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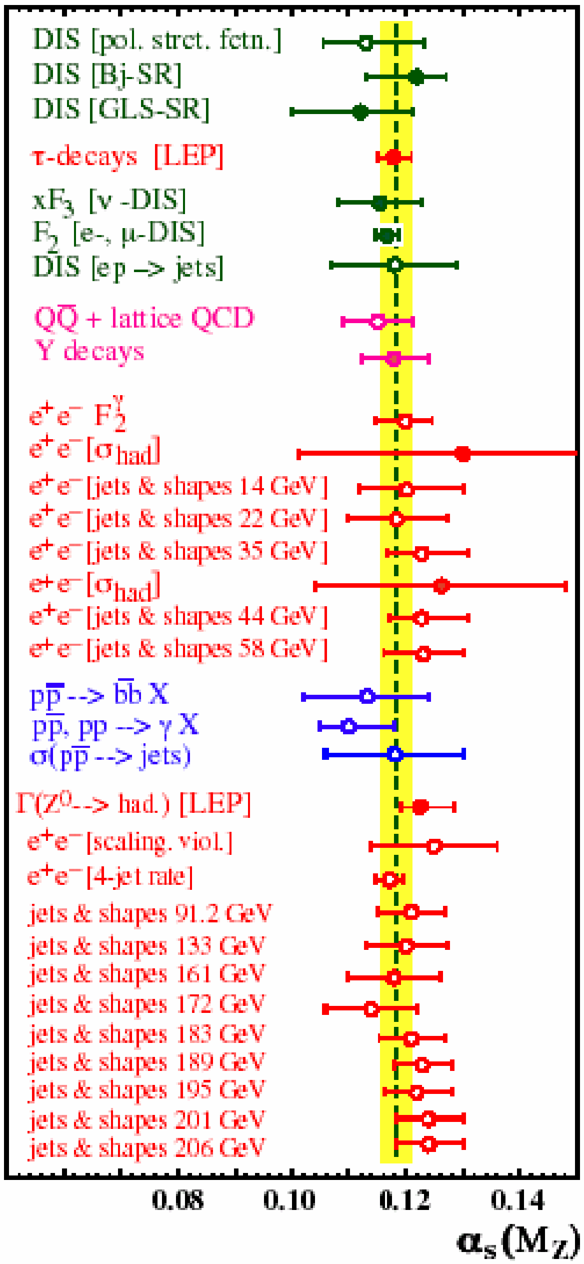
Model independent *tests of* Skyrmions and their holographic cousins

