Deep Inelastic Scattering in

Holographic AdS/QCD Models

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String Theory $\times$ Strong Interactions

Initial motivations for string theory:

- **Hadronic spectrum (Regge trajectories)**
- **Scattering amplitudes (Veneziano)**
- ...

New facts:

- **Gauge/String duality.** **Some non-perturbative aspects of QCD can be studied using the AdS/CFT correspondence: J. Maldacena, 98: Equivalence between string theory in $AdS_5 \times S^5$ space and Superconformal gauge theories SU(N) with large N on the corresponding four dimensional boundary.**

- **AdS/QCD: Phenomenological QCD like models based on AdS/CFT**

Hadronic masses from AdS/QCD (hard wall):

- ### Glueball masses

H. Boschi-Filho and N.B. , JHEP 2003, EPJC 2004

Scalar states in an AdS slice dual to scalar glueballs $J^{PC} = 0^{++}, 0^{++*}, 0^{++**}, \ldots$ with masses μ_i. The lightest mass is related to the size of the slice.

Considering an approximate gauge/string duality we found glueball masses related to zeros of Bessel functions

$$\frac{\mu_i}{\mu_1} = \frac{\chi_{2\,,\,i}}{\chi_{2\,,\,1}}$$

Results consistent with lattice data available.

- ### Spectrum of light baryons and mesons.

S. J. Brodsky, G. F. de Teramond PRL 2005 , PRL2006

QCD states with different spins dual to AdS solutions with different masses.
Very nice fit of Regge trajectories

Masses of glueball states with higher spins from AdS/QCD and comparison with Pomerons, H. Boschi-Filho, N. B. and H. L. Carrion, PRD 2006.

MOTIVATION: **Regge trajectories for Pomerons (Landshoff hep-ph/0108156)**

$$J \equiv \alpha_0 + \alpha' M^2 \approx 1.08 + 0.25\, M^2 \qquad (GeV)$$

(for baryons and mesons $J \approx 0.5 + (0.9) M^2$).

Pomerons may be related to glueballs. Recent lattice results are consistent with this interpretation (slope and intercept) H. B. Meyer and M. J. Teper, PLB 2005

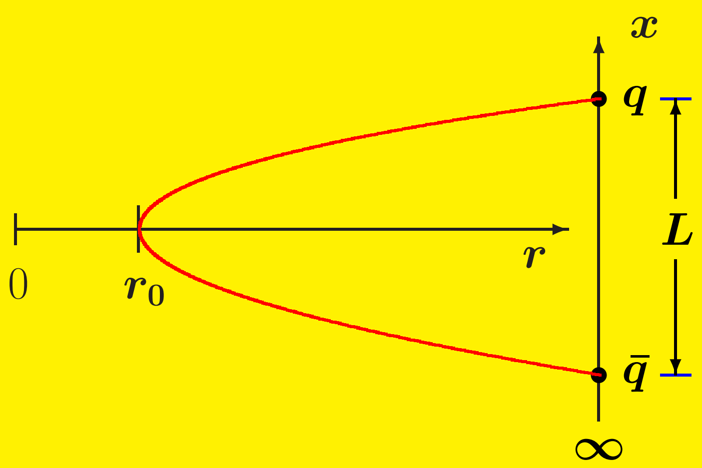
For Neumann boundary conditions in the hard wall we found a linear fit compatible with the Pomeron trajectory:

$$\alpha' = (0.26 \pm 0.02) GeV^{-2} \qquad ; \qquad \alpha_0 = 0.80 \pm 0.40$$

Wilson Loops and "quark anti-quark" potential in AdS/CFT

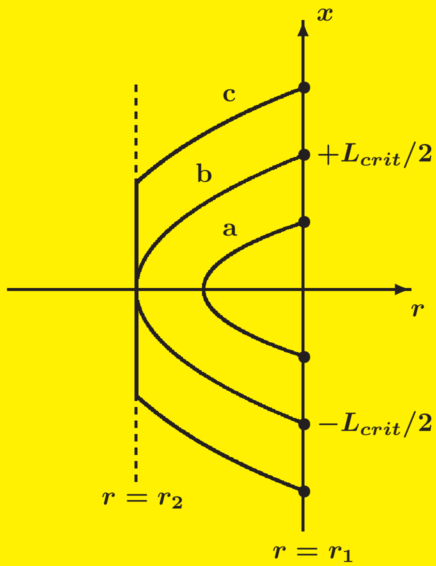
S.J..Rey and J. T. Yee EPJC 2001; J.M.Maldacena PRL 1998.

String connecting a heavy quark anti-quark stationary pair on the boundary of AdS space separated by a coordinate distance L.



