\gamma(\tau)C} \tag{37}$$

valid if

$$[A, B] = C \quad , [A, C] = -\lambda A, \quad [B, C] = \lambda B,$$
$$\alpha(\tau) = \gamma(\tau) = \sqrt{\frac{2}{\lambda}} \tanh\big(\sqrt{\frac{\lambda}{2}}\tau\big), \quad \beta(\tau) = \frac{2}{\lambda} \ln\cosh\big(\sqrt{\frac{\lambda}{2}}\tau\big). \tag{38}$$

For our case,

$$A \equiv \int\limits_{-\infty}^{+\infty} \mathrm{d}q c(q) c(-q), \;\; B \equiv \int\limits_{-\infty}^{+\infty} \mathrm{d}q c^\dagger(q) c^\dagger(-q), \;\; C = -4 \int\limits_{-\infty}^{+\infty} \mathrm{d}q c^\dagger(q) c(q),$$

$$\tau = \frac{1}{2}\gamma_D, \quad , \lambda = 8, \quad \alpha = \frac{1}{2}\tanh\left(\frac{1}{2}\mathrm{arctanh}\frac{g}{\pi}\right) \equiv \kappa. \tag{39}$$

and the vacuum state becomes

$$|\Omega\rangle = e^{-\beta(\gamma_D)} e^{-\kappa \int\limits_{-\infty}^{+\infty} \mathrm{d}p c^\dagger(p) c^\dagger(-p)} |0\rangle. \tag{40}$$

A coherent state of pairs of effective bosons (bilinear in fermion Fock operators) with zero total momentum:

$$P|\Omega\rangle = 0, \quad P = \int\limits_{-\infty}^{+\infty} \mathrm{d}pp \left[b^\dagger(p) b(p) + d^\dagger(p) d(p) \right]. \tag{41}$$

The vacuum $|\Omega\rangle$ is invariant under $U(1)$ and $U_A(1)$ transformations (i.e.

carries vanishing charge and axial charge):

$$U(\alpha)|\Omega\rangle = |\Omega\rangle, \;\; U(\alpha) = e^{i\alpha Q}, \;\; Q = \int\limits_{-\infty}^{+\infty} \mathrm{d}q \big[b^\dagger(q)b(q) - d^\dagger(q)d(q)\big],$$

$$V(\beta)|\Omega\rangle = |\Omega\rangle, \;\; V(\beta) = e^{i\beta Q_5}, \;\; Q_5 = \int\limits_{-\infty}^{+\infty} \mathrm{d}q\epsilon(q) \big[b^\dagger(q)b(q) - d^\dagger(q)d(q)\big].$$

The vacuum state $|\Omega\rangle$ corresponds to the symmetric phase (is invariant with respect to axial-vector transformations, i.e. no chiral symmetry breaking) – in contradiction with the results of Faber and Ivanov who used a Nambu–Jona-Lasinio type of ansatz for the vacuum state claiming that it is energetically favourable for the theory to exist in the broken phase. Problems: an ansatz for an exactly soluble model? true vacuum should be an eigenstate of the full Hamiltonian! $|\Omega\rangle$ is such a state, where is a phase transition in Faber and Ivanov's approach ??

TWO-POINT FUNCTION

Correlation functions calculated from the known operator solution

$$\Psi(x) = e^{(-ig/\pi)j^{+}(x)}\psi(x)e^{(-ig/\pi)j^{-}(x)}. \tag{42}$$

$\psi(x)$ is the free massless fermion field and $j^{\pm}(x)$ are the positive and negative-frequency parts of the integrated current $j(x) = j^{(+)}(x) + j^{(-)}(x)$:

$$j^{(+)}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}q \frac{c^{\dagger}(q^1)}{\sqrt{2|q^1|}} \big[e^{i\hat{q}.x} - \theta\big(\lambda - |q^1|\big)\big],$$

$$j^{(-)}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{d}q \frac{c(q^1)}{\sqrt{2|q^1|}} \big[e^{-i\hat{q}.x} - \theta\big(\lambda - |q^1|\big)\big]. \quad (43)$$

The infared regularization necessary to have meaningful objects. The scale λ introduced.