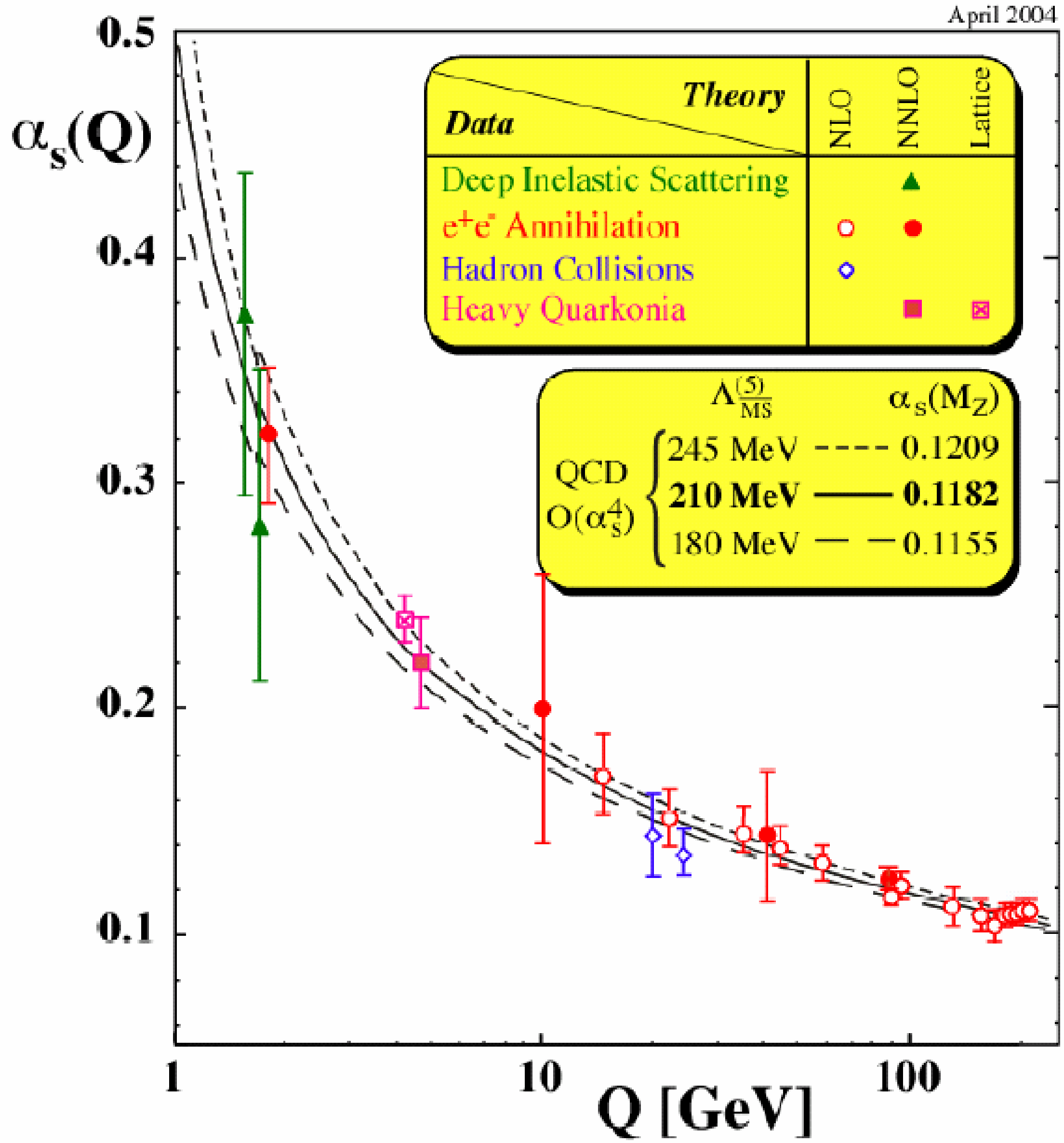
A. Cherman, T.D. Cohen, M. Nielsen, *PRL to appear*

Univ. of Maryland and Univ. of Sao Paulo

Light Cone 2009,
July 2009

QCD: strong-coupled theory





AdS/QCD models: successful at reproducing low energy hadronic observables

- two classes of AdS/QCD models

Top-down models:

arise from sting theory

D4/D8 system ⇒ gauge theory confining as QCD

AdS/CFT requires large N_c and ´t Hooft coupling

Botton-down models:

QCD large N_c ⇒ dual to a classical 5D theory

field content 5D matched

to low energy chiral symmetry of QCD

Both cases: large N_c is required

- QCD is a weakly-interacting theory of long-lived mesons in the large N_c limit. — 't Hooft, 1973
- In the large N_c limit, baryons are 'soliton-like' configurations of meson fields. — Witten, 1979
  - Baryon masses scale as N_c^1(composed of N_c quarks), interactions with mesons scale as $N_c^{1/2}$.
    - Unlike mesons, baryons are not narrow at large N_c.
    - Nucleons, deltas, become degenerate. Mass splitting scales as N_c^{-1}.
  - In contrast to the pure meson sector, meson loops make leading order contributions to baryon properties at large N_c.
- Variety of baryon models (Skyrme models) use large N_c properties of baryons for inspiration. — Skyrme, 1961
  - Baryons modeled as quantum states of slowly rotating hedgehog Skyrmions of meson fields. — Adkins, Nappi, Witten, 1983

Dashen, Manohar, Jenkins,1993-95

Model-independent relations for baryons valid at large N_c

- Goldberger-Treiman relation: $m_N g_A = f_\pi g_{\pi NN}$
- Relation is model-indepedent, and follows from chiral symmetry.
- Nucleon and delta couplings with pions are related by
- $$2g_{\pi NN} = 3g_{\pi N\Delta}$$
  - This relation follows just from the large N_c limit.
- New model-independent relation:
  - Ratio of nucleon form factors in position space, evaluated at large distances: $$\lim_{r\to\infty} \frac{G_E^{I=0}(r)G_E^{I=1}(r)}{G_M^{I=0}(r)G_M^{I=1}(r)} = \frac{18}{r^2}$$
  - Relation depends on ***both*** the large N_c and chiral limits.
  - Can serve as a useful and highly non-trivial probe of large N_c baryon models.

