A covariant investigation of Vector Mesons: dynamical properties and EM decays

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Motivations & Background

★ This investigation is a part of our study
i) for characterizing the 4D Bethe-Salpeter amplitude of mesons, by exploiting the comparison with available experimental data, in view of new data in the close future and
ii) for developing a more microscopic, but covariant approach, e.g. by exploiting more deeply the information inside the valence w.f. through the 4D mapping on the 3D LF plane and viceversa

$$\text{4D Bethe} - \text{Salpeter Eq.} \qquad\qquad \text{3D valence Eq.}$$

$$G^{-1}(K)|\Psi\rangle = 0 \qquad \longrightarrow \qquad g^{-1}(K)|\phi\rangle = 0$$

$$\downarrow G_0^{on}(K)\; G_0^{-1}(K)\;\; |\Psi\rangle = \;\; |\phi\rangle \quad \text{and} \quad |\Psi\rangle = \Pi(\mathrm{K})\;\; |\phi\rangle$$

(see, e.g. Sales et al PRC **61** (2000) and **63** (2001); Marinho et al PRD **77** 2008) $\longrightarrow$ perturbation theory for both $\Pi(K)$ and the em current operator.

★★ The main ingredients of our *phenomenological* approach are i) an Ansatz for the 4D Bethe-Salpeter vertex for Vector Mesons, having a Dirac structure and a momentum-dependent part, and ii) a Mandelstam-like formula for the electromagnetic decay constant (to accomplish a first check of our model). Notice that the *phenomenological* approach allows us to effectively jump back from 3D to 4D, avoiding (for the present time) the reconstruction of the projecting operators.

★★★ The free parameters that appears in our Ansatz of the 4D BS amplitude are fixed by comparing the valence transverse-momentum distribution (VTDM), defined through the (3D) valence wave function (obtained from our (4D) Ansatz through the exact procedure), with the one obtained within a Light-Front Hamiltonian Dynamics (LFHD) framework with constituent quarks, after solving and eigenvalue problem : this is our present dynamical input in the 3D world.

Summarizing, we avoid the reconstruction of the 4D BS amplitude through a whole knowledge of the projector $\Pi(K)$, by using a fitting procedure involving the valence w.f..

Indeed, one should first construct the 3D Green function for the given 4D Kernel, $g^{-1}(K) = g_0^{-1}(K) - w(K)$ that allows to calculate the valence w.f.. This quantity is approximate by taking a phenomenological relativistic square mass operator, that allows a nice description of the VM spectra both in the light and heavy sector (e.g. Salcedo et al EPJA **27** (2006))

★★★★ At the present stage, the comparison with experimental data is restricted to the electromagnetic width of the Vector Mesons:

$$\Gamma_{e^+e^-} = \frac{8\pi\alpha^2}{3}\frac{|f_V|^2}{M_V^3}$$

A Covariant Approach for Vector Mesons

A simple analytic form for the 4D Bethe-Salpeter (BS) amplitude of an interacting $q\bar{q}$ pair with total angular momentum $J = 1$ is adopted

$$\Psi_\lambda(k,P) = S(k,m_1)\ [\epsilon_\lambda(P) \cdot V(P)]\ \Lambda_{VM}(k,k-P)\ S(k-P,m_2)$$

$S(k,m_1) \equiv$ Dirac propagator of a constituent with mass m_1,

$$S(k,m_1) = \frac{\not{k}_{on} + m_1}{k^2 - m_1^2 + i\epsilon} + (k^- - k^-_{on})\frac{\gamma^+}{2k^+} =$$

$$= S_{on}(k,m_1) + (k^- - k^-_{on})\frac{\gamma^+}{2k^+}$$

with $k^2_{on} = m_1^2$, (and analogous decomposition can be applied to $S(k-P,m_2)$).

$P^\mu \equiv$ four-momentum of a VM with mass $P^2 = M^2$ and helicity λ

$\epsilon^\mu_\lambda(P) \equiv$ its polarization four-vector, remind $\epsilon_\lambda(P) \cdot P = 0$

$V^\mu(k,k-P) \equiv$ Dirac structure of the BS amplitude

$\Lambda_{VM}(k,k-P)$ is the momentum dependence of the BS amplitude, that acts as a Pauli-Villars regulator.

