Hadron production in a thermal model

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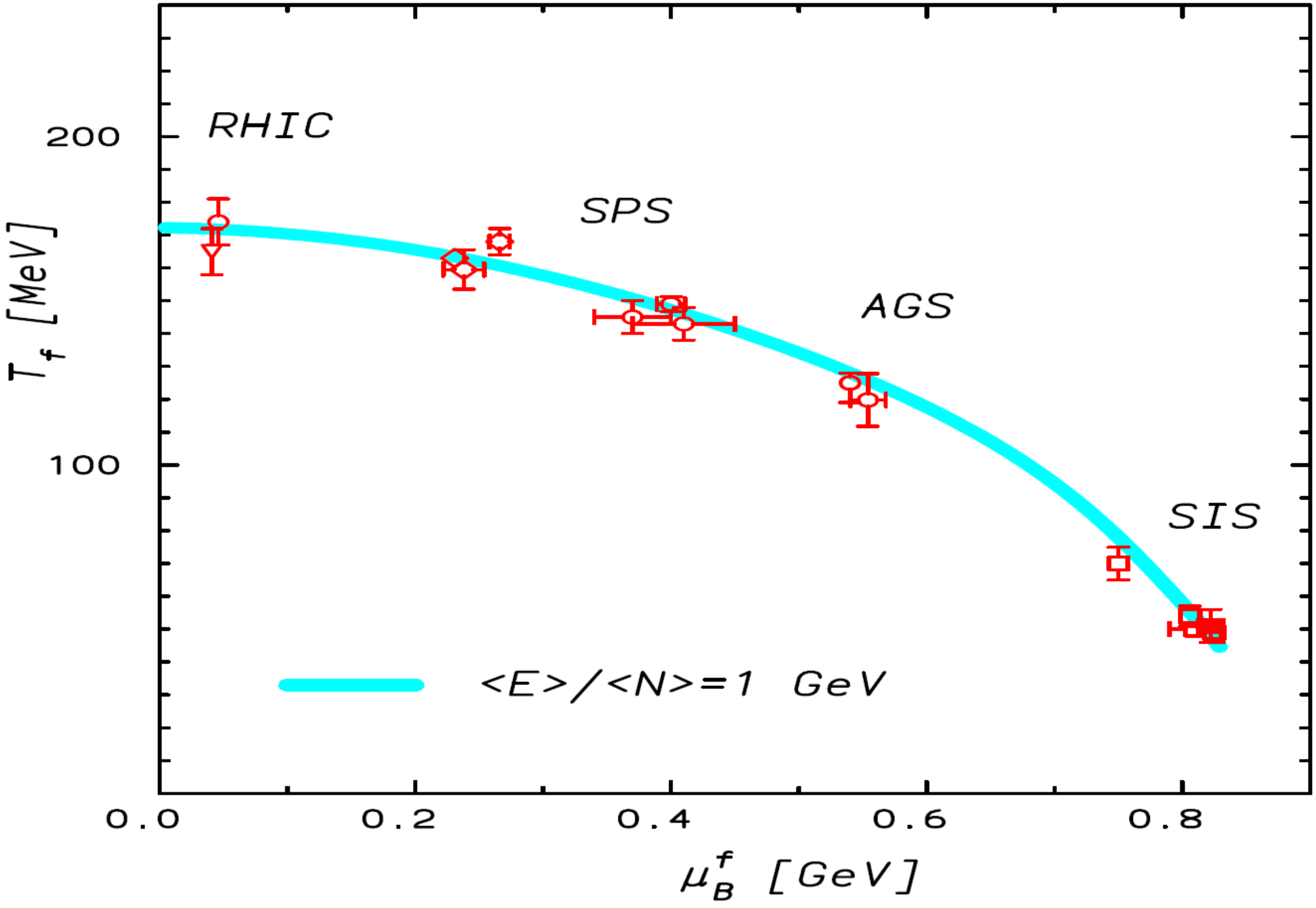
Previous works



extracted from:

P. B. Munzinger, K. Redlich, J. Stachel, arXiv:nucl-th/0304013v1

Thermal Model

- 1950 – Fermi related pion production in high energy collision to a hadron gas (HG) model construction.
- 1965 – Hagedorn predicted that hadronic mass spectrum grows exponentially with masses.
- Since 1965, more than 3200 new resonance states have been identified.
- This means that adding energy to a system, simply produces more particles, resulting in a limiting temperature $T_0 \sim 160$ MeV for a HG.