Energy: (Coulomb like Potential $\Rightarrow$ nonconfining).

$$E = -\frac{4\pi^2 R^2}{\Gamma(1/4)^4} \frac{1}{L}$$

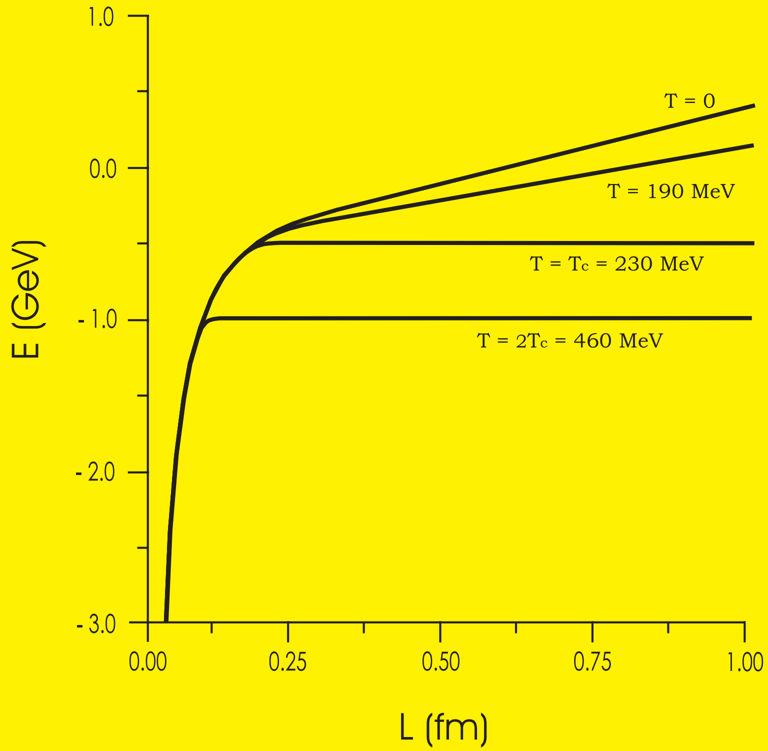


• Quark anti-quark potential from AdS/QCD (zero temperature)

H. Boschi-Filho, N.B. and C. N. Ferreira, PRD 2006.

Placing an infrared brane in AdS we found a potential with asymptotic behaviour similar to the Phenomenological Cornell potential for a quark anti-quark pair:

$$E_{Cornell}(L) = -\frac{4}{3}\frac{a}{L} + \sigma L + const. \, .$$

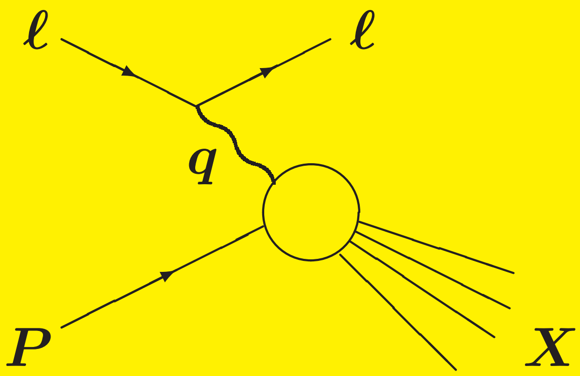


Energy as a function of quark anti-quark distance for different temperatures

- **Quark anti-quark potential at finite temperature from AdS/QCD**

H.Boschi-Filho, N.R.F.Braga and C.N.Ferreira, PRD 2006(2).

Thermal effects in gauge/string duality: black hole in AdS space dual to finite temperature gauge theory



Deep Inelastic Scattering (DIS)

Bjorken parameter: $x \equiv -q^2/2P \cdot q$. DIS: $q^2 \to \infty$, with x fixed.

Inclusive cross section is proportional to the Hadronic tensor

$$W^{\mu\nu} = i \int d^4y \, e^{iq\cdot y} \langle P, \mathcal{Q} | \left[J^\mu(y), J^\nu(0) \right] | P, \mathcal{Q} \rangle ,$$

where $J^\mu(y)$ = hadron current and $\mathcal{Q}$ = electric charge.

Structure functions (for unpolarized scattering)

$$W^{\mu\nu} = F_1(x, q^2)(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2} F_2(x, q^2)(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}) ,$$

DIS cross section is related to the Forward Compton scattering amplitude by Optical theorem ... .

DIS from gauge/string duality in the hard wall model

J. Polchinski and M. Strassler, JHEP 0305, 012 (2003)

Different kinematical regimes, depending on Bjorken parameter x:

- $x > 1/\sqrt{gN}$
- $\exp(-\sqrt{gN}) << x << (gN)^{-1/2}$
- $x < \exp(-\sqrt{gN})$.

