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QCD tests with an infrared finite gluon propagator and coupling constant

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Plan of talk

- **Infrared finite gluon propagator and coupling constant**
- **Phenomenology of IR finite gluon propagator and coupling constant: Tree level**
- **Phenomenology of IR finite gluon propagator and coupling constant: Loop level, Bjorken sum rule**
- **Concluding Remarks**

Infrared finite gluon propagator and coupling constant

Schwinger-Dyson equations (+ lattice QCD) $\longrightarrow$

1) gluon progagator is IR finite

2) coupling constant freezes as $q^2 \to 0$ (fixed-point)

How to use it? $\longrightarrow$ **D**ynamical **P**erturbation **T**heory : (Pagels and Stokar, 1979)

Amplitudes that do not vanish to all orders in perturbation theory are given by their free field values, while amplitudes that vanish as $\lambda \propto e^{-1/g^2}$ are retained, and possibly dealt with in an expansion in $g^n \lambda$

($\to$...work with dressed quantities)

How are the solutions?

$$\imath\Delta_{\mu\nu}(q) = P_{\mu\nu}\Delta(q) + \xi\frac{q_\mu q_\nu}{q^4} \; ; \; P_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \; ,$$

where $\Delta(q)$ is the gauge invariant scalar part of the gluon propagator (in Euclidean space $\longrightarrow$)

$$\Delta(Q^2) \propto \frac{1}{Q^2 + m_g^2(Q^2)}$$

We can write a new propagator $\hat{\Delta}^{-1}(Q^2)$ $\longrightarrow$ absorbs all the renormalization group logs, and form the product (renormalization group invariant)

$$\hat{d}(Q^2) = g^2\hat{\Delta}(Q^2)$$

The dynamical gluon mass $(m_g^2(Q^2))$ is given by a complicated expression falling with the momentum as $1/Q^2$

In actual calculations (Aguilar & AN) $\longrightarrow$

$$m_g^2(Q^2) = \frac{m_g^4}{Q^2 + m_g^2}$$

$m_g \approx \mathcal{O}(1.2-2)\Lambda$, with $\Lambda = \Lambda_{QCD} \approx 300$ MeV (in some cases assume $m_g^2(Q^2) \approx m_g^2$).
Do the same for the dynamical quark mass $m_q \approx \mathcal{O}(1)\Lambda$

A simple fit for the coupling constant

$$\bar{\alpha}_{sd}(q^2) = \frac{1}{4\pi b \ln[(4m_g^2 - q^2 - \imath\epsilon)/\Lambda^2]}$$

$b = (33 - 2n_f)/48\pi^2$ —— The IR fixed-point :

$$\bar{\alpha}_{sd}(0) \equiv \frac{1}{4\pi b \ln[(4m_g^2)/\Lambda^2]}$$

Phenomenology of IR finite gluon propagator and coupling constant: Tree level

Examples: Pion form factor, Pomeron and QCD models for hadronic cross sections, survival probability for rapidity gaps, gluon structure functions at small-x,$\gamma - p$ and $\gamma - \gamma$ cross sections, proton form factor, ...

Braz.J.Phys,37 (2007)306; PRD73 (2006) 074019; PLB641 (2006) 171; PRD72 (2005) 034019; Int.J.Mod.Phys.A19 (2008) 151; ....

The “new IR scale” (... gluon mass) provides a natural IR cut-off!