Definitions

$$G_E^{I=0}(r) = \frac{1}{4\pi}\int d\Omega \langle p\uparrow | J^0_{I=0} | p\uparrow\rangle,$$

$$G_M^{I=0}(r) = \frac{1}{4\pi}\int d\Omega \frac{1}{2}\varepsilon_{ij3}\langle p\uparrow | x_i J^j_{I=0} | p\uparrow\rangle,$$

$$G_E^{I=1}(r) = \frac{1}{4\pi}\int d\Omega \langle p\uparrow | J^{03}_{I=1} | p\uparrow\rangle,$$

$$G_M^{I=1}(r) = \frac{1}{4\pi}\int d\Omega \frac{1}{2}\varepsilon_{ij3}\langle p\uparrow | x_i J^{j3}_{I=1} | p\uparrow\rangle,$$

$$J^{\mu 3}_{I=1} = \mathrm{Tr}\left[J^{\mu}_{I=1}\sigma^3\right]$$

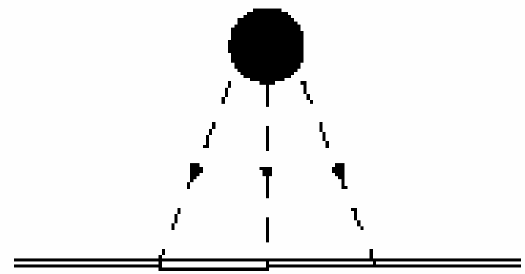
These position-space form factors can be related to the standard (experimentally accessible) momentum-space form factors by the appropriate Fourier transforms.

Model independence of the ratio

Leading large r behavior of form factors in large N_c ChiPT are mediated by pions

$$G_E^{I=0}(r)$$

$$G_M^{I=0}(r)$$

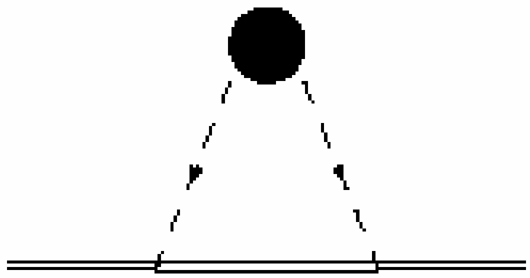

(a)Isoscalar

Three pion interaction - comes from anomaly.

$$\sim \frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3$$

$$G_E^{I=1}(r)$$

$$G_M^{I=1}(r)$$


(b)Isovector

Two pion interaction.

$$\sim \left(\frac{g_A}{f_\pi}\right)^2$$

- In the large N_c limit, leading contributions to isovector electric and isoscalar magnetic form factors are sensitive to the nucleon-delta mass splitting *~1/N_c* (Cohen PLB 359)

$$G_E^{I=1} \propto \Delta \left(\frac{g_A}{f_\pi}\right)^2 \qquad G_M^{I=0} \propto \frac{\Delta}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3$$

$$G_M^{I=1} \propto \left(\frac{g_A}{f_\pi}\right)^2 \qquad G_E^{I=0} \propto \frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3$$

$$\lim_{r\to\infty} \frac{G_E^{I=0}(r)\,G_E^{I=1}(r)}{G_M^{I=0}(r)\,G_M^{I=1}(r)}$$

ratio is independent is independent of

$g_A \quad f_\pi \quad \Delta$

Skyrme-type models

$$Ł = \frac{1}{2^6} f_\pi^2 \ \mathrm{Tr}\ [\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{2^5 e^2}\ \mathrm{Tr}\ [(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2$$

$$U = e^{\frac{i\vec{\pi}.\vec{\tau}}{f_\pi}}$$ ➡ SU(2) matrix

- Original Skyrme model includes only pion fields.
  - Baryons appear as quantum states of slowly rotating hedgehog Skyrmions of pion fields.

hedgehog ansatz: $\vec{\pi} = f(r)\vec{\tau}$

in the large *r* limite *f(r)* goes like η / r^2

in the Skyrme model: $$g_A = \frac{8\pi}{3} f_\pi^2 \eta$$

$$\lim_{r\to\infty} G^E_{I=0} = \frac{3^3}{2^9\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3 \frac{1}{r^9}$$

$$\lim_{r\to\infty} G^M_{I=0} = \frac{3\Delta}{2^9\pi^5}\frac{1}{f_\pi^3}\left(\frac{g_A}{f_\pi}\right)^3 \frac{1}{r^7}$$

$$\lim_{r\to\infty} G^M_{I=1} = \frac{1}{2^5\pi^2}\left(\frac{g_A}{f_\pi}\right)^2 \frac{1}{r^4}$$

$$\lim_{r\to\infty} G^E_{I=1} = \frac{\Delta}{2^4\pi^2}\left(\frac{g_A}{f_\pi}\right)^2 \frac{1}{r^4} .$$

$$\lim_{r\to\infty} \frac{G_E^{I=0}(r)G_E^{I=1}(r)}{G_M^{I=0}(r)G_M^{I=1}(r)} = \frac{18}{r^2}$$

