Muon g-2, Rare Decay $\pi_0 \to e^+e^-$ and DarkMatter

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Introduction

Muon g-2 (status)

Hadronic contributions within Instanton Model

Rare $\pi^0 \to e^+e^-$ Decay (status)

$\pi^0 \to e^+e^-$ Decay and Dark Matter

Conclusions

Introduction

Abnormal people are looking for traces of Extraterrestrial Guests
Abnormal Educated people are looking for hints of New Physics

Cosmology tell us that 95% of matter is not described in text-books yet

New excitements after Fermi LAT, PAMELA, ATIC, HESS and WMAP data
Interpreted as Dark Matter and/or Pulsar signals

Two search strategies:
1) High energy physics to excite heavy degrees of freedom.
No any evidence till now. LHC era has started.

2) Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

$(g-2)_\mu$ is very famous example,

$\pi_0 \to e^+e^-$ is in the list of SM test after new exp. and theor. results

That's intriguing

Anomalous magnetic moment of muon

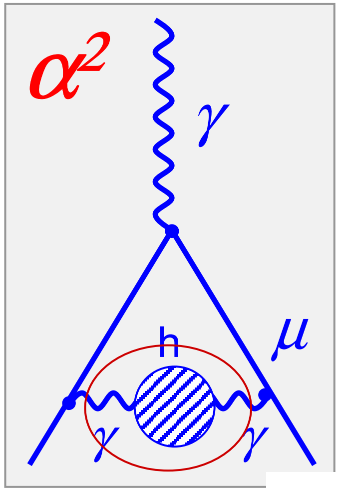
From BNL E821 experiment (1999-2006)

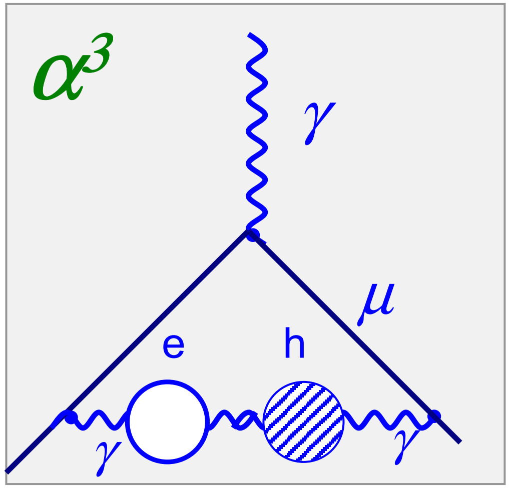
New proposals for BNL, FNAL, JPARC

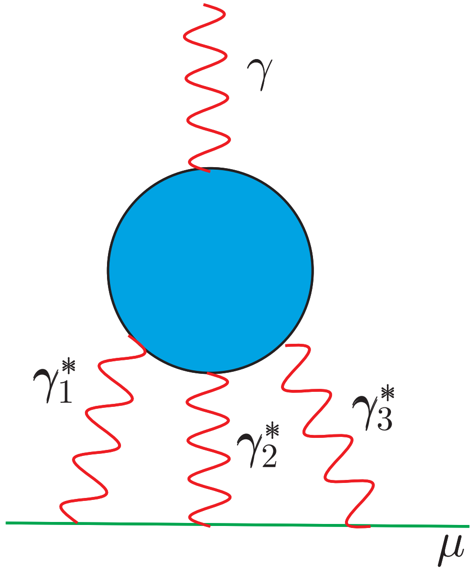
$$a_\mu^{BNL} = 11\,659\,208.0(6.3)\cdot 10^{-10}$$

Standard Model $$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Hadr} = 11\,659\,177.3(5.0)\cdot 10^{-10}$$

predicts the result which is 3.4σ below the experiment (since 2006)