Thermal Model

- 2009 - Thermal models predict at high energy heavy-ion collisions a limiting chemical freeze-out temperature T_{ch} close to T_{0}. As an example we have:
- A. Andronic et al. PLB 673 (2009) 142:

$$T = T_{\text{lim}} \frac{1}{1 + \exp(1.172 - \ln(\sqrt{s_{NN}}[\text{GeV}])/0.45)}$$

Excluded volume approach

- One uses the thermodynamically consistent approach of D. H. Rischke, *et al.*, Z. Phys. C 51, 485 (1991), as follows:

$$Z^{\rm excl}(T,\{N_i\},V) = \sum_i Z(T,N_i,V-V_0N_i)\,\theta(V-V_0N_i)$$

$$P^{\rm excl}(T,\{\mu_i\}) = \sum_i P_i^{\rm ideal}(T,\mu_i - V_0P^{\rm excl}(T,\{\mu_i\}))$$

$$= \sum_i P_i^{\rm ideal}(T,\widetilde{\mu}_i)\,.$$

Excluded volume approach

$$s^{\mathrm{excl}}(T,\{\mu_i\}) = \frac{\Sigma_j s_j^{\mathrm{ideal}}(T,\widetilde{\mu_j})}{1+V_0\Sigma_j n_j^{\mathrm{ideal}}(T,\widetilde{\mu_j})}$$

$$\epsilon^{\mathrm{excl}}(T,\{\mu_i\}) = \frac{\epsilon^{\mathrm{ideal}}(T,\widetilde{\mu_i})}{1+V_0\Sigma_j n_j^{\mathrm{ideal}}(T,\widetilde{\mu_j})}$$

Excluded volume approach

$$n_i^{\text{excl}}(T,\{\mu_i\}) = \frac{n_i^{\text{ideal}}(T,\tilde{\mu}_i)}{1 + V_0 \Sigma_j n_j^{\text{ideal}}(T,\tilde{\mu}_j)}$$

where: $$n_i^{\text{ideal}} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp\{[E_i(p)-\mu_i]/T\}\pm 1}$$

with: $$E_i(p) = \sqrt{p^2 + m_i^2}$$

Technical Remarks

- For finite volume systems the integrals from zero to infinity need corrections.
- The integrands of the thermodynamical quantities should be multiplied by

$$f(p) = 1 - \frac{3\pi}{4(pR)} + \frac{1}{(pR)^2}$$

R. Balian and C. Bloch, *Ann. Phys. (NY)* 60, 401 (1970)

X. W. Wen, et al, *Phys. Rev. C* 70, 015204 (2005)

Conservation laws

- The problem is solved in the grand canonical ensemble, imposing the constraints of baryon number, strangeness and electric charge conservation, respectively:

$$V \sum_i n_i B_i = Q_B = Z + N,$$

$$V \sum_i n_i S_i = Q_S = 0,$$

$$V \sum_i n_i I_{3i} = Q_{I_3} = (Z - N)/2,$$

where *i* runs over all particles *i* in the system, and $\{B_i, S_i, I_{3i}\}$ are the baryonic, strangeness and isospin quantum numbers of *i* particle.