In the first case the supergravity approximation is valid while in the other regimes massive string states contribute.

Prescription in the supergravity regime: relation between matrix elements of hadronic current and ten dimensional interaction action. For a scalar particle

$$\begin{aligned}\eta_\mu \langle P_X, X|\tilde{J}^\mu(q)|P, \mathcal{Q}\rangle &= (2\pi)^4\, \delta^4(P_X - P - q)\, \eta_\mu \,\langle P + q, X|J^\mu(0)|P, \mathcal{Q}\rangle \\ &= i\mathcal{Q} \int d^{10}x \sqrt{-g} A^m (\Phi_i \partial_m \Phi_X^* \,-\, \Phi_X^* \partial_m \Phi_i) \;.\end{aligned}$$

($m = AdS_5$ coordinates; $\mu =$ boundary coordinates)

Result

$$F_1(x,q^2) = 0 \;\; ; \;\; F_2(x,q^2) = \pi C_0\, \mathcal{Q}^2 \left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} x^{\Delta+1}\,(1-x)^{\Delta-2}\,,$$

where Δ is the scaling dimension of the scalar states (initial = final) and $\mathcal{Q}$ is the charge.

For fermions they found

$$F_2(x,q^2) = 2F_1(x,q^2) = C\, \mathcal{Q}^2 \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1}\,(1-x)^{\tau-2}\,,$$

where τ = twist.

• Soft Wall Model

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, PRD 2006.

Background involving AdS plus an effective dilaton field (background). The dilaton does not back-react on the metric and plays the role of a smooth infrared cut-off.

Important feature: linear Regge trajectories for scalars.

• DIS in the soft wall model

C. A. Ballon Bayona, H. Boschi-Filho and N.B., JHEP 2008

Inspired in the 5-d soft wall model, we proposed a ten dimensional prescription for the supergravity regime:

$$\begin{aligned}\eta_\mu \langle P_X, X | \tilde{J}^\mu(q) | P, \mathcal{Q} \rangle &= (2\pi)^4 \, \delta^4(P_X - P - q) \, \eta_\mu \, \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle \\ &= i\mathcal{Q} \int d^{10}x \sqrt{-g} \, e^{-\varphi} \, A^m (\Phi_i \partial_m \Phi_X^* - \Phi_X^* \partial_m \Phi_i) \, .\end{aligned}$$

where g_{MN} is $AdS_5 \times W$ but coordinate z has no hard cut off. $\varphi = cz^2$

Important: fields and masses are not the same as in the hard wall

Boundary value of the gauge field represents a virtual photon with polarization η^μ

$$A_\mu(z,y)|_{z\to 0} = \eta_\mu \, e^{iq\cdot y} \,,$$

Solutions:

$$A_\mu(z,y) = \eta_\mu \, e^{iq\cdot y} \, c \, \Gamma(1+\frac{q^2}{4c}) \; z^2 \, \mathcal{U}(1+\frac{q^2}{4c};2;cz^2)$$
$$A_z(z,y) = \frac{i}{2} \eta\cdot q \, e^{iq\cdot y} \; \Gamma(1+\frac{q^2}{4c}) \; z \, \mathcal{U}(1+\frac{q^2}{4c};1;cz^2) \,, \qquad (1)$$

where $\mathcal{U}(a;b;w) =$ conf. hypergeom. functions.

Supergravity approximation is valid when :

$$\tilde{s} < \frac{1}{\alpha'} \qquad \text{that corresponds to} \qquad x >> (gN)^{-1/2}$$

So: structure functions for scalar states in the soft wall model (in the super-gravity regime):

$$F_1 = 0 \; ; \; F_2 = 8\pi^3 \frac{\mathcal{Q}^2}{x} (\Delta - 1) \Gamma(\Delta) (\frac{q^2}{4c})^3 [\frac{\Gamma(\frac{s}{4c} + \frac{\Delta}{2} - 1)}{\Gamma(\frac{s}{4c} - \frac{\Delta}{2} + 1)}] \{\frac{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} - \frac{\Delta}{2})}{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} + \frac{\Delta}{2})}\}^2 .$$

Approximation:

$$F_2 \approx 8\,\pi^3\,\mathcal{Q}^2 (\Delta - 1)\,\Gamma(\Delta)\,(\frac{4c}{q^2})^{\Delta-1} (1-x)^{\Delta-2} x^{\Delta+1} .$$

Equivalent to the hard wall model at leading order.