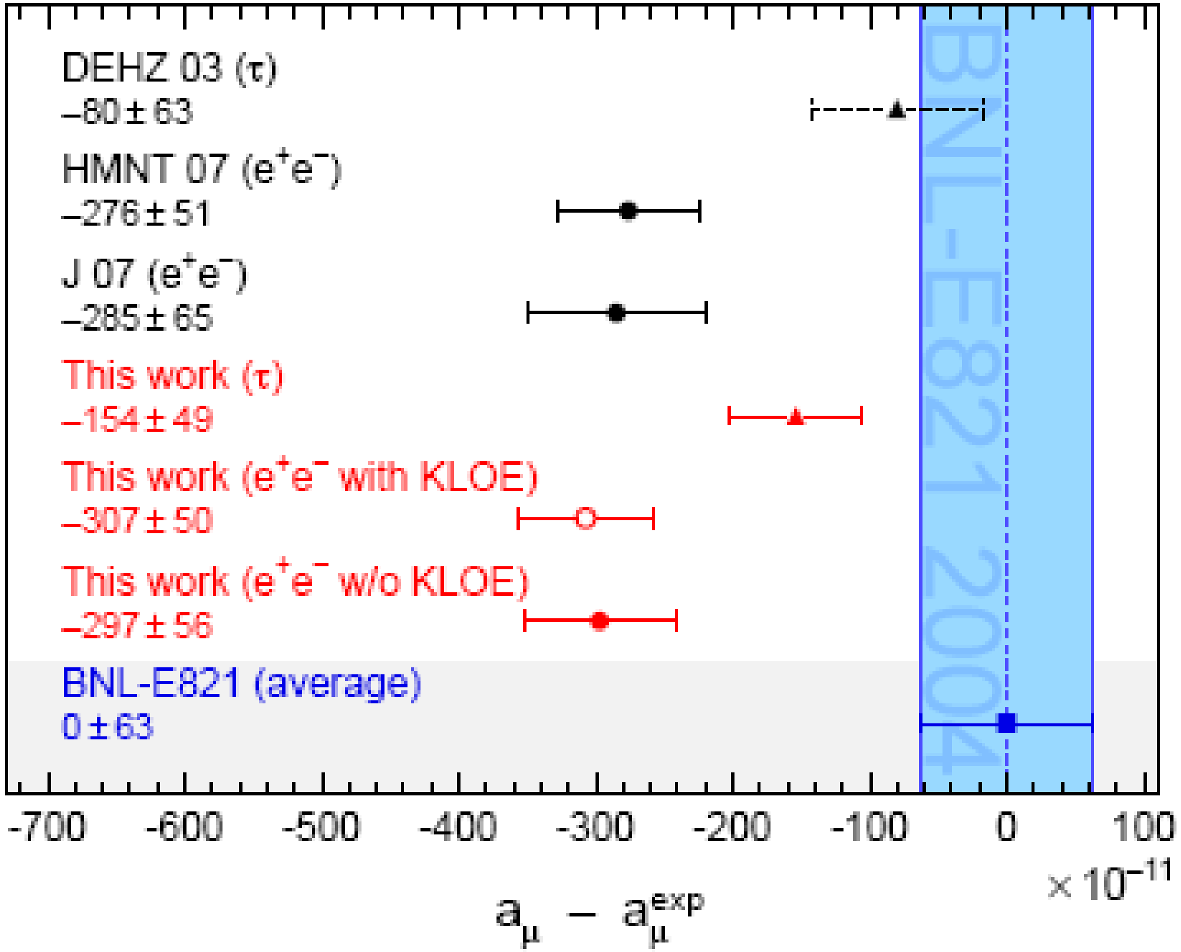


LbL to g-2

aμ(HVP)= $(6894 \pm 46)\cdot 10^{-11}$

Integral over e+e- to Hadrons cross section

$$a_\mu(\text{LbL}) \simeq (118.76 \pm 40)\times 10^{-11}$$

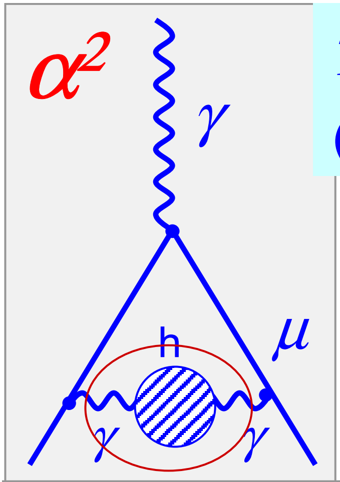


1.9 σ

3.4 σ

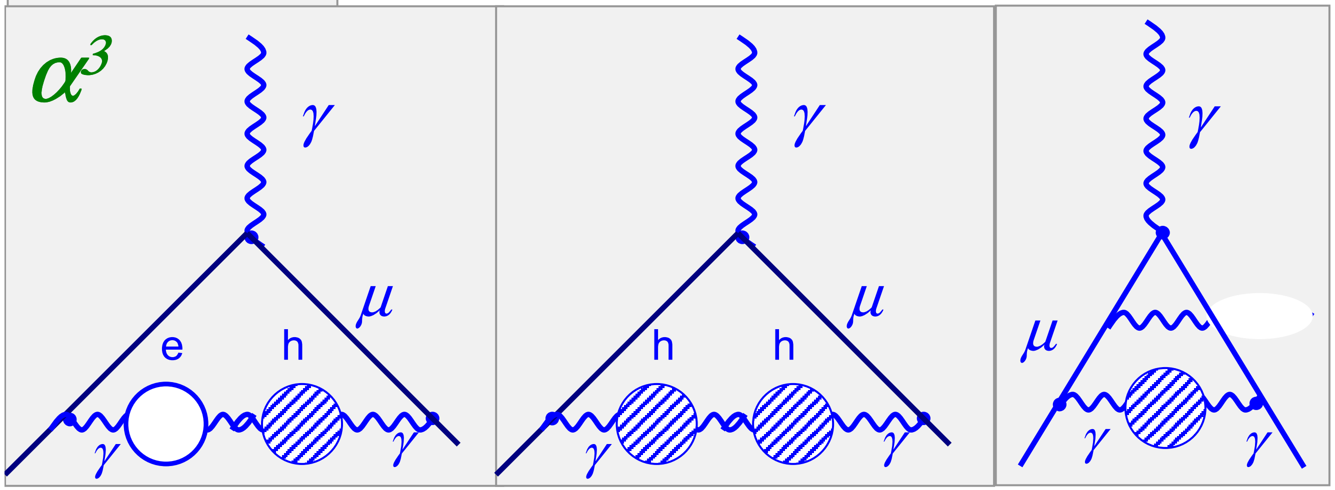
M. Davier etal., 2009

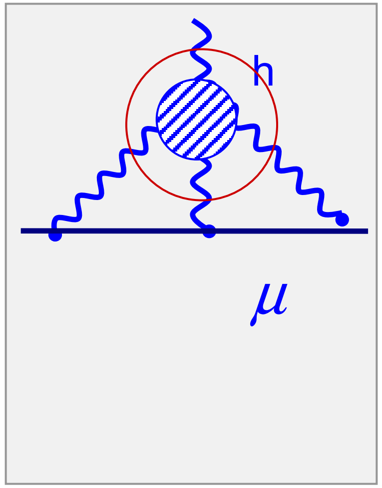
The hadronic contributions to the muon AMM (theory)

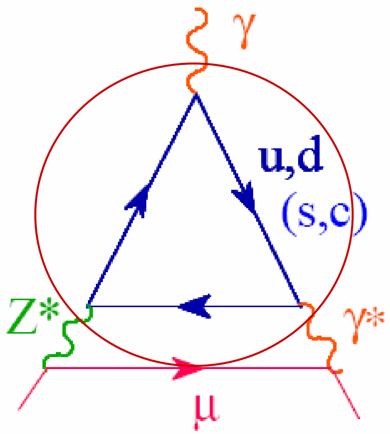


$$a_{\mu}^{\mathrm{S}} = a_{\mu}^{\mathrm{had,LO}} + a_{\mu}^{\mathrm{had,NLO}} + a_{\mu}^{\mathrm{had,LbL}}$$

Hadronic Vacuum
Polarization contributes 99% and half of error







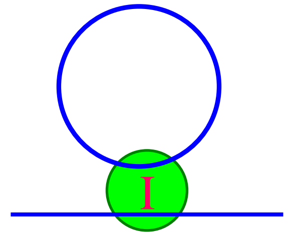
Z*γγ* effective
coupling

Light-by-light
process
contributes 1%
and half of error

Instanton model

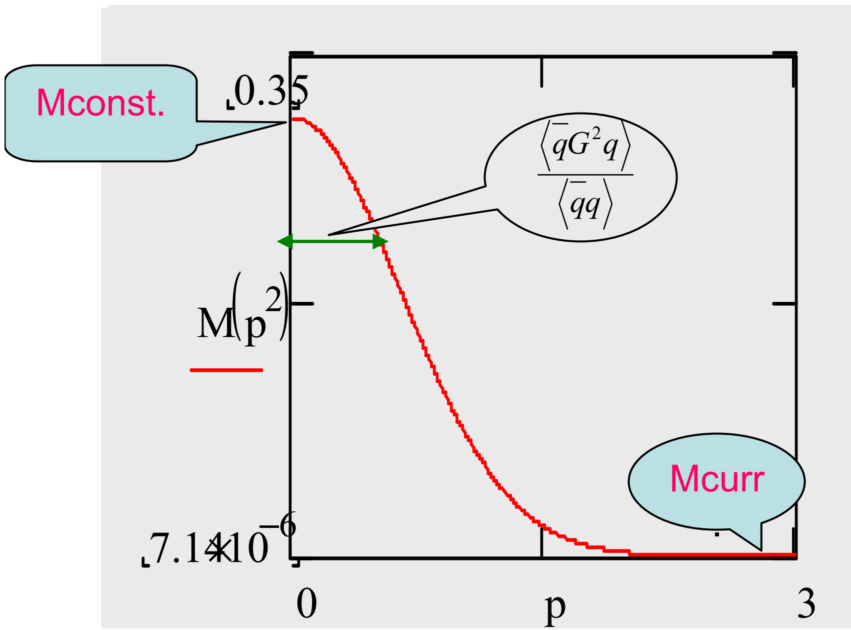
The **dressed quark propagator** is defined as