Table 1
Comparison of experimental particle ratios and thermal model calculations for $T = 174$ MeV, $\mu_B = 46$ MeV at $\sqrt{s} = 130$ GeV. Also shown in parentheses are model predictions for particle ratios at $\sqrt{s} = 200$ GeV with $T = 177$ MeV, $\mu_B = 29$ MeV

Ratio	Model calculation $\sqrt{s} = 130$ (200) GeV	Exp. data $\sqrt{s} = 130$ GeV	Exp.	Ref.
$\bar{p}/p$	0.629 (0.752)	0.65 ± 0.07	STAR	[17]
		0.64 ± 0.07	PHENIX	[22]
		0.60 ± 0.07	PHOBOS	[24]
		0.64 ± 0.07	BRAHMS	[26]
$\bar{p}/\pi^-$	0.078 (0.089)	0.08 ± 0.01	STAR	[18]
π^-/π^+	1.007 (1.004)	1.00 ± 0.02	PHOBOS	[24]
		0.95 ± 0.06	BRAHMS	[25]
K^-/K^+	0.894 (0.932)	0.88 ± 0.05	STAR	[19]
		0.78 ± 0.13	PHENIX	[23]
		0.91 ± 0.09	PHOBOS	[24]
		0.89 ± 0.07	BRAHMS	[25]
K^-/π^-	0.145 (0.147)	0.149 ± 0.02	STAR	[19]
$K^{\star 0}/h^-$	0.037 (0.036)	0.06 ± 0.017	STAR	[20]
$\overline{K^{\star 0}}/h^-$	0.032 (0.033)	0.058 ± 0.017	STAR	[20]
$\bar{\Lambda}/\Lambda$	0.753 (0.842)	0.77 ± 0.07	STAR	[20]
$\bar{\Xi}/\Xi$	0.894 (0.942)	0.82 ± 0.08	STAR	[21]
Λ/h^-	0.040 (0.039)			
$\Lambda/K^{\star 0}$	1.086 (1.091)			
Ξ^-/Λ	0.123 (0.127)			
Ξ^-/K^-	49.8×10^{-3} (50.2×10^{-3})			
$\Xi^+/\bar{\Lambda}$	0.145 (0.142)			
Ξ^+/π^+	6.51×10^{-3} (7.01×10^{-3})			
Ω/Ξ	0.196 (0.197)			
Ω^+/Ω^-	0.898 (0.941)			
Ω^-/π^-	1.47×10^{-3} (1.50×10^{-3})			

QHD model

- We assumed that the chemical freeze-out can be described as a mixture of the lightest baryons and mesons. We use the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\mathrm{QHD}} + \mathcal{L}_{\pi K \rho},$$

where

$$\mathcal{L}_{\mathrm{QHD}} = \mathcal{L}_b + \mathcal{L}_m,$$

with

$$\mathcal{L}_b = \sum_j \bar{\psi}_j \{\gamma_\mu [i\partial^\mu - g_{\omega j}\omega^\mu - g_{\rho j} I_{3j} \rho^{0\mu}] - (m_j - g_{\sigma j}\sigma)\}\psi_j,$$

$$\mathcal{L}_m = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{b}{3}(g_{\sigma N}\sigma)^3 - \frac{c}{4}(g_{\sigma N}\sigma)^4 - \frac{1}{4}(\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}(\partial_\mu \rho_\nu^0 - \partial_\nu \rho_\mu^0)^2 + \frac{1}{2} m_\rho^2 \rho_\mu^0 \rho^{0\mu}.$$

- We have applied this model for the baryonic octet multiplet. D. P. Menezes *et al*, PRC76 (2007) 064902.

TABLE II. Particle ratios: experimental vs those obtained by relativistic mean field models. The last two columns refer to models with noninteracting particles.