Comparison of models at leading order. Non trivial compensation of factors:

Sum over final hadronic states:

$$\sum_X \delta(M_X^2 + (P+q)^2) = \frac{1}{4c} \qquad \text{Soft wall.}$$

$$\sum_X \delta(M_X^2 + (P+q)^2) = \frac{1}{2\pi s^{1/2}\Lambda} \qquad \text{Hard wall.}$$

Amplitudes:

$$\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle_{HW} \sim \Lambda^{-1/2} s^{1/4} \langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle_{SW},$$

Remark: in the elastic limit $x \to 1$ our matrix elements of the hadronic currents are consistent with the elastic form factor calculated by

S. J. Brodsky and G. F. de Teramond PRD 2008

In all the previous analysis the initial and final hadrons had the same conformal dimension: $\Delta' = \Delta$.

Deep inelastic scattering when final states have higher conformal dimension: $\Delta' > \Delta$. (hard wall model)

C.A. Ballon Bayona, H. Boschi-Filho, N.B. , JHEP 2008

$\Delta' \neq \Delta$ affects current conservation $\Leftrightarrow$ transversality of scattering amplitude.

In the supergravity regime

$$(2\pi)^4 \, \delta^4(P_X - P - q) \, \eta_\mu \, \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle \; = \; \int d^{10}x \sqrt{-g} A^m \, j_m \, ,$$

where the 5-d scalar current is

$$j_m \; = \; i \, \mathcal{Q} (\Phi_i \partial_m \Phi_X^* \; - \; \Phi_X^* \partial_m \Phi_i)$$

and $A_m = (A_z, A_\mu)$ (Kaluza-Klein gauge field).

For the scalar

$$\frac{1}{\sqrt{-g}} \, \partial_m (\sqrt{-g} \, \partial^m \Phi \,) \; - \; m_5^2 \Phi \; = \; 0 \, .$$

The 5-d mass is related to Δ of the boundary operator dual to Φ:

$$m_5^2 = \frac{\Delta(\Delta - 4)}{R^2}.$$

When $\Delta' > \Delta$ the current is not conserved

$$\frac{1}{\sqrt{-g}} \partial_m(\sqrt{-g}\, j^m) = \frac{i\mathcal{Q}}{R^2} \Phi_i \Phi_X^* [\Delta'(\Delta' - 4) - \Delta(\Delta - 4)] \neq 0 \,.$$

This would lead to non transverse $T^{\mu\nu}$

Phenomenological approach: modified five dimensional hadronic current:

$$\tilde{j}_\mu \equiv j_\mu - i \frac{(P_X - P)_\mu}{(P_X - P)^2} \frac{R^2}{z^2} \frac{1}{\sqrt{-g}} \partial_n(\sqrt{-g}\, j^n).$$

This current leads to a transverse $T^{\mu\nu}$.

New prescription for the four dimensional current matrix element:

$$(2\pi)^4 \delta^4(P_X - P - q)\, \eta_\mu \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle = \int d^{10}x \sqrt{-g} A^m \tilde{j}_m \,.$$

Contributions to the structure functions of final hadrons with Δ'

$$
\begin{aligned}
F_1^{\Delta'}(x,q^2) &= 0 \\
F_2^{\Delta'}(x,q^2) &= \pi^2\, 2^{2\Delta} |C_i|^2 |C_X|^2 \mathcal{Q}^2 (\frac{\Lambda^2}{q^2})^{\Delta-1} x^{1-\Delta'} (1-x)^{\Delta'-2} [\frac{\Gamma(\frac{\Delta'+\Delta}{2})\Gamma(\frac{\Delta'+\Delta}{2}-1)}{\Gamma(\Delta'-1)}]^2 \\
&\times \; [F(\frac{\Delta'+\Delta}{2}, \frac{\Delta'+\Delta}{2}-1; \Delta'-1; -\frac{1-x}{x})]^2 \,,
\end{aligned}
$$

where $F(a,b\,;c\,;\omega)$ is the Gauss hypergeometric function.

The structure functions involve the sum over all possible values of Δ' (inclusive cross section)

$$
F_2(x,q^2) = F_2^{\Delta'=\Delta}(x,q^2) + \sum_{\Delta'>\Delta} F_2^{\Delta'}(x,q^2)\,.
$$

Conformal dimension Δ $\Leftrightarrow$ minimum number of constituents of the state.

The minimum number of constituents of a hadron is 2.

So, $\rho \equiv (\Delta' - \Delta)/2$ represents the number of extra hadrons (mesons) that could be present in the "final state".