- In all known cases leading long distance physics in Skyrme-like models precisely reproduces the results of large N_c chiral pert. theory.
- Ratio dependes only on long-distance physics which is the same in all Skyrme-type models

5D Skyrmions

Pomarol-Wulzer holographic baryon model

Pomarol, Wulzer, NPB809:347-361,2009.

Panico, Wulzer, arXiv:0811.2211

- Bottom-up AdS/QCD model: uses hard-wall AdS background, with two *U(2)* 5D gauge fields, L_M R_M, associated with 4D left and right quark currents (AdS/CFT dictionary).
    - Chiral symmetry broken by choice of IR boundary conditions for 5D gauge fields.
    - Has a 5D CS term to get anomaly physics right.

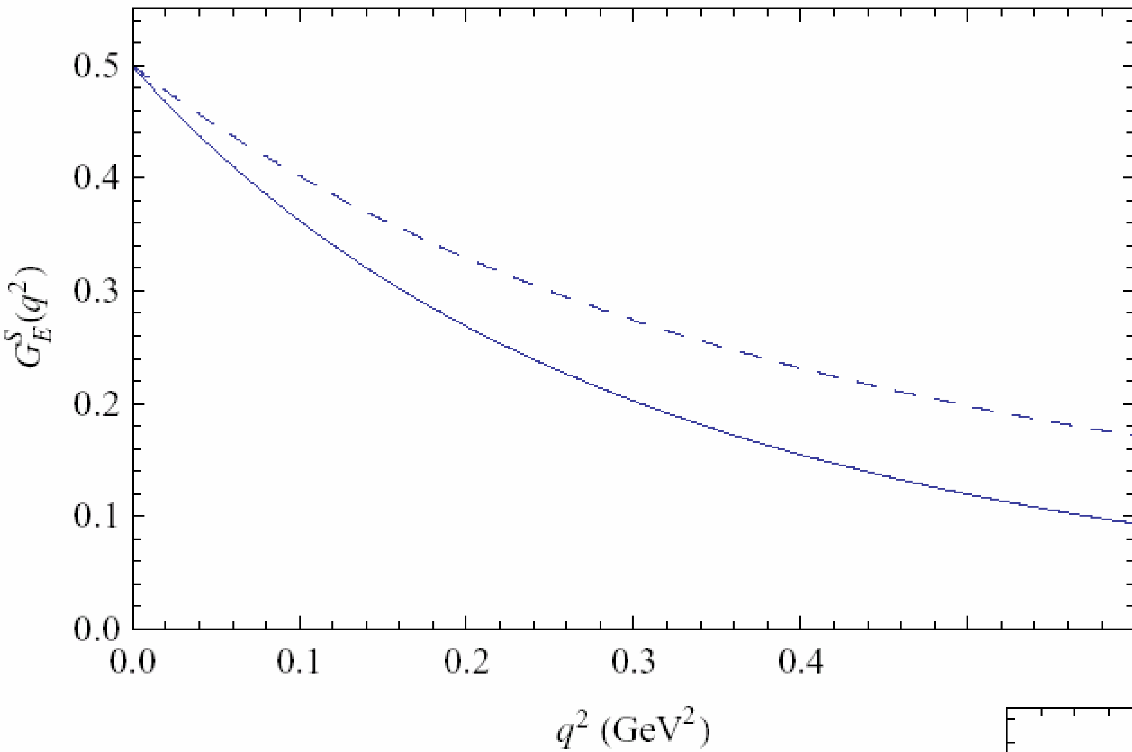
$$S = -\frac{M_5}{2}\int d^5x\sqrt{g}\ \mathrm{Tr}\ [\mathbf{L}^2_{MN} + \mathbf{R}^2_{MN}] + \frac{-iN_c}{24\pi^2}\int_{5D}[\omega_5(\mathbf{L}) - \omega_5(\mathbf{R})]$$