$$S^{-1}(p) = \hat{p} - M(p)$$



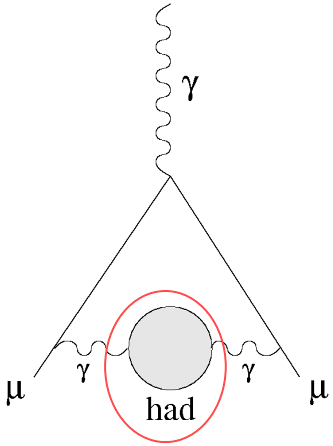
$$M(p) = M_q f^2(p)$$

f(p) is related to the quark zero mode in the *instanton* field



Incorporates soft momentum physics and smoothly transits to high-momentum regime

Leading Order Contribution to Muon Anomalous Magnetic Moment

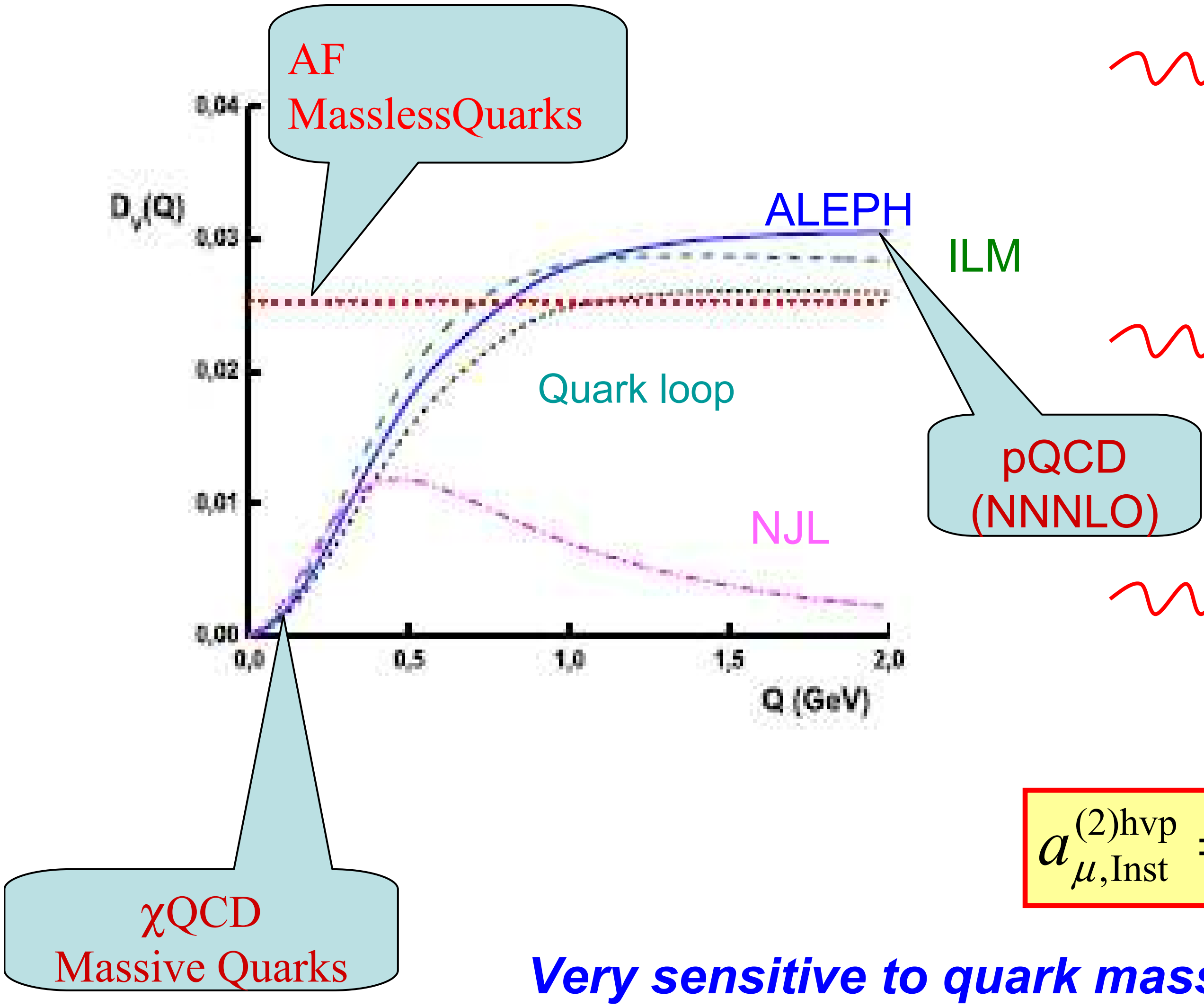


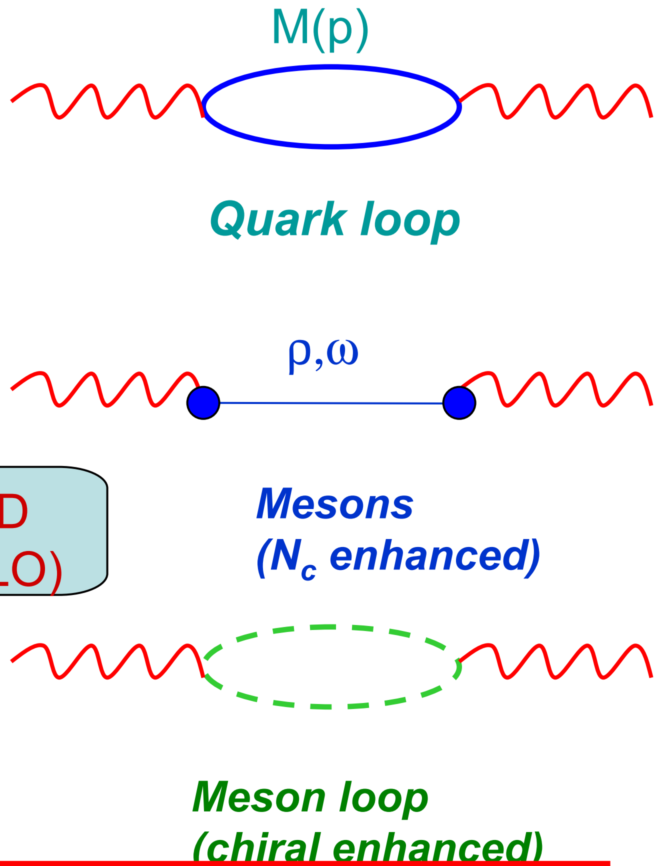
$$a_\mu^{(2)\mathrm{hvp}} = \frac{8}{3}\alpha^2 \int_0^1 dx \frac{(1-x)(1-x/2)}{x} D_V\left(\frac{x^2}{1-x} m_\mu^2\right) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \frac{K(t)}{t} \rho_V(t)$$

Adler function is defined as

$$D_V(Q^2) = -Q^2 \frac{d\Pi_V(Q^2)}{dQ^2} = \int_0^\infty dt \frac{Q^2}{(t+Q^2)^2} \rho_V(t)$$

NχQM Adler function and ALEPH data (AD, PRD, 2004)



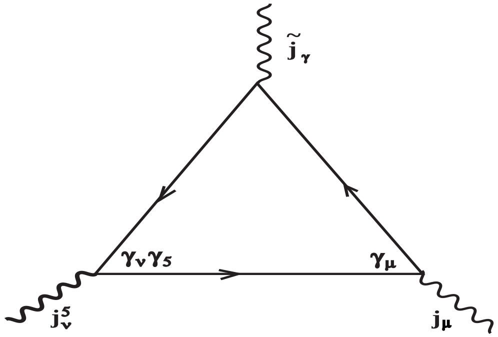


$$a_{\mu,\mathrm{Inst}}^{(2)\mathrm{hvp}} = (633 \pm 50) \cdot 10^{-10}$$

Very sensitive to quark mass: Mq=200-240 MeV

$$a_{\mu}^{(2)\mathrm{hvp}} = (689.1 \pm 4.3) \cdot 10^{-10}, e^+e^-$$

V^*AV correlator and muon AMM



For **specific** kinematics: $q_2=q$ is arbitrary, $q_1 \to 0$
only 2 structures exists in the triangle amplitude

$$T_{\mu\nu\lambda}(q_1,q_2) = w_T\left(q_2^2 q_1^\rho \varepsilon_{\rho\mu\nu\lambda} + q_2^\nu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\mu\lambda} + q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu}\right) - w_L q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu},$$

The amplitude is transversal with respect to vector current

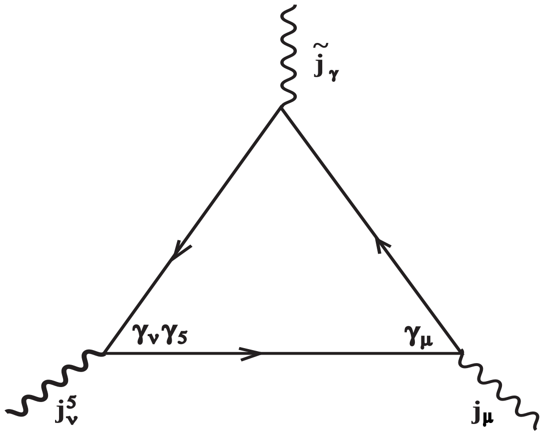
$$q_1^\mu T_{\mu\nu\lambda} = q_2^\nu T_{\mu\nu\lambda} = 0$$

but longitudinal with respect to axial-vector current

$$q_2^\lambda T_{\mu\nu\lambda} = -\left(w_L q_2^2\right) \cdot q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu}$$