The two-point function defined as

$$\langle vac|\Psi(x)\overline{\Psi}(y)|vac\rangle. \quad (44)$$

What is $|vac\rangle$? As a rule, the perturbative vacuum state taken. Commuting the fermion operators through the exponentials and the exponentials themselves, one arrives at

$$\langle vac|\Psi(x)\overline{\Psi}(y)|vac\rangle = e^{\frac{g^2}{\pi} D^{(+)}(x-y)} e^{-4\frac{g}{\sqrt{\pi}} D^{(+)}(x-y)} \langle 0|\psi(x)\overline{\psi}(y)|0\rangle. \quad (45)$$

Here, with $\mu = e^{\gamma_E}\lambda$,

$$D^{(+)}(x) = \frac{1}{2\pi}\int \frac{dk}{2|k|}\theta(|k| - \lambda)e^{-ik.x} = -\frac{1}{4\pi}\ln\big(- \mu^2 x^2 + ix^0\epsilon\big) \tag{46}$$

What will be

$$\langle\Omega|\Psi(x)\overline{\Psi}(y)|\Omega\rangle? \tag{47}$$

All this nice, BUT:

We have not worked with the Hamiltonian of the Thirring model!

The correct one is

$$H = \int_{-\infty}^{+\infty} \mathrm{d}x \Big[- i\Psi^\dagger\alpha^1\partial_1\Psi - \frac{1}{2}g\big(j^0 j^0 - j^1 j^1\big)\Big] \tag{48}$$

I simply took $\Psi(x) = \psi(x)$ in the kinetic term. One should instead insert the solution $\Psi(x) = \exp\big(-ig/\sqrt{\pi} j(x)\big)\psi(x)$ into the kinetic term. This generates the term $-i\alpha^1\psi^\dagger\partial_1\psi$ we had before AND changes the interaction term from $j^0 j^0 - j^1 j^1$ to $j^0 j^0 + j^1 j^1$. This mistake done also by Faber and Ivanov and probably by other authors.

Correct Heisenberg equations generated with the above Hamiltonian

$$i\partial_0\Psi = -i\alpha^1\partial_1\Psi + gj^0\Psi - g\alpha^1 j^1\Psi \equiv -[H, \Psi]. \tag{49}$$

With the bosonized current

$$j^\mu(x) = -\frac{i}{\sqrt{2\pi}}\int \frac{dk^1}{\sqrt{2k^0}} k^\mu \big\{ c(k^1)e^{-i\hat{k}.x} - c^\dagger(k^1)e^{i\hat{k}.x} \big\}, \tag{50}$$

we obtain

$$H_{int} = \frac{g}{\pi} \int_{-\infty}^{+\infty} \mathrm{d}k |k^1| c^\dagger(k^1) c(k^1). \tag{51}$$

Free massless bosons with "renormalized" energy $\frac{g}{\pi}|k^1|$. H_0 will also contribute for the one-boson state $c^\dagger(k^1)|0\rangle$ and H_{int} for free fermion states.

The other aspects:

canonical quantization may not always be valid for interacting fields:

$$\left\{\Psi(x), \Psi^\dagger(y)\right\} = Z^{-1}\delta(x-y), \quad Z^{-1} = \exp\left(g^2 D^{(+)}(0)\right)$$

disagreement with Johnson's

$[\Psi(x), j^0(y)] = a\delta(z-y)\Psi(y), \; a = (1 - \frac{g}{\pi})$. He found a^{-1}.

SL AND LF DERIVATIE COUPLING MODEL

The classical Lagrangian density

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^{\mu} \overleftrightarrow{\partial_{\mu}} \Psi - m\overline{\Psi}\Psi + \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}\mu^2\phi^2 - g\partial_{\mu}\phi J^{\mu}, \quad J^{\mu} = \overline{\Psi}\gamma^{\mu}\Psi. \quad (52)$$

For $m = 0$ known as the Schroer's model, for axial vector current interaction as Rothe-Stamatescu model ($m \neq 0, \mu = 0$).

Euler-Lagrange eqs.

$$\begin{aligned} i\gamma^{\mu}\partial_{\mu}\Psi &= m\Psi + g\partial_{\mu}\phi\gamma^{\mu}\Psi, \\ \partial_{\mu}\partial^{\mu}\phi + \mu^2\Phi &= g\partial_{\mu}J^{\mu} = 0. \end{aligned} \quad (53)$$

Free scalar field, enters into the operator solution

$$\Psi(x) =: e^{ig\phi(x)} : \psi(x), \quad i\gamma^{\mu}\partial_{\mu}\psi(x) = m\psi(x). \tag{54}$$