$$M_5 \sim \mathcal{O}(N_c^1)$$

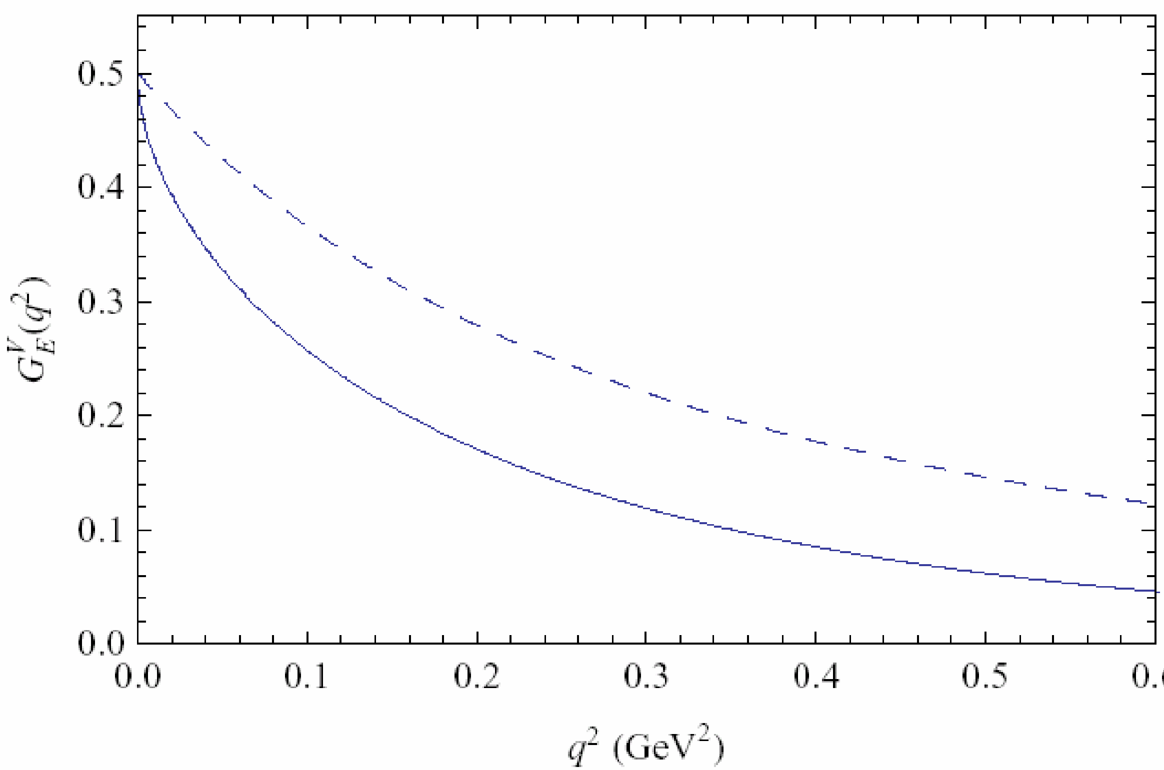
- this model looks *quite* different from 4D Skyrme models: there are no explicit pion fields.
- Baryons appear as quantum states of slowly rotating Skyrmion-like hedgehog configurations of the 5D gauge field.
- Skyrmions are stabilized by the CS term.

	Experiment	AdS_5	Deviation
M_N	940 MeV	1130 MeV	20%
μ_S	0.44	0.34	30%
μ_V	2.35	1.79	31%
g_A	1.25	0.70	79%
$\sqrt{\langle r^2_{E,S}\rangle}$	0.79 fm	0.88 fm	11%
$\sqrt{\langle r^2_{E,V}\rangle}$	0.93 fm	∞	
$\sqrt{\langle r^2_{M,S}\rangle}$	0.82 fm	0.92 fm	12%
$\sqrt{\langle r^2_{M,V}\rangle}$	0.87 fm	∞	
$\sqrt{\langle r^2_{A}\rangle}$	0.68 fm	0.76 fm	12%
μ_p/μ_n	-1.461	-1.459	0.1%

g_A = 0.65 in the original Skyrme model



0.0
0.1
0.2
0.3
0.4
0.5
$G_E^S(q^2)$
q^2 (GeV^2)