This is famous **Adler-Bell-Jackiw anomaly**

V^*AV amplitude



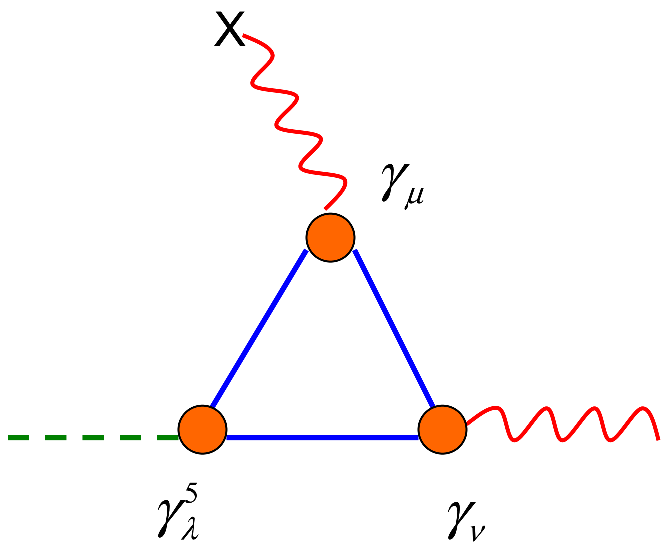
In local theory for quarks with constant mass one gets

$$w_L = 2w_T = \frac{2N_C}{3}\int_0^1 dx \frac{x(1-x)}{x(1-x)q^2+m^2} \underset{m\to 0}{\Rightarrow} \frac{2N_C}{3}\frac{1}{q^2}$$

- ***Perturbative nonrenormalization of wL (Adler-Bardeen theorem, 1969)***
- ***Nonperturbative nonrenormalization of wL ('t Hooft duality condition, 1980)***
- ***Perturbative nonrenormalization of wT (Vainshtein theorem, 2003)***
- ***Nonperturbative corrections to wT at large q are O($1/q^6$) (De Rafael et.al., 2002)***
- ***Absence of Power corrections to wT at large q in chiral limit in Instanton model (Dorokhov,2005)***
- ***Massive corrections (Teryaev, Pasechnik; Jegerliner, Tarasov, 2005; Melnikov,2006)***

N^nLO QCD =0 for all n>0

Anomalous wL structure (NonSinglet)



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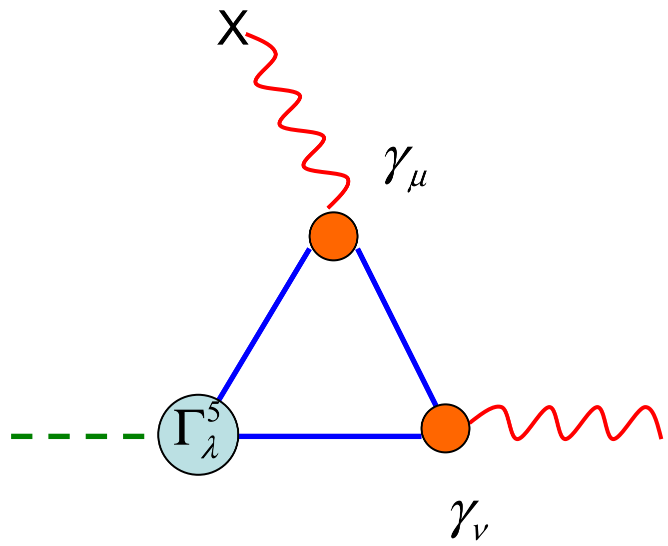
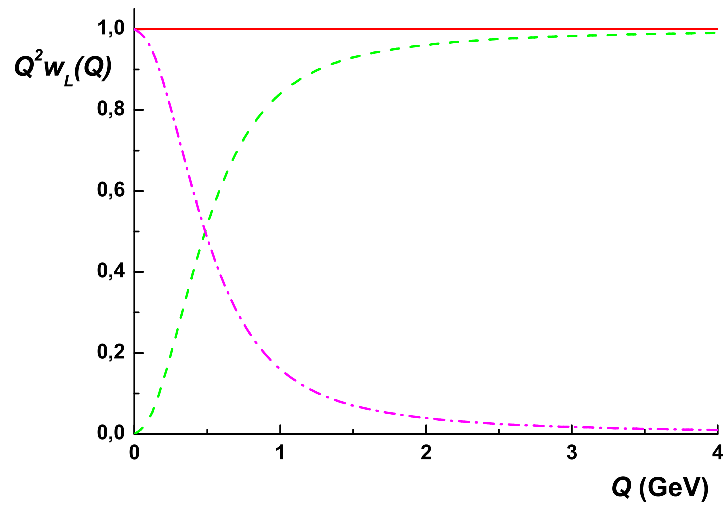


+ rest

Diagram with Local vertices

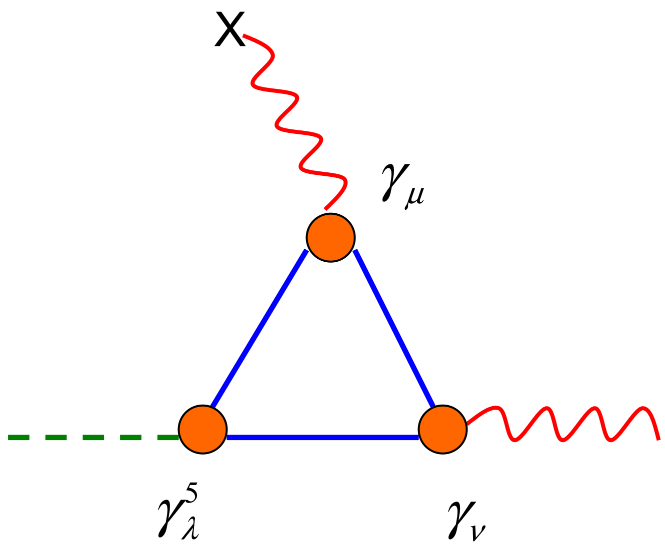
Diagram with NonLocal Axial vertices



$$w_L^{(3)} = \frac{2N_C}{3}\frac{1}{q^2}$$

In accordance with Anomaly and 't Hooft duality principle (massless Pion states in triplet)

Anomalous wL structure (Singlet)



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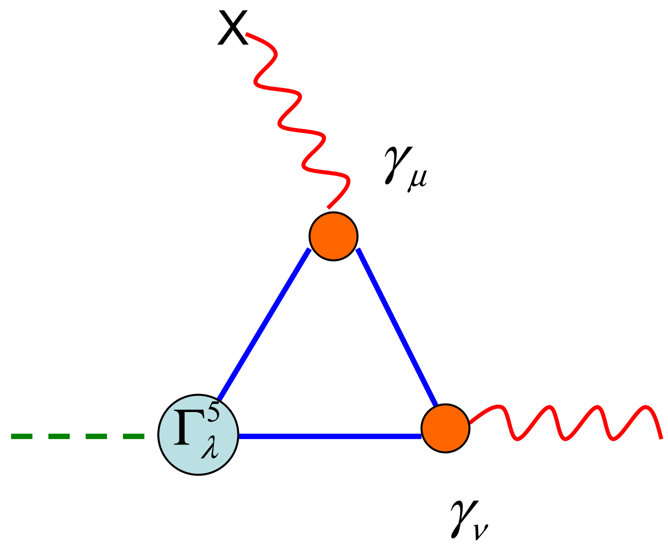
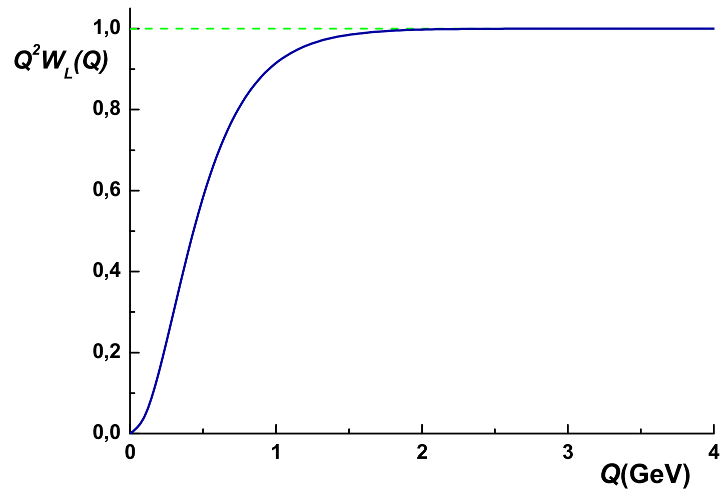