$\psi(x)$ quantized as before,

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \frac{dk^1}{\sqrt{2\omega(k^1)}} \left[a(k^1)e^{-i\hat{k}.x} + a^{\dagger}(k^1)e^{i\hat{k}.x}\right]. \tag{55}$$

Conjugate momenta

$$\Pi_{\phi} = \partial_0\phi(x) - gJ^0, \;\; \Pi_{\Psi} = \frac{i}{2}\Psi^{\dagger}(x), \;\; \Pi_{\Psi^{\dagger}} = -\frac{i}{2}\Psi(x). \tag{56}$$

The Hamiltonian

$$
\begin{aligned}
H &= H_0 + H^{'}, \\
H_0 &= \int_{-\infty}^{+\infty} \mathrm{d}x \big[\frac{1}{2}\Pi_\phi^2 + \frac{1}{2}(\partial_1\phi)^2 + \frac{1}{2}\phi^2\big], \\
H^{'} &= \int_{-\infty}^{+\infty} \mathrm{d}x \big[-i\Psi^\dagger\alpha^1\partial_1\Psi + m\Psi^\dagger\gamma^0\Psi + g\partial_1 J^1\big].
\end{aligned}
\tag{57}
$$

If the kinetic term taken directly for the free field $\psi(x)$, the interacting

Hamiltonian becomes

$$H_{int} = \frac{g}{2\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} \mathrm{d}k \big[c^\dagger(k^1)a(k^1) + a^\dagger(k^1)c(k^1) + a^\dagger(k^1)c^\dagger(k^1) + a(k^1)c(k^1)\big]. \tag{58}$$

It can be diagonalized (for $m = 0$) by a Bogoliubov transformation implemented by means of a unitary operator $U = \exp(iS)$ with

$$S(\gamma) = -i \int\limits_{-\infty}^{+\infty} \mathrm{d}k\gamma(k)\big[c^\dagger(k^1)a^\dagger(-k^1) - c(k^1)a(-k^1)\big] \tag{59}$$

using auxiliary operator

$$T = \int_{-\infty}^{+\infty} \mathrm{d}k \big[|k^1| + \omega(k^1)\big] c^(k) c(k). \tag{60}$$

The physical vacuum is then

$$|\Omega\rangle = N \exp\big[\int_{-\infty}^{+\infty} \mathrm{d}k \gamma(g) c^\dagger(-k^1) a^\dagger(k^1)\big] |0\rangle. \tag{61}$$

The correct procedure is again to insert the solution for the interacting fermion field $\Psi(x)$ into the kinematic term. The interaction cancels in the

Hamiltonian and we find that

$$H = H^0 + H_f, \;\; H_f = \int_{-\infty}^{+\infty} \mathrm{d}x \big[-i\psi^\dagger \alpha^1 \partial_1 \psi + m\psi^\dagger \gamma^0 \psi \big], \tag{62}$$

i.e. the full Hamiltonian is just the sum of free Hamiltonians of the massive scalar and fermion fields. Correct Heisenberg equations generated with this Hamiltonian. Physical vacuum coincides with the Fock vacuum.

The LF analysis (including Dirac-Bergmann quanization) is in progress. The calculated correlation functions in both schemes agree.

Remark: Non-trivial physics (anomalous axial vector current, anomalous dimension of the fermion field, etc.) found in the case of the

Rothe-Stamatescu model where the interaction term is

$$L_{int} = -g\partial_\mu \phi \overline{\Psi} \gamma^\mu \gamma^5 \Psi. \tag{63}$$

Based on ”gauge invariant” current:

$$j_\epsilon^\mu \overline{\Psi}(x+\epsilon)\gamma^\mu \Psi(x) \exp\left(ig \int_x^{x+\epsilon} d\tau_\lambda \epsilon^{\lambda\nu} \partial_\nu \phi(x)\right) - VEV. \tag{64}$$

GAUGE INVARIANCE?

SUMMARY

- New features in the maassless Thirring model found, no broken phase present

- The correct Hamiltonians of the massless Thirring model and the derivative coupling model have Fock vacua as their physical vacuum states

- Correlation functions for the SL and LF massive derivative coupling model coincide, but theory is too simple

- analyze Federbush model: two species of massive fermions, similar structure of the operator solution as the massless Thirring model. DIFFERENCE: exponential of a massive integrated current - normal ordering insufficient, triple-dot ordering instead. 4-point correlation function does not factorize into product of two two-point functioins but is an independent quantity with a new type of singularity for coinciding points.