0.0
0.1
0.2
0.3
0.4
0.5
0.6
$G_E^V(q^2)$
q^2 (GeV^2)

in the large r limit

$$\lim_{r\to\infty} G_E^{I=0} = -\frac{\beta^3 L^6}{\pi^2}\frac{1}{r^9}$$

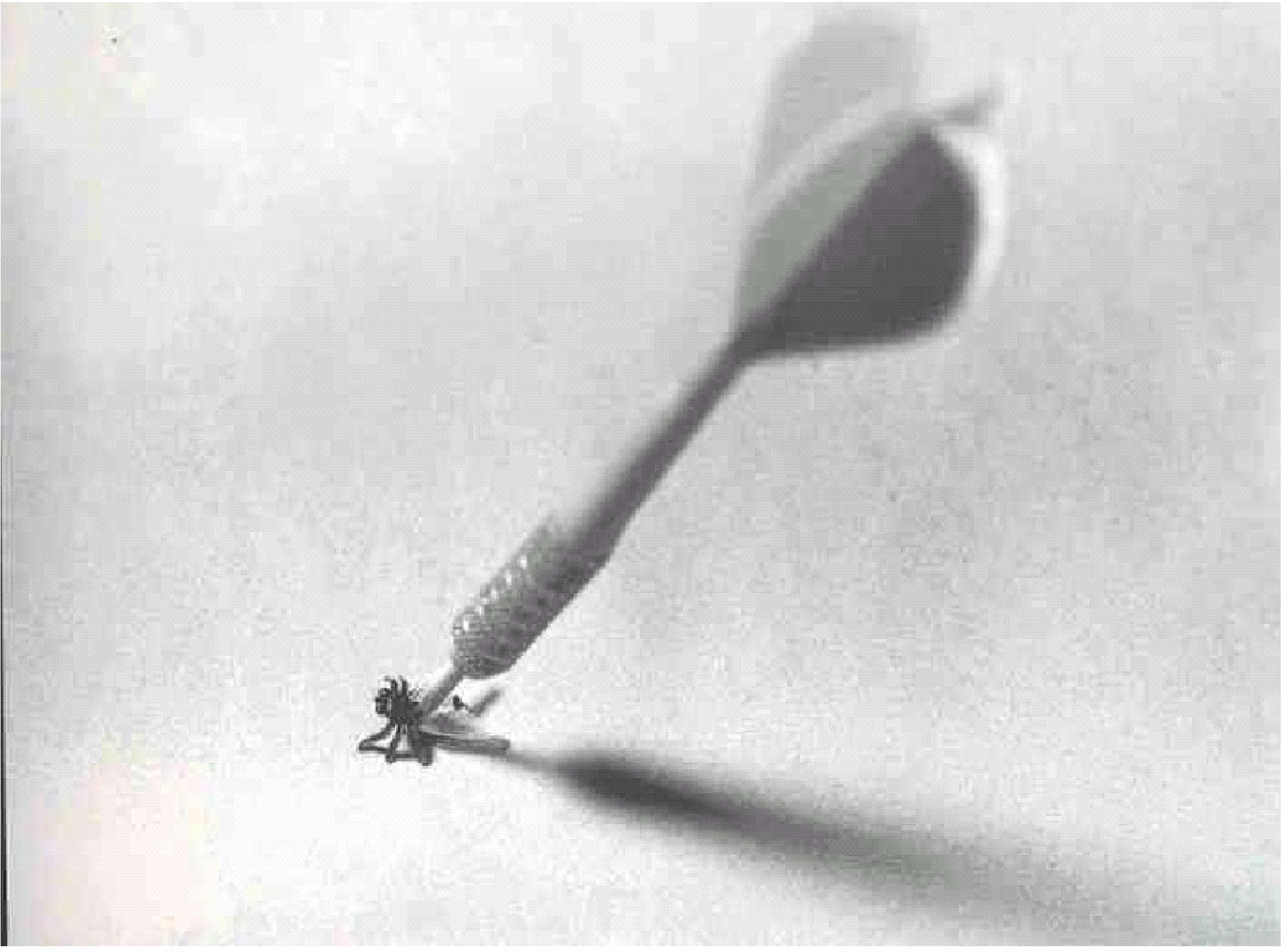
$$\lim_{r\to\infty} G_M^{I=0} = \frac{\beta^3 L^6}{6\pi^2\mathcal{I}}\frac{1}{r^7}$$

$$\lim_{r\to\infty} G_E^{I=1} = \frac{8\beta^2}{3\mathcal{I}} M_5 L^3 \frac{1}{r^4}$$