+ rest

Diagram with Local vertices

Diagram with NonLocal Axial vertices



$$q^2 w_L^{(0)}(q^2)\Big|_{q^2\to 0} = 0$$

In accordance with Anomaly and 't Hooft duality principle (no massless states in singlet channel due to $U_A(1)$ anomaly)

wLT in the Instanton Model (NonSinglet)

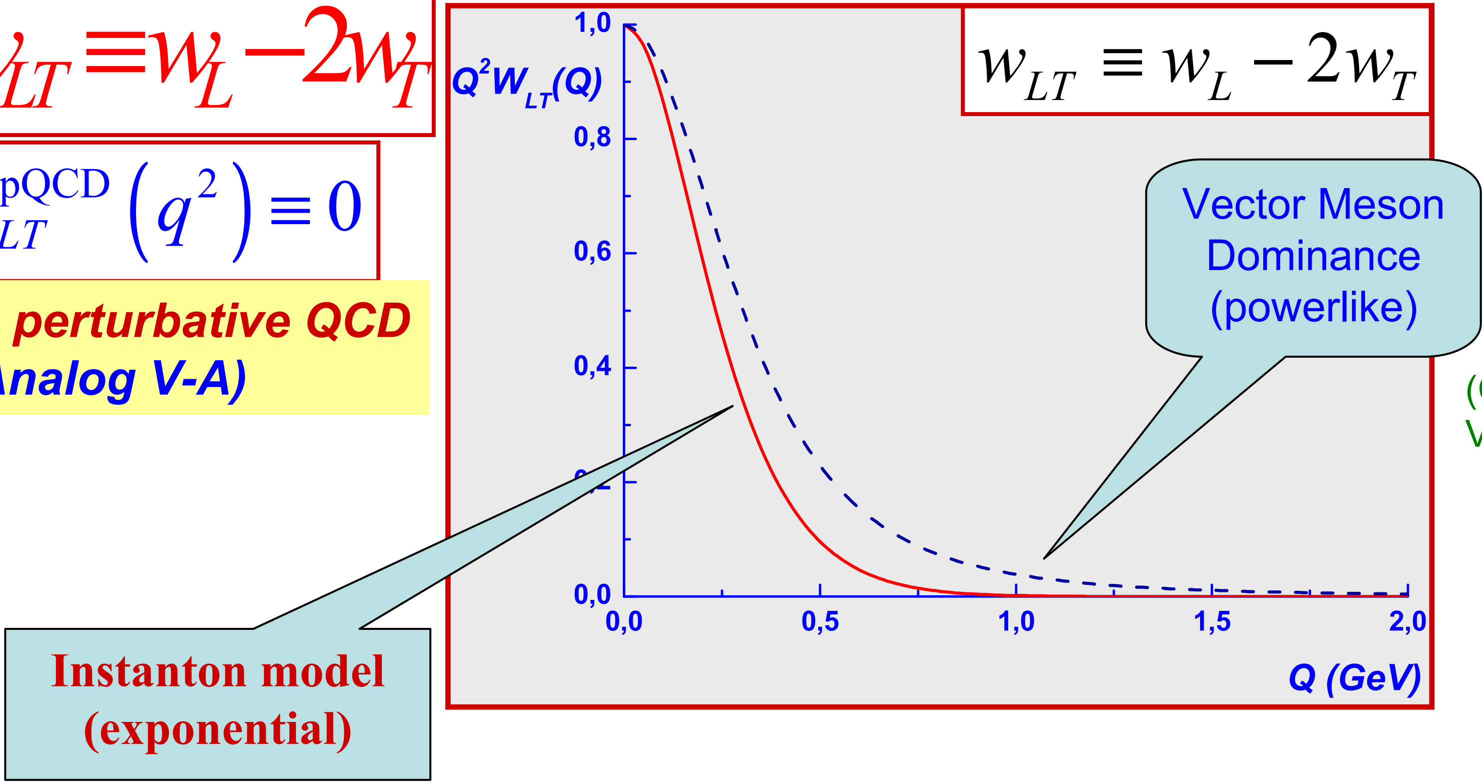
In local theory for quarks with constant mass one gets

$$w_L = 2w_T = \frac{2N_C}{3}\int_0^1 dx \frac{x(1-x)}{x(1-x)q^2+m^2} \underset{m\to 0}{\Rightarrow} \frac{2N_C}{3}\frac{1}{q^2}$$

$$w_{LT} \equiv w_L - 2w_T$$

$$w_{LT}^{\mathrm{pQCD}}\left(q^2\right) \equiv 0$$

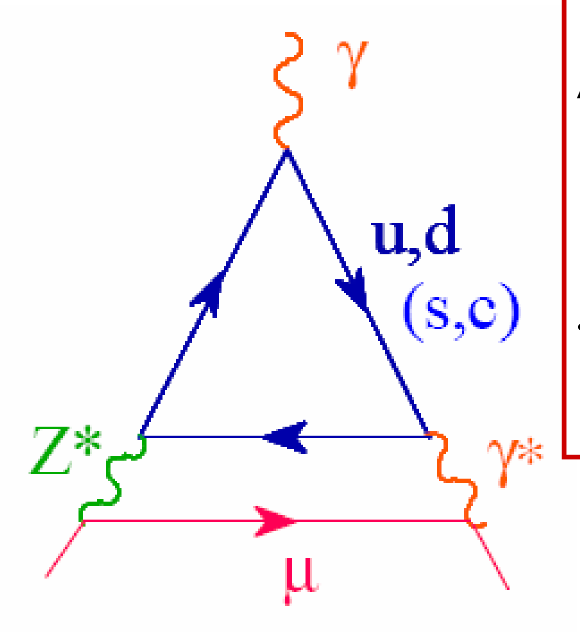
In perturbative QCD
(Analog V-A)



(Czarnecki, Marciano, Vainshtein, 2003)

- Absence of Power corrections at large q in chiral limit in Instanton model

Z*γγ* contribution to a_μ



$$\Delta a_\mu^{EW} = 2\sqrt{2}\,\frac{\alpha}{\pi} G_\mu m_\mu^2 i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 2kp} \cdot$$

$$\cdot \left[\frac{1}{3} \left(1 + \frac{2(kp)^2}{k^2 m_\mu^2} \right) \left(w_L - \frac{m_Z^2}{m_Z^2 - k^2} w_T \right) + \frac{m_Z^2}{m_Z^2 - k^2} w_T \right]$$

Perturbative QCD
(Anomaly cancelation)

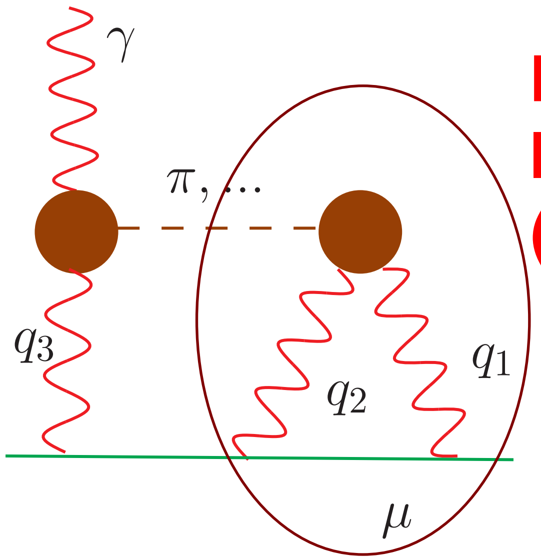
$$\Delta a_\mu^{\mathrm{EW,pQCD}} = 0$$

VMD + OPE
(Czarnecki, Marciano, Vainshtein, 2003)

$$\Delta a_\mu^{\mathrm{EW}} \approx -2.02 \cdot 10^{-11}$$

Instanton model:
(Dorokhov, 2005)

$$\Delta a_\mu^{\mathrm{EW}} = -1.48 \cdot 10^{-11}$$



Pion pole contribution within Instanton model (A.D., W. Broniowski PRD 2008)

$$a_\mu^{\mathrm{LbL},\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_3}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 q_3^2 \left[(p+q_1)^2 - M^2\right]\left[(p-q_3)^2 - M^2\right]}$$