All terms can be related to the $\rho = 0$ ($\Delta' = \Delta$) case:

$\rho = 1$ case ("one extra meson") is simple:

$$F_2^{(\rho=1)}(x, q^2) = F_2^{(\rho=0)}(x, q^2)\, x^{-2}(1-x)^2 \,.$$

$\rho = 2$ (maximum of 2 "extra mesons"):

$$F_2^{(\rho=2)}(x, q^2) = F_2^{(\rho=0)}(x, q^2)\, \Delta^2\, (1-x)^{-2\Delta}\, \mathcal{S}^{\,2}$$

where

$$\mathcal{S} \equiv -1 + \frac{1}{x} + (\Delta + 1)\ln x + \sum_{n=2}^{\Delta+1} \binom{\Delta+1}{n} \frac{(-1)^n}{(n-1)}[1 - x^{n+1}]\,,$$

The next terms: $\rho \geq 3$ can be calculated in a similar way.

When should these corrections with $\Delta' > \Delta$ be important?

• When the center of mass energy $\sqrt{s}$ is much larger than the hadronic mass scale we expect more hadrons (more constituents) in the final state.

$s \sim q^2/x$ **So, this corresponds to small x.**

Estimate of maximum number of hadrons in final state (mass Λ).

$$N_{max} \approx \frac{\sqrt{s}}{\Lambda} \approx (\frac{q^2}{x\Lambda^2})^{1/2}.$$

This places the limit:

$$0 \le \rho \le (N_{max} - 1).$$

So, we have a sum over a finite number of terms that leads to

$$F_2(x, q^2) \approx \pi C_0 \, \mathcal{Q}^2 \left(\frac{q^2}{\Lambda^2}\right)^{1/2} x^{-1/2}.$$

Comparison with geometric scaling.

For scattering at small x ($x < 0.01$) the observed total cross section depends on x and q^2 through the variable:

$$\mathcal{T} = q^2 x^\lambda / q_0^2 \, x_0^\lambda$$

where $q_0 = 1GeV$ and $x_0 = 3 \times 10^{-4}$ and $0.3 < \lambda < 0.4$

A. M. Stasto, K. J. Golec-Biernat and J. Kwiecinski, PRL 2001

E. Iancu, K. Itakura and L. McLerran, NPA 2002

...

In terms of the structure function F_2, geometric scaling means that

$$\sigma(q^2, x) = 4\pi^2 \alpha_{EM} \frac{F_2(x, q^2)}{q^2}.$$

depends just on $\mathcal{T} = q^2 x^\lambda / q_0^2 \, x_0^\lambda$ with $0.3 < \lambda < 0.4$.

For $\Delta' = \Delta$ (initial and final hadrons with the same dimension) in supergravity at small x

$$\frac{F_2^{\Delta'=\Delta}(x,q^2)}{q^2} \sim (q^2\, x^{\lambda})^{-\Delta}\,,$$

with $\lambda = -\frac{\Delta+1}{\Delta}$??

While our new result, adding up all allowed Δ' implies (at small x)

$$\frac{F_2^{\Delta'=\Delta}(x,q^2)}{q^2} \sim (q^2 x^{\lambda})^{-1/2}\,,$$

with $\lambda = 1$ So: scaling similar to geometric scaling.

Deep Inelastic Scattering in D3-D7 scenario

C. A, Ballon Bayona, H. Boschi-Filho and N.B., JHEP 2008

D3-D7 system consists of $AdS_5 \times S^5$ space with addition of N_f coincident D7 brane probes. This corresponds to adding flavour to AdS/CFT

$AdS_5 \times S^5$ space:

$$ds^2_{10} \equiv g_{MN}\, dx^M dx^N = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}[dr^2 + r^2\,[d\tilde{\theta}_1{}^2 + \sin^2\tilde{\theta}_1 d\tilde{\theta}_2{}^2$$
$$+ \sin^2\tilde{\theta}_1 \sin^2\tilde{\theta}_2(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2 d\varphi^2)]]\,.$$

$r = R^2/z$.

Cilyndrical coordinates

$$\rho \equiv r\sin\tilde{\theta}_1 \sin\tilde{\theta}_2\,,\; w_5 \equiv r\sin\tilde{\theta}_1\cos\tilde{\theta}_2\,,\; w_6 \equiv r\cos\tilde{\theta}_1\;,$$

the metric is rewritten as

$$ds^2_{10} = \frac{\rho^2 + w_5^2 + w_6^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + w_5^2 + w_6^2}[dw_5^2 + dw_6^2 + d\rho^2 + \rho^2 d\Omega_3^2]\;,$$

where $d\Omega_3^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\varphi^2$.

