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Effects of the Chemical Potential in Two-Dimensional Quantum Field Theories

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Outline

- The model
- Finite Temperature Results
- Finite Chemical Potential Results
- Alternative Derivation of the Results
- Imaginary Time Formalism

The Model

- Chemical arise when we have a conserved charge and we are dealing with a grand canonical ensemble, where:

$$H \rightarrow H - \mu Q,$$

- Massless fermions in an Abelian gauge field background

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi + \mu\bar{\psi}\gamma^{0}\psi,$$

- The effective action for the abelian gauge fields is obtained by integrating out the fermion fields in the path integral. At zero temperature, it can be evaluated in closed form and contains only quadratic terms in the gauge fields.

The Model

- This result holds even in the presence of a nonzero chemical potential, which is best seen in the mixed space, where the dependence of the propagator on the chemical potential is given by an overall phase which gives unity around a closed loop.

PRD 72, 085006 (2005)
PRD 73, 065010 (2006)

$$\Delta^{(T=0)}(p_E, E) = \frac{1}{p^2 + m^2} = \frac{1}{p_E^2 + E^2},$$

$$\Delta^{(T=0)}(\tau, E) = \int \frac{dp_E}{2\pi} \frac{e^{-ip_E\tau}}{p_E^2 + E^2}$$

$$\Delta^{(T=0,\mu)}(\tau, E) = e^{\mu\tau}\Delta^{(T=0=\mu)}(\tau, E),$$

Finite Temperature Results

- We work in the real time formalism. In this case, the propagator acquires a 2 x 2 matrix structure. For example, in the closed time path formalism the (++) component of the propagator has the form

$$S(p) = \frac{1}{\not{p}} + 2i\pi \not{p} n(|p^0|)\,\delta(p^2)$$

where n(|p^0|) is the Fermi-Dirac distribution function,

$$n(|p^0|) = \frac{1}{e^{\beta|p^0|}+1}.$$

Finite Temperature Results

- At finite temperature, the effective action does not have a closed form. It has been shown that all the even point functions are nonzero even though at zero temperature the effective action is only quadratic in the background fields. In particular, the nonvanishing (even) *n*-point amplitudes where found to have a very simple structure of the form PRD 59, 105011 (1999)

$$\Gamma^{\mu_1\mu_2\cdots(T)}_{a_1a_2\cdots}(p_1, p_2, \ldots) \propto [\delta(p_{1-})\delta(p_{2-})\cdots \times u^{\mu_1}_{-}u^{\mu_2}_{-}\cdots] + [\delta(p_{1+})\delta(p_{2+})\cdots \times u^{\mu_1}_{+}u^{\mu_2}_{+}\cdots] \quad (1)$$

with the light-cone variables and null vectors given by:

$$u^{\mu}_{\pm} \equiv (1, \mp 1), \qquad A_{\pm} = A^0 \pm A^1 = A \cdot u_{\pm},$$

Finite Chemical Potential Results

- Next, we generalize the previous results, and evaluate the complete effective action for the model, by explicitly evaluating the *n*-point functions.

- The components of the propagator at finite temperature in the presence of a chemical potential change. The (++) component of the propagator in the closed time path formalism is given by:

$$S_{++}^{(\beta,\mu)}(p) = \bar{\not{p}}\left[\frac{1}{\bar{p}^2 + i\epsilon} + 2\pi i n_F(p_0\,\mathrm{sgn}(\bar{p}_0)\delta(\bar{p}^2)\right],$$

where we have identified a new four vector with:

$$\bar{k}_\mu = (k_0 + \mu, -\vec{k}).$$

Finite Chemical Potential Results

- The simplest nontrivial calculation is that for the two-point function arising from the fermion loop diagram, which in the momentum space has the form:

$$i\Gamma_{++}^{\mu\nu(\beta,\mu)}(p) = -e^2 \int \frac{d^2k}{(2\pi)^2} \mathrm{Tr}\gamma^\mu S_{++}^{(\beta,\mu)}(k+p)\gamma^\nu S_{++}^{(\beta,\mu)}(k).$$