One problem - gluons in the initial or final state:

1) Phase space

2) Sum of gluon polarization?

example: $g + g \to g + g$ (Luna, A.N., ... Phys.Rev.D72:034019,2005) (sum over 2 degrees of polarization)

How to deal with this ????

non-leptonic annihilation B mesons decays [A.Natale & C.Zanetti, hep-ph/0803.0154 (in press)]

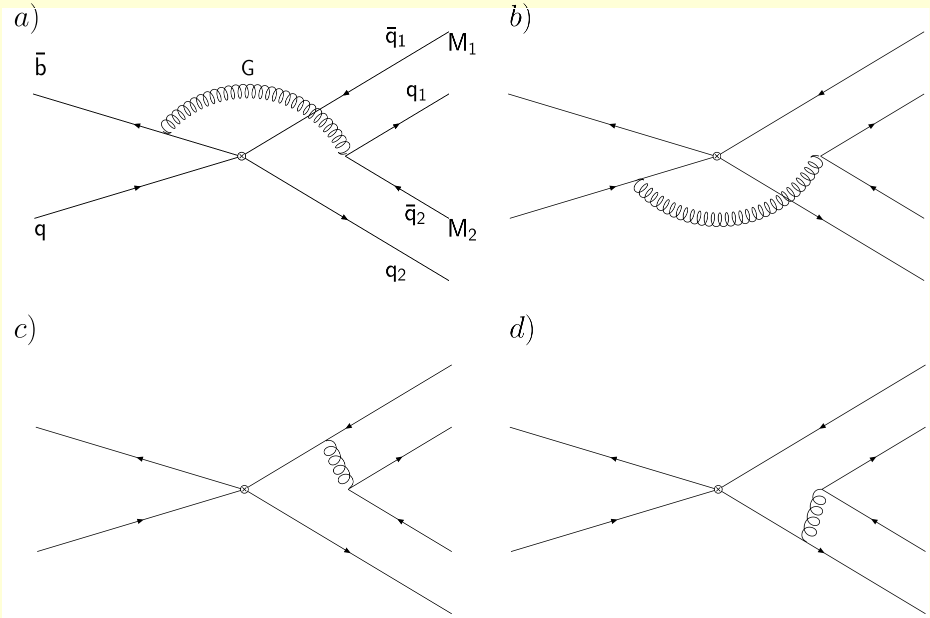

Figure 1: two-body non-leptonic annihilation B decays

(end-point divergences...) $\int \frac{dx}{x} = \ln \frac{m_B}{\Lambda_h}(1+\rho e^{i\phi}) \quad 0 \leq \rho \leq 1$

$\mathcal{B}r(B_d^0 \to K^+K^-) \times 10^8 \to\ 7.18$, $\mathcal{B}r(B_d^0 \to D_s^- K^+) \times 10^5 \to\ 1.98$, ...

(compatible with existent data - without any *ad hoc* cutoff)

Phenomenology of IR finite gluon propagator and coupling constant: Loop level, the Bjorken sum rule

Work in progress with A. C. Aguilar and J. Papavassiliou

Pinch Technique is fundamental in order to apply DPT at loop level

Pinch Technique provide skeleton expansion as proposed by Brodsky et al. (PRD **63**, 094017 (2001))