$$\lim_{r\to\infty} G_M^{I=0} = -\frac{8\beta^2}{9} M_5 L^3 \frac{1}{r^4}$$

$$\lim_{r\to\infty} \frac{G_E^{I=0}(r)G_E^{I=1}(r)}{G_M^{I=0}(r)G_M^{I=1}(r)} = \frac{18}{r^2}$$

large N_c and chiral physics handled correctly in Pomarol-Wulzer model



Baryons as Holographic Instantons

Sakai-Sugimoto model

Hata et al, hep-th/0701280

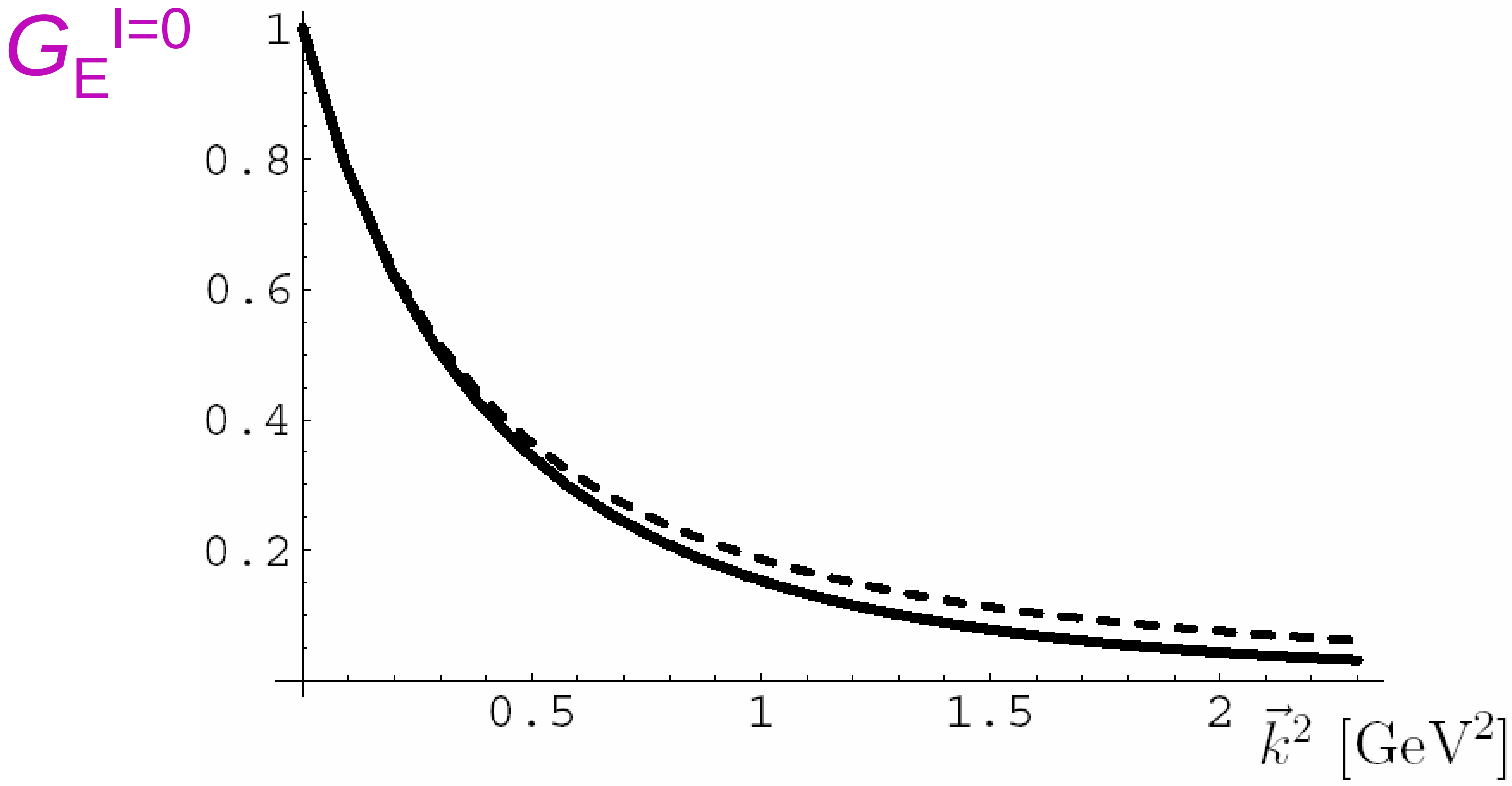
Hashimoto, Sakai, Sugimoto, PTP120:1093-1137,2008