- The integration over k_0 using the delta function is straightforward and with some algebra, the temperature dependent photon two-point function is obtained to be:

$$i\Gamma_{++}^{\mu\nu(T,\mu)}(p) = -\frac{e^2}{4}[\delta(p_-)u_-^\mu u_-^\nu + \delta(p_+)u_+^\mu u_+^\nu] \times \int dk^1 \,\mathrm{sgn}(k^1)\mathrm{sgn}(k^1+p^1) I_2^{(T,\mu)},$$

Finite Chemical Potential Results

where:

$$
\begin{aligned}
I_2^{(T,\mu)} = [&(n_F((k^1 - \mu)\mathrm{sgn}(k^1)) + n_F((k^1 + p^1 - \mu) \\
&\times \mathrm{sgn}(k^1 + p^1)) - 2n_F((k^1 - \mu)\mathrm{sgn}(k^1)) \\
&\times n_F((k^1 + p^1 - \mu)\mathrm{sgn}(k^1 + p^1))) \\
&+ (\mu \rightarrow -\mu)].
\end{aligned}
$$

Finite Chemical Potential Results

- We note here that the chiral two-point function can be obtained from the photon two-point function because of gamma matrix identities. Thus the temperature chiral two-point function can be obtained to be:

$$\Gamma_{5,++}^{\mu\nu(T,\mu)}(p) = \epsilon_{\lambda}^{\mu}\Gamma_{++}^{\lambda\nu(T,\mu)}(p).$$
$$\propto \epsilon^{\mu\lambda}[\delta(p_-)u_{-\lambda}u_-^{\nu} + \delta(p_+)u_{+\lambda}u_+^{\nu}]$$
$$\equiv -\delta(p_-)u_-^{\mu}u_-^{\nu} + \delta(p_+)u_+^{\mu}u_+^{\nu},$$

Finite Chemical Potential Results

As a result, we obtain:

$$p_\mu \Gamma^{\mu\nu(T,\mu)}_{5,++}(p) \propto -\delta(p_-)p_- u^\nu_- + \delta(p_+)p_+ u^\nu_+ \equiv 0.$$

- This shows that there is no contribution to the chiral anomaly at finite temperature and nonzero chemical potential. The absence of a temperature dependent contribution to the chiral anomaly is well known and the discussion in the next section will clarify why the chemical potential does not modify the anomaly.

Finite Chemical Potential Results

PRD 78, 065005 (2008)

- Using the other components of the fermion propagator we can also evaluate the other components of the photon-self energy at finite temperature and chemical potential. For example, the temperature dependent part of the (+-) component of the two point function was obtained to be:

$$i\Gamma_{+-}^{\mu\nu(T,\mu)}(p) = -i\Gamma_{++}^{\mu\nu(T,\mu)}(p).$$

- It follows now that the temperature dependent part of the retarded two-point function in the presence of a nonzero chemical potential identically vanishes, namely,

$$i\Gamma_{R}^{\mu\nu(T,\mu)} \equiv i\Gamma_{++}^{\mu\nu(T,\mu)} + i\Gamma_{+-}^{\mu\nu(T,\mu)} = 0.$$

Finite Chemical Potential Results

- When we calculate the higher point functions, we find that a pattern emerges: the structure of the amplitudes always decomposes into two parts,

$$
\begin{aligned}
i\Gamma^{\mu_1\cdots\mu_n(T,\mu)}_{+\cdots+}(p_1,\ldots,p_{n-1}) = {} & \frac{(-ie)^n(\pi)^{n-2}}{4}[\delta(p_{1-})\cdots\delta(p_{(n-1)-})u^{\mu_1}_{-}\cdots u^{\mu_n}_{-} + \delta(p_{1+})\cdots\delta(p_{(n-1)+})u^{\mu_1}_{+}\cdots u^{\mu_n}_{+}] \\
& \times \int dk^1\,\mathrm{sgn}(k^1)\mathrm{sgn}(k^1+p^1)\mathrm{sgn}(k^1+P^1_{T(3)})\cdots\mathrm{sgn}(k^1+P^1_{T(n)})I^{(T,\mu)}_n \\
& + \text{all permutations off external momenta,} \quad \text{for } n \geq 2,
\end{aligned}
$$