The Dirac structure is the familiar one (transverse to P^μ), viz

$$V^\mu(P) = \frac{M}{M+m_1+m_2}\left[\gamma^\mu - \frac{P^\mu \not{P}}{M^2} + \imath \frac{1}{M}\,\sigma^{\mu\nu} P_\nu\right] \rightarrow$$

$$\epsilon_\lambda(P)\cdot V(P) = \frac{M}{M+m_1+m_2}\,\not{\epsilon}_\lambda\left[1 - \frac{\not{P}}{M}\right]$$

Comments:

- in principle one should have two different $\Lambda_{VM}(k, k-P)$'s, one for each Dirac structure appearing in $\epsilon_\lambda(P)\cdot V(P)$, but we assumed such a simplifying form since .....
- ....the adopted form for the Dirac structure leads to the expected Melosh Rotation factor for a ${}^3S_1$ system, in the limit of non interacting systems, i.e. $M \rightarrow M_{free}$ or $P^\mu \rightarrow P^\mu_{free}$.

For the present preliminary calculations, the momentum dependence has the following simple form: only single poles $\rightarrow$

$$\Lambda_{VM}(k,k-P) = \mathcal{N}\ \left[k^2 - m_1^2 + (P-k)^2 - m_2^2\right]\ \times$$
$$\Pi_{i=1,3} \frac{1}{\left[k^2 - m_{R_i}^2 + \imath\epsilon\right]\left[(P-k)^2 - m_{R_i}^2 + \imath\epsilon\right]}$$

m_{R_i}, $i = 1,2,3$ are the free parameters of our Ansatz, to be determined by comparing the valence transverse-momentum distribution (TDM) to a TDM, calculated in a phenomenological model that reproduces the VM spectrum

$\mathcal{N}$ is the normalization factor, that can be derived by imposing the standard normalization for the Bethe-Salpeter amplitude, but in Impulse Approximation (namely with free propagators for the constituents)

$$2P^+ =\ N_c \int \frac{d^4k}{(2\pi)^4}\ \bar{\Lambda}_{VM}(k,P)\ \mathbf{T}^+_{\lambda,\lambda}(P,P,k)\ \Lambda_{VM}(k,P) \times$$
$$\frac{1}{\left[(k-P)^2 - m_2^2 + \imath\epsilon\right]}\frac{1}{\left[(k-P)^2 - m_2^2 + \imath\epsilon\right]}\frac{1}{(k^2 - m_1^2 + \imath\epsilon)}$$

with

$$\mathbf{T}^+_{\lambda,\lambda}(P,P,k) = Tr\left[\epsilon_\lambda(P)\ \cdot V(k,k-P)\ (\not k - \not P + m_2)\gamma^+ \times\right.$$
$$\left.(\not k - \not P + m_2)\ \epsilon^*_\lambda(P) \cdot \bar{V}(k,k-P)\ (\not k + m_1)\right] =$$
$$= -2(P^+ - k^+)\left[\mathcal{T}^+(P,P,k,on) + \frac{(k^- - k^-_{(1)})}{2}\mathcal{T}^+(P,P,k,ist)\right]$$

and $N_c = 3$

note the cancellation of the dependence upon the helicity, as it should do

Comments:

The form chosen for $\Lambda_{VM}(k, k-P)$ allows one

- to implement the correct symmetry under the exchange of the quark momenta (for equal mass constituents)
- to regularize the integrals present in our approach (valence wave function, decay constant, TDM, ...)
- to avoid any free propagation of the constituents due to the numerator in $\Lambda_{VM}(k, k-P)$ that exactly cancels out the free propagation, that appears in the denominator of the valence wave function.

The valence transverse-momentum distribution and the longitudinal distribution

As announced, to determine m_{R_i}, we exploit the valence TDM, through the following three steps

★ Evaluation of the valence wave function

$$\Phi^{val}_{VM}(\xi, \mathbf{k}_\perp; m_{R_i}) = \int dk^- \ S_{on}(k, m_1) \ [\epsilon_\lambda(P) \cdot V(P)] \ \times$$
$$\Lambda_{VM}(k, k-P) \ S_{on}(k-P, m_2)$$

$\xi = k^+/P^+$.