Localization of the D7 branes: $w_5 = 0$, $w_6 = L$. Induced metric:

$$ds_8^2 = G_{ab} dx^a dx^b = \frac{\rho^2 + L^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{\rho^2 + L^2} (d\rho^2 + \rho^2 d\Omega_3^2,) \, . \qquad (2)$$

Note: $\rho^2 + L^2 = r^2$ so on the brane $L \leq r < \infty$.

This corresponds to an induced infrared cut off: $m_h = L/R^2$.

Scalar mesons: fluctuations of the D7 branes in the transversal directions w_5, w_6.

Perturbing the D-brane action we find the field solutions. ...

Initial and final hadronic solutions are respectively

$$\Phi_i = C_{0,\ell} e^{ip\cdot y} \mathcal{Y}^\ell(\Omega) \frac{\rho^\ell}{(\rho^2 + L^2)^{\ell+1}}$$

$$\Phi_X = C_{n,\ell} e^{ip\cdot y} \mathcal{Y}^\ell(\Omega) \frac{\rho^\ell}{(\rho^2 + L^2)^{n+\ell+1}} F(-n-\ell-1, -n; \ell+2; -\frac{\rho^2}{L^2}) \,. \qquad (3)$$

Sum over the masses of final states for the D3-D7 model:

$$\sum_X \delta(M_X^2 + (P+q)^2) \approx \frac{1}{4m_h^2(2n+2\ell+3)} = \frac{1}{4m_h^2\sqrt{s+m_h^2}} \,. \qquad (4)$$

Gauge field in the AdS bulk

$$S_{int} = 2\mu_7(R\pi\alpha')^2 \int d^8x \sqrt{-G} \, \frac{v^\alpha A^{\tilde{m}}}{\rho^2 + L^2} (\partial_\alpha \Phi^* \partial_{\tilde{m}} \Phi + \partial_{\tilde{m}} \Phi^* \partial_\alpha \Phi) \,. \qquad (5)$$

$$F_1 = 0 \; ; \; F_2 = k_7 \frac{\Gamma(2\ell+4)}{\Gamma^4(\ell+2)} \frac{\Gamma(n+2\ell+3)}{\Gamma(n+1)} \left(\frac{q^2}{m_h^2}\right)^2 \frac{\mathcal{I}_{bulk}^2}{x} \qquad (6)$$

where $k_7 = 32\pi^7 \mathcal{Q}^2 L^4 \alpha'^4 \mu_7^2$.

Gauge field on the D7 brane

$$\langle S_{int} \rangle \,=\, 2i\,\mathcal{Q}\,(R\pi\alpha')^2\,\mu_7 \int d^8x\sqrt{-G}\,G^{\mu\nu}A_\mu\frac{1}{\rho^2+L^2}\,(\Phi_i\partial_\nu\Phi_X^* \,-\, \Phi_X^*\partial_\nu\Phi_i\,)\,, \tag{7}$$

Structure functions

$$F_1 = 0 \;;\;\; F_2 = \bar{k}_7\,\frac{\Gamma(2\ell+4)}{\Gamma^4(\ell+2)}\frac{\Gamma(n+2\ell+3)}{\Gamma(n+1)}\left(\frac{q^2}{4m_h^2}\right)^3\frac{1}{\cosh^2\left(\pi\sqrt{\frac{q^2}{4m_h^2}-\frac{1}{4}}\;\right)}\frac{\mathcal{I}^2_{brane}}{x}\,, \tag{8}$$