$$\left[F_{\pi_0^*\gamma^*\gamma^*}\left(q_3^2; q_1^2, q_2^2\right) \frac{G_P}{g_{\pi q}^2 \left(1 - G_P J_{PP}\left(q_3^2\right)\right)} F_{\pi_0^*\gamma^*\gamma^*}\left(q_3^2; q_3^2, 0\right) T_1\left(q_1, q_3; p\right) + perm. \right]$$

$$a_\mu^{\pi,\mathrm{LbL}} = 6.27 \cdot 10^{-11}$$

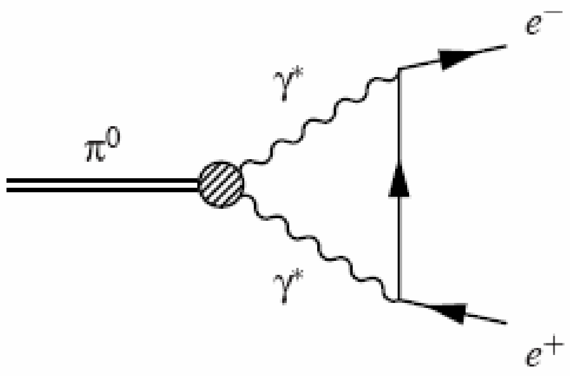
Full kinematic dependence

Correct QCD asymptotics

Complete calculations are in progress

Rare Pion Decay $\pi^0 \rightarrow e^+e^{--}$ *from KTeV*

PRD (2007)



Lowest order diagram

One of the simplest process for THEORY

From KTeV E799-II EXPERIMENT at Fermilab experiment (1997-2007)

$$B^{\mathrm{KTeV}}_{\pi \rightarrow e^+ e^-} = (7.49 \pm 0.29 \pm 0.25) \cdot 10^{-8}$$

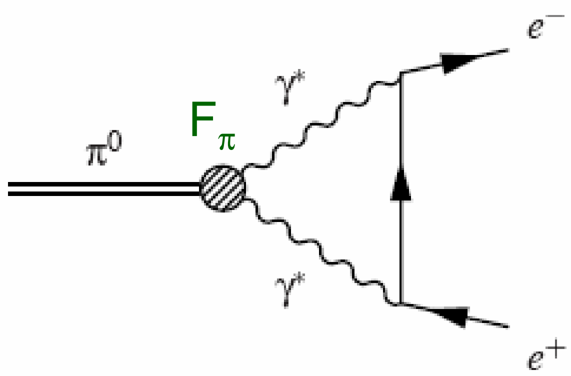
99-00' set,

The result is based on observation of 794 candidate π_0 → e+e- events using K_L → 3π_0 as a source of tagged π_0s.
The older data used 275 events with the result:

$$B^{\mathrm{KTeV}}_{\pi \rightarrow e^+ e^-}(\mathrm{old}) = (7.04 \pm 0.46 \pm 0.28) \cdot 10^{-8}$$

97' set

Classical theory of $\pi^0 \to e^+e^-$ decay



Drell (59'), Berman,Geffen (60'), Quigg,Jackson (68')

Bergstrom,et.al. (82') Dispersion Approach

Savage, Luke, Wise (92') χPT

$$R\left(\pi^0 \to e^+e^-\right) = \frac{B\left(\pi^0 \to e^+e^-\right)}{B\left(\pi^0 \to \gamma\gamma\right)} = 2\beta\left(m_\pi^2\right)\left(\frac{\alpha}{\pi}\frac{m_e}{m_\pi}\right)^2\left[\left(\mathrm{Re}\,\mathcal{A}\left(m_\pi^2\right)\right)^2 + \left(\mathrm{Im}\,\mathcal{A}\left(m_\pi^2\right)\right)^2\right]$$

$$\beta\left(q^2\right) = \sqrt{1 - 4\frac{m_e^2}{q^2}}$$

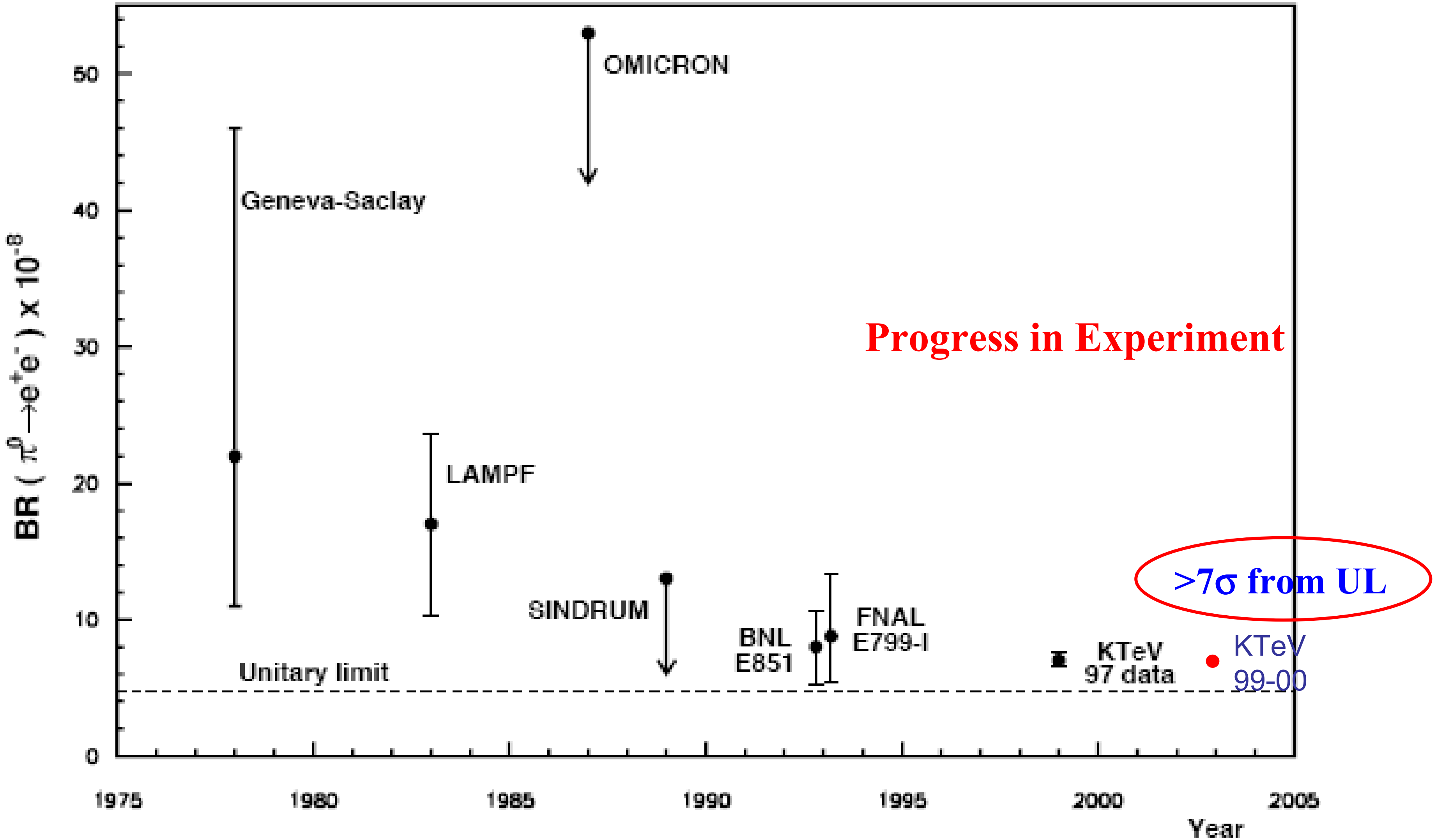
$$\mathcal{A}\left(q^2\right) = \frac{2i}{q^2}\int\frac{d^4k}{\pi^2}\frac{q^2k^2 - (qk)^2}{\left(k^2 + i\varepsilon\right)\left((k-q)^2 + i\varepsilon\right)\left((k-p)^2 - m_e^2 + i\varepsilon\right)}F_\pi\left(k^2, (k-q)^2\right)$$

$$\mathrm{Im}\,\mathcal{A}\left(q^2\right) = \frac{\pi}{2\beta\left(q^2\right)}\ln\left(\frac{1-\beta\left(q^2\right)}{1+\beta\left(q^2\right)}\right)$$