★★ Definition of the valence TDM, $n(k_\perp)$, and the valence probability, $P_{q\bar{q}}$,

$$n_{VM}(k_\perp) = \frac{N_c}{(2\pi)^3 \ [P^+]^2 \ P_{q\bar{q}}} \int_0^{2\pi} d\theta_{k_\perp} \ \times$$
$$\int_0^1 \frac{d\xi}{\xi \ (1-\xi)} \ \mathcal{T}^+(P, P, k, on) \ |\Phi^{val}_{VM}(\xi, \mathbf{k}_\perp; m_{R_i})|^2$$

$$P_{q\bar{q}} = \frac{N_c}{(2\pi)^3 \ [P^+]^2} \int_0^1 \frac{d\xi}{\xi \ (1-\xi)} \ \times$$
$$\int d\mathbf{k}_\perp \ \mathcal{T}^+(P, P, k, on) \ |\Phi^{val}_{VM}(\xi, \mathbf{k}_\perp; m_{R_i})|^2$$

$n_{VM}(k_\perp)$ is normalized as: $\int k_\perp \ dk_\perp \ n_{VM}(k_\perp) = 1$.

★★★ In a Light-front Hamiltonian Dynamics (LFHD) approach in a frame where $\mathbf{P}_\perp = \mathbf{0}$, one has

$$n^{HD}_{VM}(k_\perp) = \int_0^{2\pi} d\theta_{\hat{k}_\perp} \int_0^1 \frac{d\xi \; M_0^2}{\xi(1-\xi)} \; |\psi_{VM}(\xi, \mathbf{k}_\perp)|^2$$

where $\psi_{VM}(\xi, \mathbf{k}_\perp)$ is an eigenfunction of a relativistic squared mass operator. This means that the spectrum of VM's, not only the ground state but also the first four excited states, are reproduced.

The free parameters in Λ_{VM} are obtained by minimizing the difference

$$|n_{VM}(k_\perp) - n^{HD}_{VM}(k_\perp)|$$

To avoid poles in the evaluation of valence TMD we have taken $m_{R_i} \geq Max\{M_V/2, mq\}$

★★★ Definition of the valence longitudinal distribution, $q(x)$,

$$q_V(x) = \frac{N_c}{(2\pi)^3\,[P^+]^2\,P_{q\bar{q}}} \int \frac{d\mathbf{k}_\perp}{x\,(1-x)}\,\mathcal{T}^+(P,P,k,on)\;\times$$
$$|\Phi^{val}_{VM}(\xi,\mathbf{k}_\perp;m_{R_i})|^2$$

normalized as $\int_0^1 dx\; q_V(x) = 1$

The Mandelstam formula for EM decay constant

The EM decays constant of a neutral VM, f_V, is obtained through the transition matrix element of the EM current, viz

$$\langle 0|J^\mu(0)|P,\lambda\rangle = \imath\sqrt{2} f_V \epsilon^\mu_\lambda$$

The EM decay width is

$$\Gamma_{e^+e^-} = \frac{8\pi\alpha^2}{3}\frac{|f_V|^2}{M_V^3}$$

The transition matrix element is *microscopically* approximated *à la* Mandelstam

$$\langle 0|J^\mu(0)|P,\lambda\rangle = \mathcal{F}_{VM}\,\frac{N_c}{(2\pi)^4}\int d^4k\ \times$$

$$\frac{\Lambda_{VM}(k,k-P,m_1,m_2)}{(k^2-m_1^2+\imath\epsilon)\,[(P-k)^2-m_2^2+\imath\epsilon]}\times$$

$$\mathrm{Tr}[\epsilon_\lambda(P)\cdot V(P)\,(\not k-\not P+m_2)\gamma^\mu(\not k+m_1)]$$

where

$$\mathcal{F}_\rho = \frac{(Q_u-Q_d)}{\sqrt{2}} \qquad \mathcal{F}_\phi = Q_s \qquad \mathcal{F}_{J/\Psi} = Q_c$$

with Q_i the quark charge.