Ratio	Exp. data	Exp.	NL3 [7]	TM1 [8]	GM1 [9]	GM3 [9]	TW [3]	DDME1 [5]	Octet	Octet+decuplet
$\bar{p}/p$	0.65 ± 0.07	STAR	0.650	0.646	0.626	0.597	0.656	0.663	0.661	0.649
	0.64 ± 0.07	PHENIX								
	0.60 ± 0.07	PHOBOS								
	0.64 ± 0.07	BRAHMS								
$\bar{p}/\pi^-$	0.08 ± 0.01	STAR	0.075	0.072	0.072	0.063	0.076	0.074	0.039	0.041
π^-/π^+	1.00 ± 0.02	PHOBOS	0.998	0.991	0.999	1.00	1.01	1.01	1.00	1.01
	0.95 ± 0.06	BRAHMS								
K^-/K^+	0.88 ± 0.05	STAR	0.912	0.911	0.907	0.905	0.896	0.900	0.961	0.941
	0.78 ± 0.13	PHENIX								
	0.91 ± 0.09	PHOBOS								
	0.89 ± 0.07	BRAHMS								
K^-/π^-	0.149 ± 0.02	STAR	0.234	0.234	0.242	0.243	0.228	0.227	0.232	0.235
$\bar{\Lambda}/\Lambda$	0.77 ± 0.07	STAR	0.681	0.680	0.666	0.644	0.663	0.675	0.687	0.689
$\bar{\Xi}^-/\Xi^-$	0.82 ± 0.08	STAR	0.746	0.747	0.735	0.711	0.739	0.748	0.714	0.732
K^{0*}/h^-	0.06 ± 0.017	STAR	0.058	0.059	0.063	0.064	0.064	0.063	0.060	0.061
$\bar{K}^{0*}/h^-$	0.058 ± 0.017	STAR	0.053	0.054	0.057	0.058	0.056	0.056	0.057	0.057
$\bar{\Omega}/\Omega$			0.693	0.699	0.715	0.723	0.585	0.586	—	0.784
$\bar{\Omega}/\pi^-$			0.001	0.001	0.002	0.002	0.001	0.001	—	0.001
Λ/h^-			0.021	0.020	0.021	0.020	0.023	0.022	0.013	0.014
Ω/Ξ^-			0.172	0.178	0.195	0.211	0.173	0.174	—	0.250
λ/K^{0*}			0.351	0.341	0.331	0.301	0.363	0.353	0.226	0.228
$\bar{\Xi}^-/\Lambda$			0.226	0.226	0.227	0.221	0.296	0.295	0.241	0.222
$\bar{\Xi}^-/\bar{\Lambda}$			0.332	0.332	0.341	0.343	0.330	0.327	0.312	0.322
$\Xi^-/\bar{K}^-$			0.047	0.045	0.050	0.049	0.048	0.045	0.027	0.030
$\bar{\Xi}^-/\bar{K}^-$			0.035	0.034	0.035	0.033	—	—	0.019	0.022
T (MeV)			149	149	152.0	152.8	146.6	146.2	146.4	148.8
μ_B (MeV)			47.5	46.5	47.5	48.0	62.8	57.0	30.5	32.5
$\rho \times 10^{-3}$ (fm^{-3})			8.37	8.03	9.62	9.95	4.90	4.45	2.41	4.77
χ^2			23.94	24.43	27.99	33.19	22.18	21.83	45.44	41.63
Radius (fm)			22.4	22.7	21.38	21.14	26.77	27.65	33.91	27.02

- Hadronic SU(3) $\sigma - \omega$ approach improves very much the previous discussed QHD-NL models.

  D. Zschiesche, *et. al*, *Phys. Lett. B* 547, 7 (2002)

- New ingredient: baryon and meson masses vary in medium.