where $\bar{k}_7 = 32\pi^9\mathcal{Q}^2L^4\alpha'^4\mu_7^2\,.$

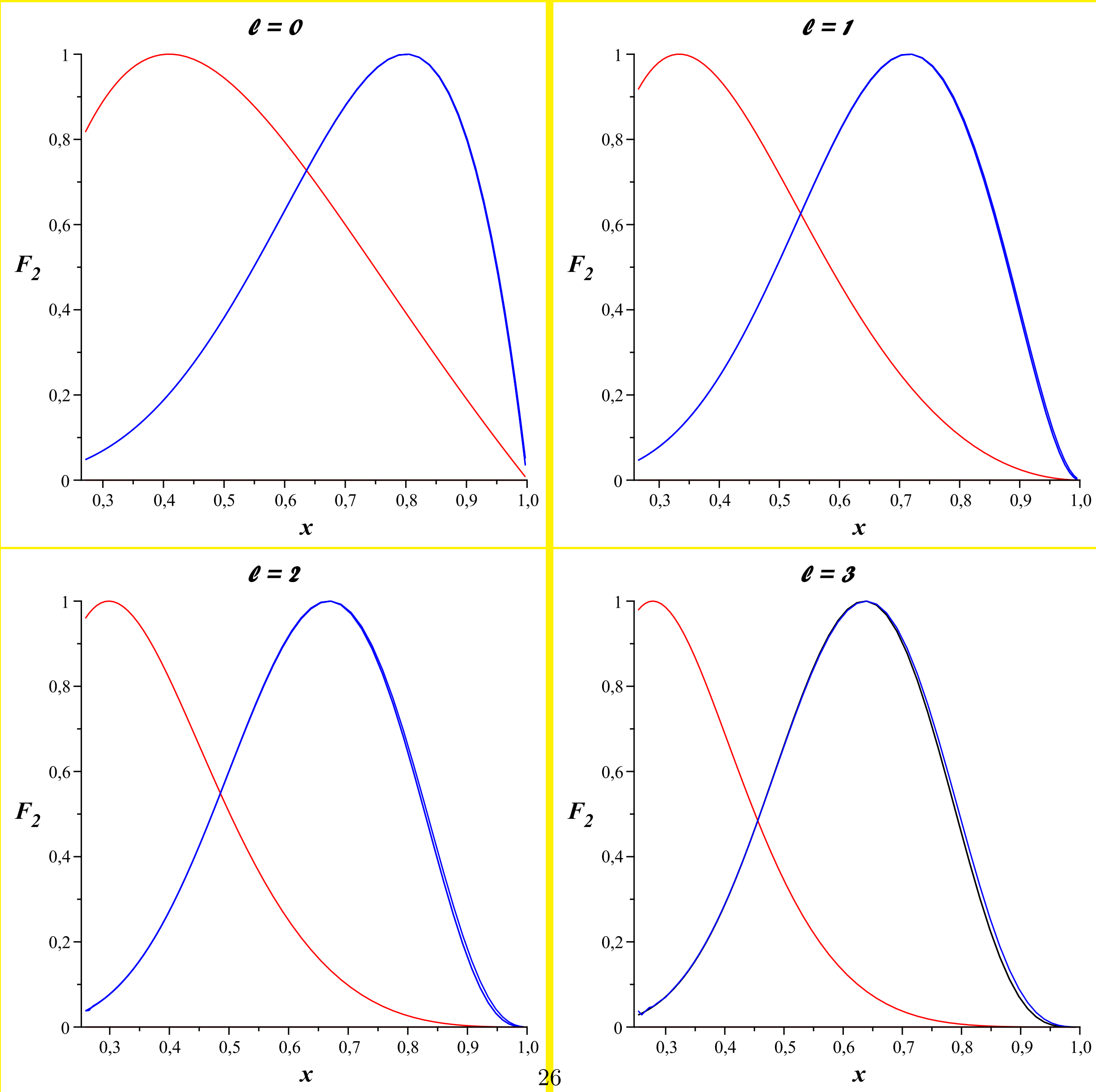


Figure 1: **The structure functions F_2 versus the Bjorken variable x for $\ell = 0, 1, 2, 3$ and $Q = 75$. The red lines represent the brane results. The blue lines represent the other models.**