The EM decay constants of light and heavy VM's

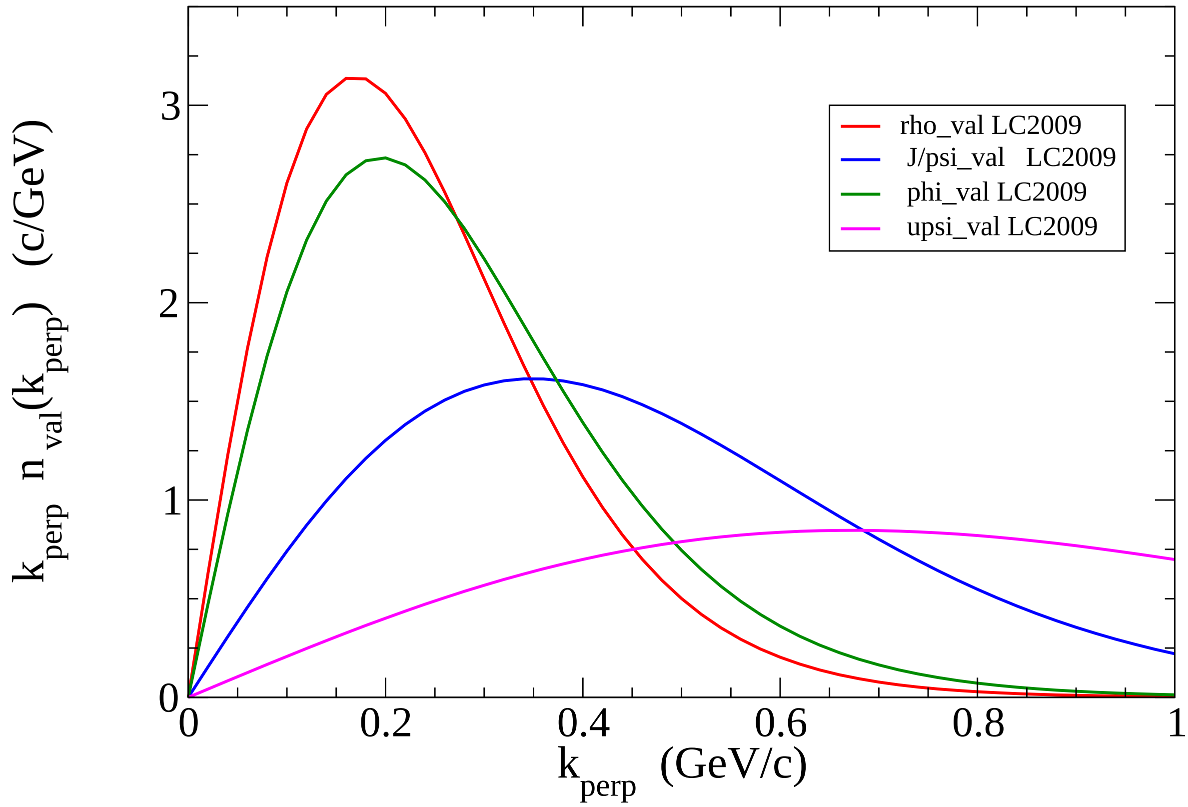
Preliminary results for both valence probability, $P_{q\bar{q}}$, and em decay widths, $\Gamma_{e^+e^-}$, are shown for the adapted version of the ITA model, that contains a Coulomb-like term and a confining one.

Preliminary VM em decay widths and valence probabilities.