Finite Chemical Potential Results

where:

$$
\begin{aligned}
I_n^{(T,\mu)} = & [([n_F((k^1 - \mu)\mathrm{sgn}(k^1)) + n_F((k^1 + p_1^1 - \mu)\mathrm{sgn}(k^1 + p_1^1)) + \text{linear permutations}] \\
& - 2[n_F((k^1 - \mu)\mathrm{sgn}(k^1))n_F((k^1 + p_1^1 - \mu)\mathrm{sgn}(k^1 + p_1^1)) + \text{quadratic permutations}] \\
& + 2^2[n_F((k^1 - \mu)\mathrm{sgn}(k^1))n_F((k^1 + p_1^1 - \mu)\mathrm{sgn}(k^1 + p_1^1))n_F((k^1 + p_1^1 + p_2^1 - \mu)\mathrm{sgn}(k^1 + p_1^1 + p_2^1)) \\
& + \text{cubic permutations}] + \cdots) + (-1)^n(\mu \rightarrow -\mu)].
\end{aligned}
$$

- The symmetry (antisimmetry) of all the amplitudes under the change of sign of the chemical potential is manifest. In the limit of vanishing chemical potential all the odd point amplitudes vanish and the even point amplitudes reduce to those already calculated.

Alternative Derivation of the Results

- The particular structure of the amplitudes involving products of light-cone delta functions as well as light-cone (null) vectors is quite puzzling. In order to better understand the structure of the *n*-point functions in this model, we present an alternative derivation in terms of one-dimensional fermions.

- We decompose the fermions in the theory into left- and right-handed components as:

$$\psi_L = P_L \psi = \tfrac{1}{2}(1 - \gamma_5)\psi, \qquad \psi_R = P_R \psi = \tfrac{1}{2}(1 + \gamma_5)\psi,$$

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Alternative Derivation of the Results

- The Lagrangian density can be written in terms of these components as:

$$\mathcal{L} = i\psi_R^\dagger(\partial_- - i\mu + ieA_-)\psi_R + i\psi_L^\dagger(\partial_+ - i\mu + ieA_+)\psi_L,$$

- The lagrangian density of the theory effectively becomes a sum of two decoupled lagrangian densities. As a consequence, the amplitudes will involve only the (+) components or the (-) components of the photon field.

Alternative Derivation of the Results

- Propagators:

$$S_R^{(\beta,\mu)}(p) = \bar{p}_+ P_R \Big[\frac{1}{\bar{p}_+ \bar{p}_- + i\epsilon} + 2\pi i n_F(p_0 \,\mathrm{sgn}(\bar{p}_0)) \delta(\bar{p}_+ \bar{p}_-) \Big],$$

$$S_L^{(\beta,\mu)}(p) = \bar{p}_- P_L \Big[\frac{1}{\bar{p}_+ \bar{p}_- + i\epsilon} + 2\pi i n_F(p_0 \,\mathrm{sgn}(\bar{p}_0)) \delta(\bar{p}_+ \bar{p}_-) \Big],$$

Alternative Derivation of the Results

- Two-point function for the (+) photon component:

$$i\Gamma^{(\beta,\mu)}_{++}(p) = -e^2 \int \frac{d^2k}{(2\pi)^2} \mathrm{Tr} S^{(\beta,\mu)}_L(k+p) S^{(\beta,\mu)}_L(k)$$

$$= -\frac{e^2}{4}\delta(p_+) \int dk^1 \,\mathrm{sgn}(k^1)\mathrm{sgn}(k^1+p^1) I^\mu_2$$

- In general:

$$\Gamma_{+++\cdots+}(p_i) \propto \delta(p_{1+})\delta(p_{2+})\cdots.$$

Imaginary Time Formalism

- We have found for the retarded two-point function that:

$$i\Gamma_R^{\mu\nu(T,\mu)} \equiv i\Gamma_{++}^{\mu\nu(T,\mu)} + i\Gamma_{+-}^{\mu\nu(T,\mu)} = 0.$$

- QUESTION: How does the nontrivial structure for the Feynman amplitudes found in this model can be obtained in the imaginary time formalism?