The Imaginary part is Model Independent;
Unitary limit

From condition $|A|^2 \geq (\mathrm{Im}\, A)^2$ one has the ***unitary limit***

$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\mathrm{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.75 \cdot 10^{-8}.$$

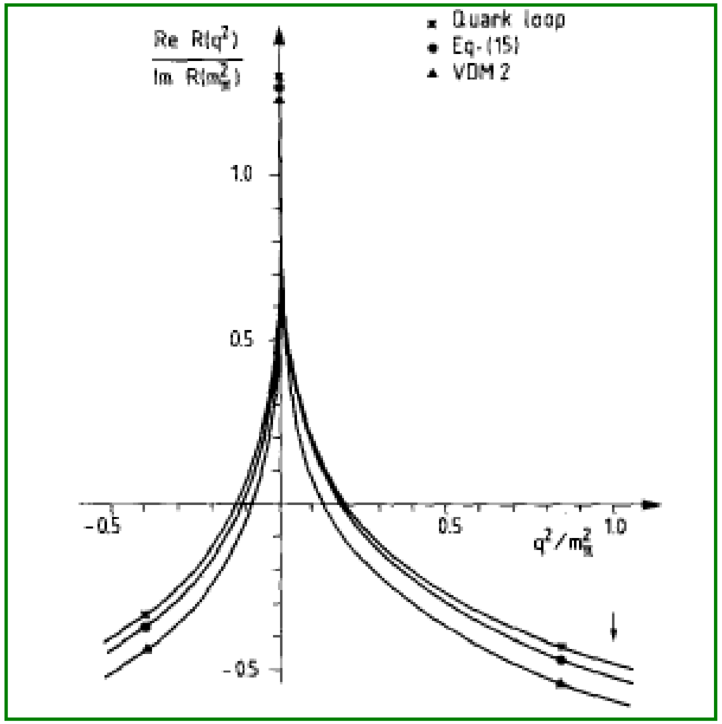


Still no intrigue

1. Dispersion approach (Bergstrom et.al.(82))

$$\mathrm{Re}\,\mathcal{A}\left(q^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \frac{q^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\mathcal{A}\left(s\right)}{s\left(s-q^2\right)}.$$

$$\mathrm{Re}\,\mathcal{A}\left(q^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \frac{1}{\beta_e\left(q^2\right)}\left[\frac{1}{4}\ln^2\left(y\right) + \frac{\pi^2}{12} + \mathrm{Li}_2\left(-y\right)\right], \qquad y = \frac{1-\beta_e\left(q^2\right)}{1+\beta_e\left(q^2\right)}.$$



The Real Part is known up to Constant

This Constant is the Amplitude in Soft Limit $q^2 \to 0$

In general it is determined in Model Dependent way

$$\mathrm{Re}\,\mathcal{A}\left(m_\pi^2\right) = \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12} \approx \mathrm{Re}\,\mathcal{A}\left(q^2=0\right) + 31.9$$

I. The Decay Amplitude in Soft limit $q^2 \to 0$

$$\mathrm{Re}\, A\left(m_\pi^2\right) = \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12} + 3\ln\left(\frac{m_e}{\mu}\right) + \chi_P\left(\mu\right) + \mathrm{O}\left(\frac{m_e^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}\right)$$

$$\chi_P\left(\mu\right) = -\frac{5}{4} - \frac{3}{2}\left[\int_0^{\mu^2} dt \frac{F_\pi\left(t,t\right)-1}{t} + \int_{\mu^2}^{\infty} dt \frac{F_\pi\left(t,t\right)}{t}\right]$$

$$m_e^2 << m_\pi^2 << \Lambda^2 \approx m_\rho^2$$

$$a_\mu^{(2)\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s)$$

The unknown constant is expressed as inverse moment of Pion Transition FF at spacelike momenta !!!

Still no intrigue

Thus the amplitude is fully reconstructed!
in terms of moments of Pion Transition FF

A. E. Dorokhov and M. A. Ivanov, Phys. Rev. D **75** (2007) 114007

S1

II. CLEO data and Lower Bound on Branching

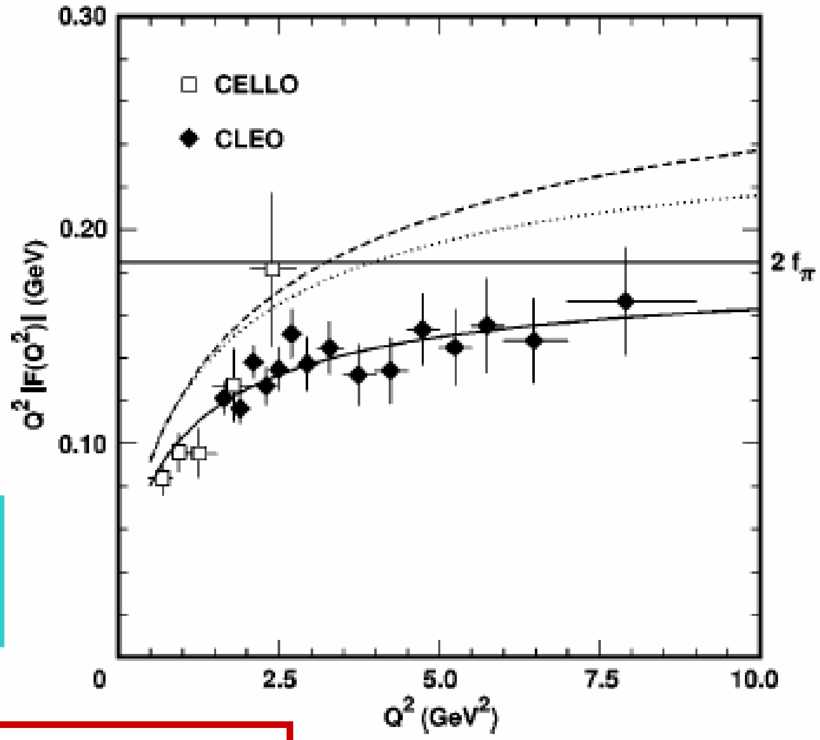
Use inequality $F_\pi(t,t) < F_\pi(t,0)$ at spacelike $t > 0$

and CLEO data (98')

$$F_\pi^{\mathrm{CLEO}}(t,0) = \frac{1}{1 + t / s_0^{\mathrm{CLEO}}}$$

$$s_0^{\mathrm{CLEO}} = (776 \pm 22 \quad \mathrm{MeV})^2$$

$$\mathrm{Re}\,\mathcal{A}(q^2 = 0) > -\frac{3}{2}\ln\left(\frac{s_0^{\mathrm{CLEO}}}{m_e^2}\right) - \frac{5}{4} = -23.2 \pm 0.1$$



$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\mathrm{CLEO}}\left(\pi^0 \to e^+e^-\right) = (5.91 \pm 0.02) \cdot 10^{-8}$$

$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\mathrm{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.75 \cdot 10^{-8}.$$

$$R^{\mathrm{KTeV}} = (7.58 \pm 0.40) \cdot 10^{-8}$$

Intrigue appears

Slide 21

S1 The asymetric ff is fixed rather accurately. Space like form factors are smooth functions, all known form factors (pion, nucleon) are well parametrized by monopole form, so if normalization is known, the derivative at zero (radius) is known and tail is fixed by CLEO then we are quite confident with ff. Moreover is in agreemnet with OPE prediction.

Another point is that the effect is logarithmic, and we don't need to know precise form of ff, but rather its characteristic scale.