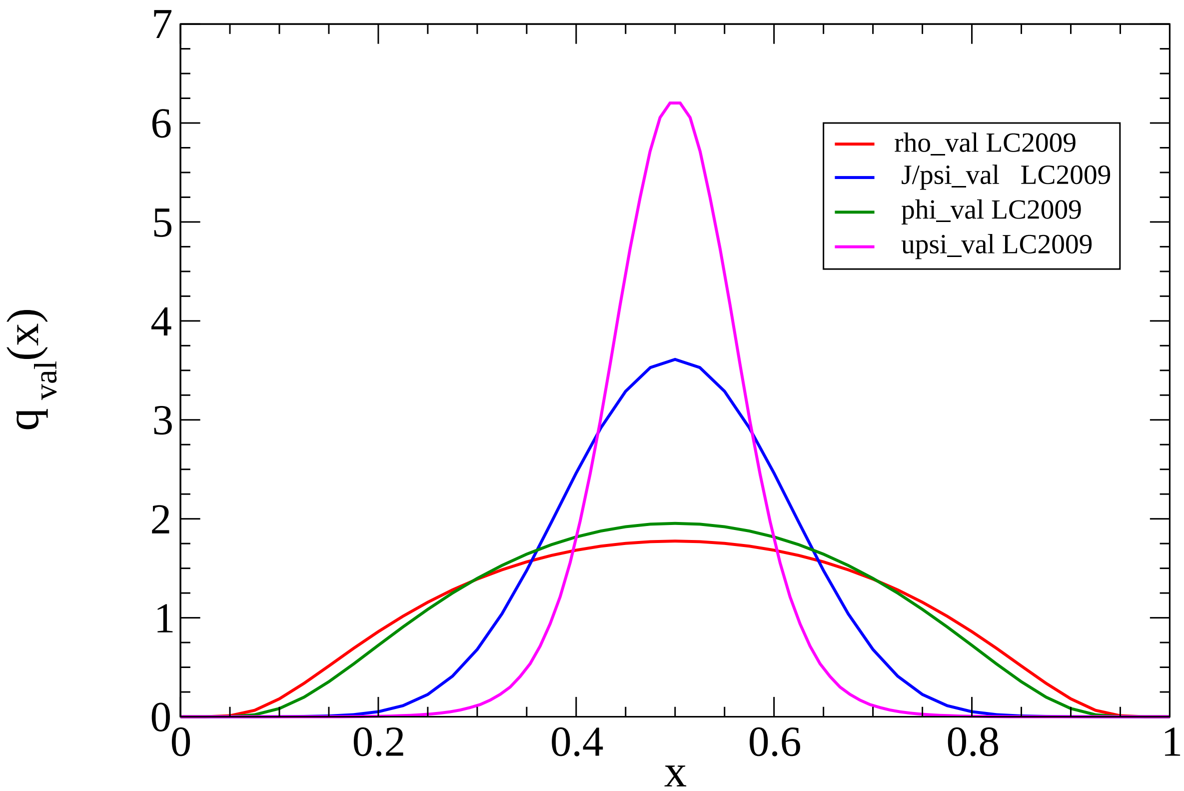
VM	$m_q(MeV)$	M^{th}_{VM} (MeV)	M^{exp}_{VM} (MeV)
ρ	345	769	775.5 ± 0.4
ϕ	460	1010	1019.455 ± 0.020
J/ψ	1583	3070	3096.916 ± 0.011
Υ	5140	9499	9460.30 ± 0.268

Preliminary VM valence probabilities and EM decay widths.

VM	$P_{q\bar{q}}$	$\Gamma^{th}_{e^+e^-}$ (keV)	$\Gamma^{exp}_{e^+e^-}$ (keV) (PDG09)
ρ	1.00	6.40	7.04 ± 0.06
ϕ	0.988	1.396	1.270 ± 0.040
J/ψ	0.722	2.595	5.55 ± 0.14
Υ	0.542	0.383	1.340 ± 0.018



Valence transverse momentum distributions for a constituent inside ρ, ϕ, J/ψ and Υ vs the quark transverse momentum. Solid lines: fits, obtained by using our analytic Ansatz, to the same quantity, evaluated within the ITA model,



Parton distributions from only the $q\bar{q}$ component, for a constituent inside ρ, ϕ, J/ψ and Υ. Solid lines: obtained by using our analytic Ansatz, after applying the fitting procedure to $n_{val}(k_\perp)$,

Preliminary Conclusions & Future Work

- we have started to apply our covariant approach to VM's, by exploiting an Ansatz for the 4D BS amplitude that prevents the free propagation of the constituents
- a dynamical input is included through a fit of the valence transverse distribution, corresponding to our 4D Ansatz, to the transverse-momentum distribution obtained from a 3D eigenfunction of a squared mass operator, reproducing the VM spectra, at large extent.
- A preliminary, but encouraging, calculations of the valence probability (a priori, it was not sure that such quantities were $P_{q\bar{q}} \leq 1$) and the electromagnetic decay widths
- The analysis must be completed by calculating other quantities, like the chiral-even TDM, and even GPD's (as we did for the pion), in order to better investigate the prediction power of our approach
- A more sophisticate $q\bar{q}$ potential should be tested, in order to improve the dynamical input of our model