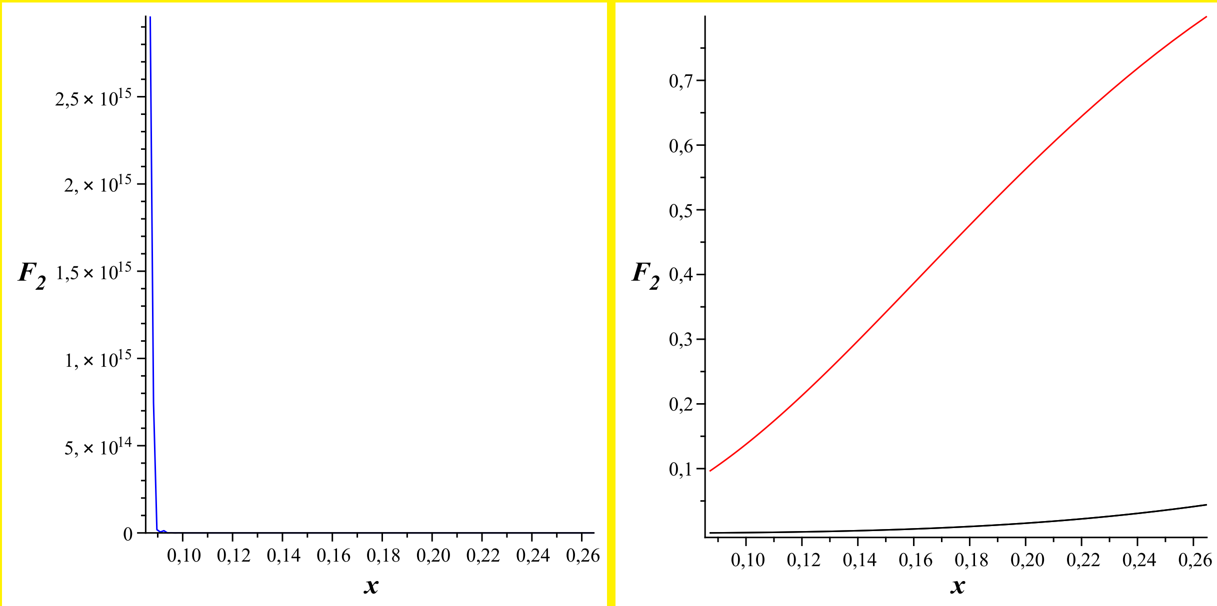


Figure 2: **The structure functions F_2 at small x for $\ell = 0$ and $Q = 75$. The first plot corresponds to the bulk case (blue). In the second plot the red line represents the brane structure function, while the black line corresponds to the hard and soft wall models.**

Singular behaviour of the structure function for small x when the gauge field is in the bulk

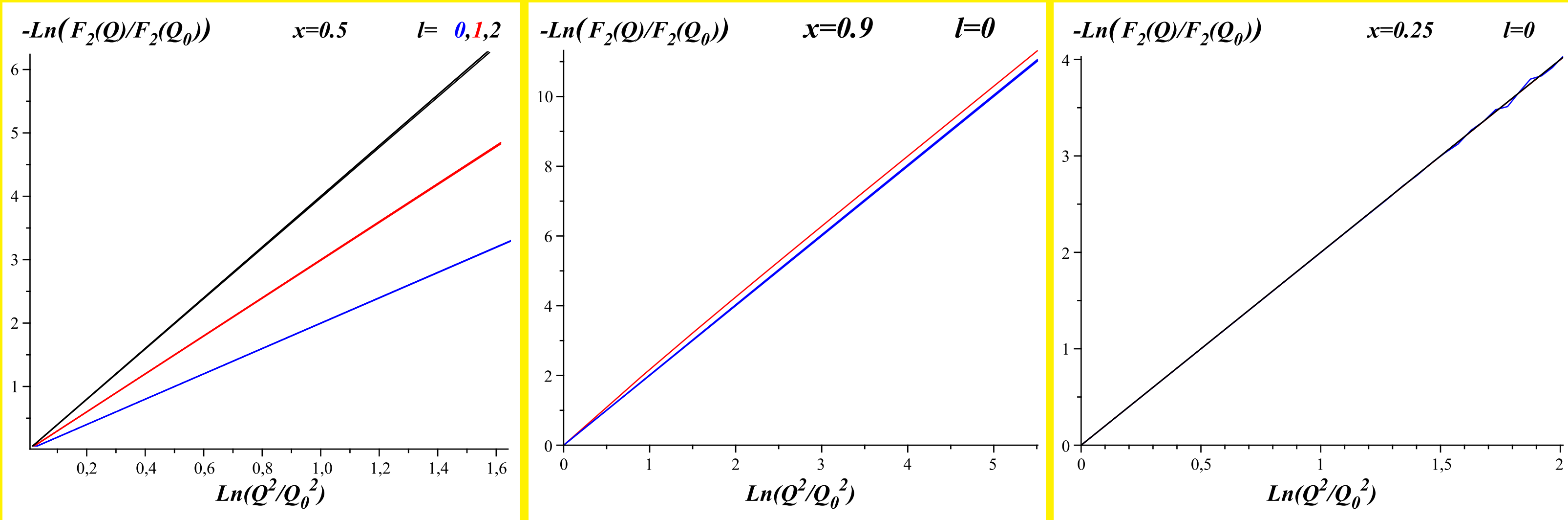


Figure 3: $\boldsymbol{Q^2}$ **dependence of the structure functions $\boldsymbol{F_2}$ for the four models considered. In the first plot, with $\boldsymbol{x = 0.5}$, $\boldsymbol{Q_0 = 45}$, we have $\boldsymbol{\ell = 0}$ ((blue line), $\boldsymbol{\ell = 1}$ (red line) and $\boldsymbol{\ell = 2}$ (black line). Each line in this plot represents the coinciding results for the four models. In the second plot $\boldsymbol{x = 0.9}$ $\boldsymbol{\ell = 0}$ and the red line represents the brane case while the blue line represents the other three models. In the third plot, with $\boldsymbol{x = 0.25}$ and $\boldsymbol{\ell = 0}$, all the structure functions coincide except for the bulk case that oscillates for higher values of $\boldsymbol{Q}$.**