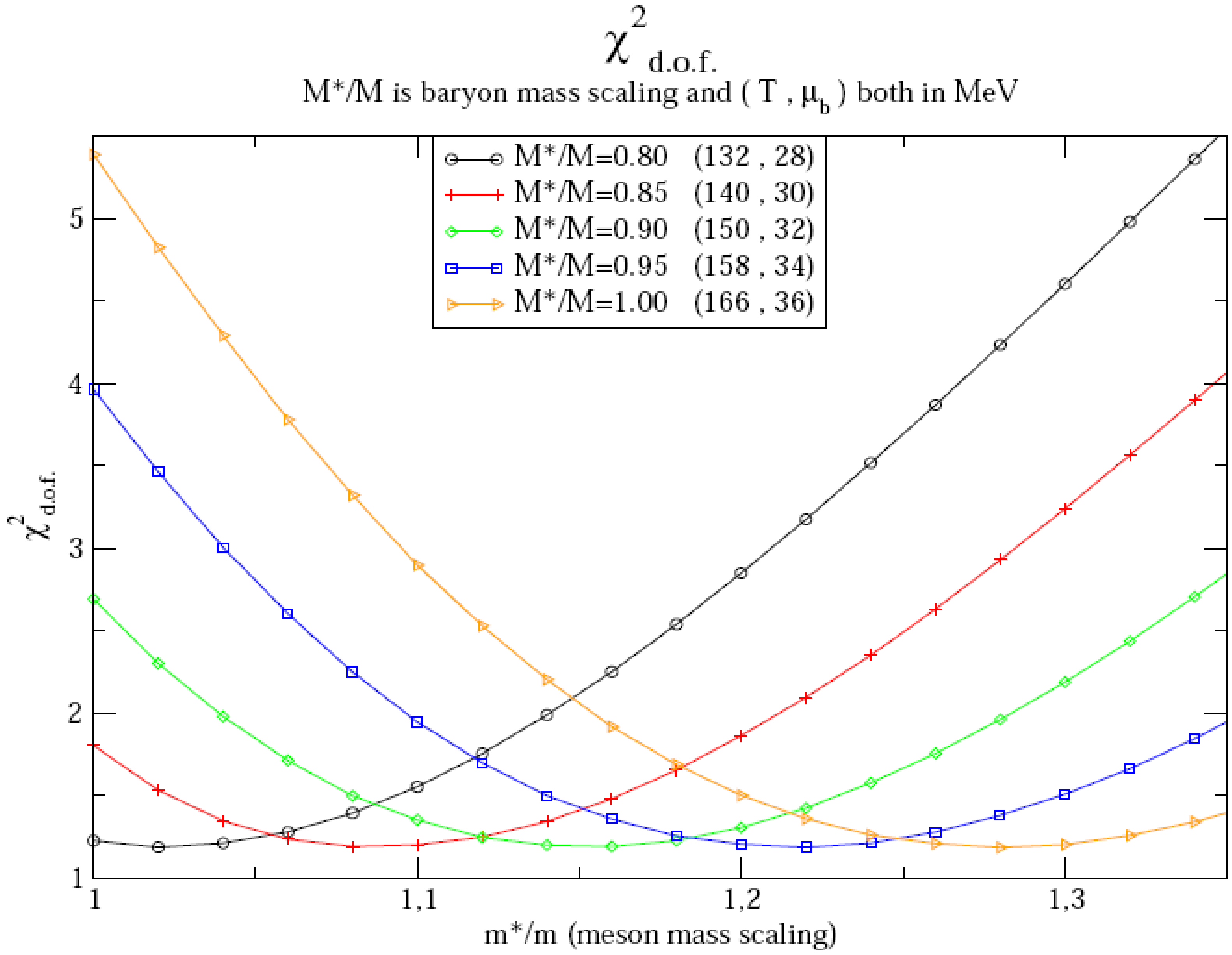
- How would work thermal models themselves without excludent volume but with $M \rightarrow M^*$ and $m \rightarrow m^*$?

Au-Au collision

- Inspired from the SU(3) σ–ω model we treat the collision without excluded volume but let the hadronic masses to vary.
- Our particles in the model are the simple multiplets. Baryonic (octet+decuplet) and the pseudo-scalar and vector meson nonets.
- The fitting of experimental hadronic ratios are presented in terms of χ^2.

$$\chi^2 = \sum_i \frac{\left(\mathscr{R}_i^{\text{exp.}} - \mathscr{R}_i^{\text{model}}\right)^2}{\sigma_i^{\,2}}$$