Sasha; 09.04.2008

III. $F_\pi(t,t)$ general arguments

Let $$F_\pi(t,t) = \frac{1}{1+t/s_1}$$ *then* $$\mathrm{Re}\,\mathcal{A}^{\text{theory}}(q^2=0) = -\frac{3}{2}\ln\left(\frac{s_1}{m_e^2}\right) - \frac{5}{4}$$

1. From $$-\frac{\partial F_\pi(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_\pi(t,0)}{\partial t}\bigg|_{t=0}$$ *one has* $$s_1 = s_0/2$$

2. From OPE QCD (Brodsky, Lepage)

$$F_\pi^{\text{OPE}}(t,0)\Big|_{t\to\infty} = 8\pi^2 f_\pi^2 \frac{1}{t},$$

$$F_\pi^{\text{OPE}}(t,t)\Big|_{t\to\infty} = \frac{8\pi^2 f_\pi^2}{3}\frac{1}{t}$$

one has $$s_1^{OPE} = s_0^{OPE}/3$$

F(t,0) → F(t,t) reduces to rescaling

It follows $$\mathrm{Re}\,A^{\text{theory}}(q^2=0) = -21.9 \pm 0.3$$

$$B^{\text{KTeV}} = (7.49 \pm 0.39)\cdot 10^{-8}$$

$$B^{\text{theory}}(\pi^0 \to e^+e^-) = (6.2 \pm 0.1)\cdot 10^{-8}$$

3.3σ below data!!

It would required change of s_0 scale by factor more then 10!

Now it's intriguing!

A. $F_\pi(t,t)$ QCD sum rules (V.Nesterenko, A.Radyushkin, YaF 83')

$$F_\pi^{\mathrm{QCDsr}}(t,t) = 2\int_0^{s_0^{\mathrm{QCDsr}}} ds \int_0^1 dx \frac{x(1-x)t^2}{[x(1-x)s+t]^3} + \text{v.c.},$$

From

$$-\frac{\partial F_\pi(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_\pi(t,0)}{\partial t}\bigg|_{t=0}$$

and

$$\langle r^2\rangle_{\pi^0\gamma^*\gamma^*}^{\mathrm{QCDsr}} = -6\,\frac{\partial F_\pi^{\mathrm{QCDsr}}(t,t)}{\partial t}\bigg|_{t=0} = \frac{12}{s_0^{\mathrm{QCDsr}}}$$

one has

$$s_0^{\mathrm{QCDsr}} = s_0^{\mathrm{CLEO}}$$

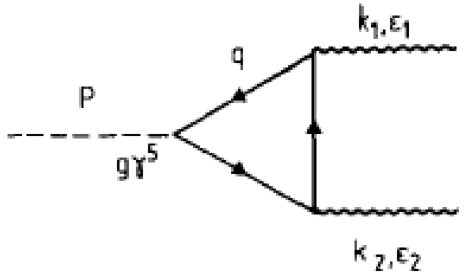
$$\mathrm{Re}\,A^{\mathrm{QCDsr}}\left(q^2=0\right) = -\frac{3}{2}\ln\left(\frac{s_0^{\mathrm{CLEO}}}{\mathrm{m}_e^2}\right) + \frac{1}{4} = -21.7 \pm 0.1,$$

$$s_1^{\mathrm{QCDsr}} = \frac{s_0^{\mathrm{QCDsr}}}{e}$$

Nicely confirms general arguments!

$$\mathrm{Re}\,A^{\mathrm{theory}}\left(q^2=0\right) = -21.9 \pm 0.3$$

C. $F_\pi(t,t)$ Quark Models *(Bergstrom 82')*



Constituent constant Quark mass

$$F_\pi^{\text{quark}}(t,t) = \frac{2M_q^2}{\beta_q(t)\,t} \ln\left(\frac{\beta_q(t)+1}{\beta_q(t)-1}\right), \qquad \beta_q(t) = \sqrt{1+4\frac{M_q^2}{t}}$$

$$\text{Re}\,\mathcal{A}_{\text{QM}}(q^2=0) = 3\ln\left(\frac{m_e}{M_q}\right) - \frac{17}{4}$$

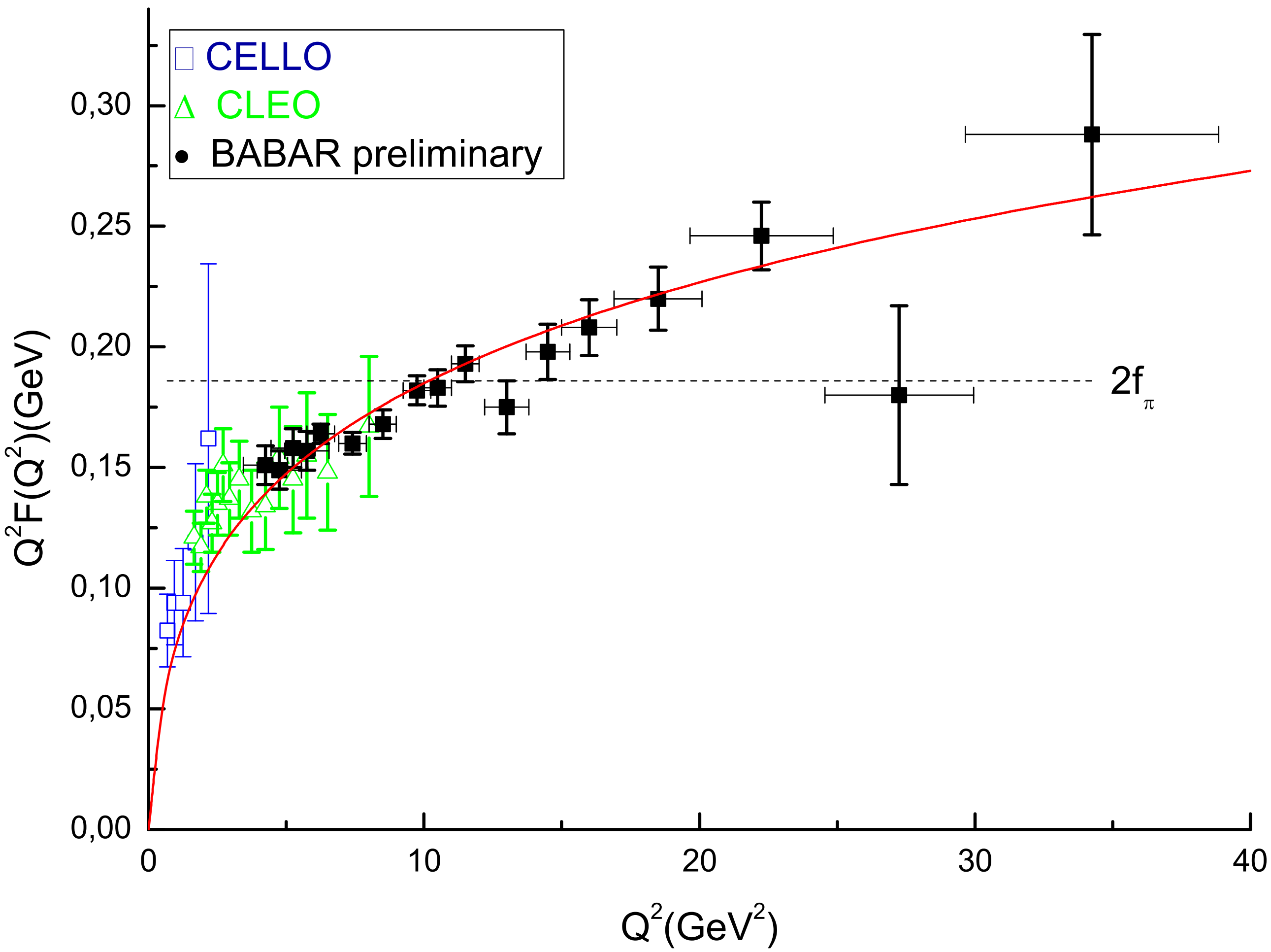
M_Q=135MeV

$$A_{\text{QM}}(0) = -21.0$$

$$\text{Re}\,A^{\text{theory}}\left(q^2=0\right) = -21.9 \pm 0.3$$

$$A^{KTeV}(0) = -18.6 \pm 0.9$$

$$F_\pi^{\text{quark}}(t,t) \underset{t\to\infty}{=} \frac{2M_q^2}{t} \ln\left(\frac{t}{M_q^2}\right)$$ strongly violate QCD 1/t



CELLO
CLEO
BABAR preliminary
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0
10
20
30
40
2f_π
Q^2F(Q^2)(GeV)
Q^2(GeV^2)

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