- Top-down AdS/QCD model: baryons are described as instantons in a 5D Yang-Mills and CS theory, formulated in the D4/D8 model.

$$S = -\kappa \int d^4x dz \left(\mathrm{Tr}\, \frac{\mathbf{F}_{\mu\nu}^2}{2}(1+z^2)^{-1/3} + (1+z^2)\mathbf{F}_{\mu z}^2 \right) + \frac{N_c}{24\pi^2}\int \omega_5(\mathbf{A})$$

- A_M :5D U(N_f) field, F_{MN}: field strength, $\omega_5(A)$:CS 5-form
- Model expected to make sense when N_c and the 't Hooft coupling are large.

- CS term stabilizes the instanton size to be of order $\lambda^{-1/2}$
- at large λ , higher-derivative terms are **not** suppressed, and the *1/* λ expansion is not well-justified for instantons.
- Important to test if the model, as implemented, is consistent with large N_c chiral physics.



	SS model	Skyrmion[14)]	experiment
$\langle r^2 \rangle_{I=0}^{1/2}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2 \rangle_{M,I=0}^{1/2}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2 \rangle_{E,\mathrm{p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.875\ \mathrm{fm})^2$
$\langle r^2 \rangle_{E,\mathrm{n}}$	0	$-\infty$	$-0.116\ \mathrm{fm}^2$
$\langle r^2 \rangle_{M,\mathrm{p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.855\ \mathrm{fm})^2$
$\langle r^2 \rangle_{M,\mathrm{n}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.873\ \mathrm{fm})^2$
$\langle r^2 \rangle_{A}^{1/2}$	0.537 fm	–	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	–1.34	–1.31	–1.91
$\left\lvert \frac{\mu_p}{\mu_n} \right\rvert$	1.63	1.43	1.46
g_A	0.734	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	–	$4.2 \sim 6.5$

in the large *r* limit

$$\lim_{r\to\infty} G_E^{I=0}(r) = \frac{g_{v^1}\psi_1(0)}{4\pi r} e^{-\rho_1 r}$$

$$\lim_{r\to\infty} G_M^{I=0}(r) = \frac{9\pi r}{16\pi\lambda N_c} g_{v^1}\psi_1(0)\rho_1 e^{-\rho_1 r}$$

$$\lim_{r\to\infty} G_E^{I=1}(r) = \frac{g_{v^1}\psi_1(0)}{4\pi r} e^{-\rho_1 r}$$

$$\lim_{r\to\infty} G_M^{I=1}(r) = \frac{N_c}{12\pi}\sqrt{\frac{2}{15}} g_{v^1}\psi_1(0)\rho_1 \; e^{-\rho_1 r}$$

$$\lim_{r\to\infty} \frac{\tilde{G}_E^{I=0}\tilde{G}_E^{I=1}}{\tilde{G}_M^{I=0}\tilde{G}_M^{I=1}} = \frac{\lambda\sqrt{40/3}}{\pi\rho_1^2 r^2}$$

ρ_1 ⇨ related with meson rho mass

- Model does not satisfy large N_c relation.
  - Ratio depends on model parameters.
- Model fails because it does not treat chiral symmetry correctly.
  - Another signal: the isovector charge radius is finite in the model. It is known to be infinite in chiral perturbation theory in the chiral limit.
- These results suggest that the Sakai-Sugimoto instanton model fails to correctly describe the long-range part of large N_c baryon physics.

- origem of problem: large λ expansion.
- SS model: pion nucleon coupling scales as

$$\frac{g_A}{f_\pi} \sim \sqrt{\frac{N_c}{\lambda}}$$

- pion loops contributions to nucleon properties are discarded if the large λ limit is taken prior to the large N_c limit.
- It is necessary to go beyond large λ limit in the SS model to capture large N_c chiral physics.

Summary

- We discussed a model-independent large N_c relation for baryons.

$$\lim_{r\to\infty} \frac{G_E^{I=0}(r)G_E^{I=1}(r)}{G_M^{I=0}(r)G_M^{I=1}(r)} = \frac{18}{r^2}$$

- Relation probes consistency of implementation of chiral and large N_c physics in baryon models.
  - In this case, the probe reveals that some holographic models get large N_c chiral physics right, while others do not.
  - This situation illustrates the utility of large N_c analysis as a diagnostic tool for probing models.
- Relation should be checked in other new large N_c baryon models.