Mass scaling fittings



Ratio	Set1	Set2	Set3	Set4	Th**	Exp. Data	Exp.
$\bar{p}/p$	0.719	0.653	0.655	0.658	0.629	0.65±0.07	STAR
						0.64±0.07	PHENIX
						0.60±0.07	PHOBOS
						0.64±0.07	BRAHMS
$\bar{p}/\pi^-$	0.039	0.077	0.078	0.079	0.078	0.08±0.01	STAR
π^-/π^+	1.006	1.015	1.015	1.015	1.007	1.00±0.02	PHOBOS
						0.95±0.06	BRAHMS
K^-/K^+	0.959	0.872	0.871	0.871	0.894	0.88±0.05	STAR
						0.78±0.13	PHENIX
						0.91±0.09	PHOBOS
						0.89±0.07	BRAHMS
K^-/π^-	0.231	0.177	0.177	0.176	0.145	0.149±0.02	STAR
K^{*0}/h^-	0.055	0.034	0.033	0.033	0.037	0.06±0.017	STAR
$\bar{K^{*0}}/h^-$	0.052	0.029	0.029	0.029	0.032	0.058±0.017	STAR
$\bar{\Lambda}/\Lambda$	0.749	0.748	0.751	0.754	0.753	0.77±0.07	STAR
Ξ^+/Ξ^-	0.781	0.858	0.861	0.864	0.894	0.82±0.08	STAR
T (MeV)	144	166	158	150	174		
μ_B (MeV)	24	36	34	32	46		
μ_S (MeV)	3.17	11.91	11.35	10.78			
R_{FB} (fm)	26.3	16.3	17.1	18.0			
E/ρ	0.72	0.91	0.87	0.82			
χ^2_{dof}	5.40	1.185	1.186	1.191	0.81		

Table 3: Comparison of experimental particle ratios with the ones obtained from the different masses scalings: M^*/M for baryons and m^*/m for mesons. Respectively: Set 1 shows (1.00,1.00), set 2 shows (1.00,1.28), set 3 shows (0.95,1.22) and set 4 shows (0.90,1.16). Experimental values of Au-Au collisions at $\sqrt{s} = 130$ MeV. Th**: from P. Braun-Munziger *et al*, PLB 518 (2001) 41.

Thermodynamical results

M^*/M	m^*/m	χ^2_{dof}	T	μ_B	μ_S	ε/P	ε/T^4	s/T^3	ε/ρ	R_{FB}	ρ
1.00	1.00	5.400	144	24	3.2	5.05	2.49	2.96	0.72	26.3	0.19
1.10	1.41	1.184	182	39	16.0	5.38	2.55	3.00	0.99	15.0	0.37
1.05	1.35	1.188	174	38	12.6	5.38	2.55	3.00	0.95	15.6	0.32
1.00	1.28	1.185	166	36	11.9	5.38	2.58	3.03	0.91	16.3	0.28
0.95	1.22	1.186	158	34	11.4	5.39	2.58	3.03	0.87	17.1	0.24
0.90	1.16	1.191	150	32	10.8	5.40	2.58	3.04	0.82	18.0	0.21
0.85	1.08	1.192	140	30	9.7	5.37	2.54	3.00	0.76	19.7	0.17
0.80	1.02	1.187	132	28	9.1	5.37	2.54	3.00	0.72	20.9	0.14
C1		0.78	171	48	11.1	6.43	6.27		1053		0.66
C2		0.77	153	51	9.4	6.39	4.53		931		0.35
$\frac{3}{2}kT$		3.86	166	30	5.5	6.13	1.58		1033		0.15

Table 4: Some thermodynamics quantities obtained by own model. T, μ_B, μ_S are given in MeV. R_{FB} is given in fm. ρ is given in fm^{-3}. ε, P, s are energy density, pressure and entropy density. C1, C2 are taken from: PLB 547 (2002) 7.

Conclusions

- In order to obtain a good fit to the ratio data, the hadronic in medium effects are important.
- In the hadronic masses scaling we studied, curiously, $(m^*/m)/(M^*/M)$ is ~ 1.28 for all the best Au-Au hadronic ratio fittings.
- Only good ratio fittings, however, are not enough to select different models.
- Notice that, different values of m^*/m and M^*/M with good ratio fittings left us with a considerable uncertainty regarding the temperature (182-132 MeV), 38%, and chemical potential (39-28 MeV), 39%.

Conclusions

- In particular, how near or how far from the QGP transition is the collision studied? From the model dependence we found it is difficult to answer.

- ε/P, ε/T^4 and s/T^3 are almost constant in our results, suggesting another scale besides the $(m^*/m)/(M^*/M)$ one.

- More experimental data, particularly regarding the fireball radius, total density, and eventually thermodynamics quantities should be important to select among different models.

- Others heavy-ion A-A collision studies would help to construct $(m^*/m)/(M^*/M)$ as a function of density and temperature.