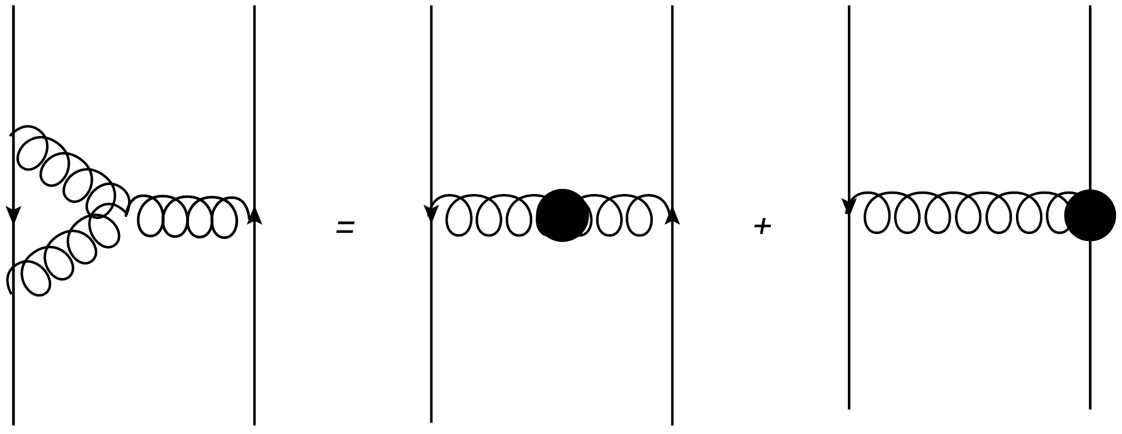

Figure 2: One of the several contributions for quark-quark scattering at one-loop where a typical non-Abelian contribution can be separated into contributions to the vertex and propagator. The black blob indicates the pinched parts that enter into the vertex and propagator at 1-loop level.

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx\ \left[g_1^p(x, Q^2) - g_1^n(x, Q^2)\right]$$

$g_1^p(g_1^n)$: first spin structure function for the proton (neutron).

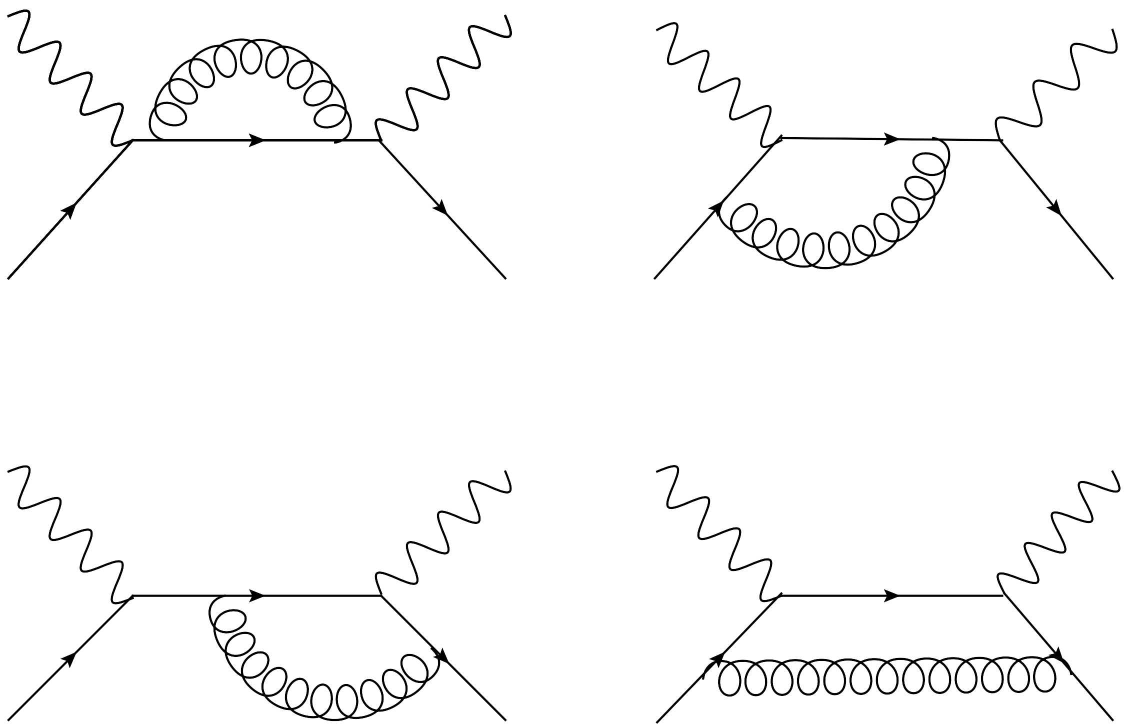

Figure 3: Perturbative corrections to the deep inelastic structure functions at 1-loop.

QCD correction up to third order in α_s $(n_f = 3)$

$$\Gamma_1^{p-n}(Q^2) = \frac{1}{6}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]$$

Pert. QCD $\rightarrow$ DPT dressed quantities...

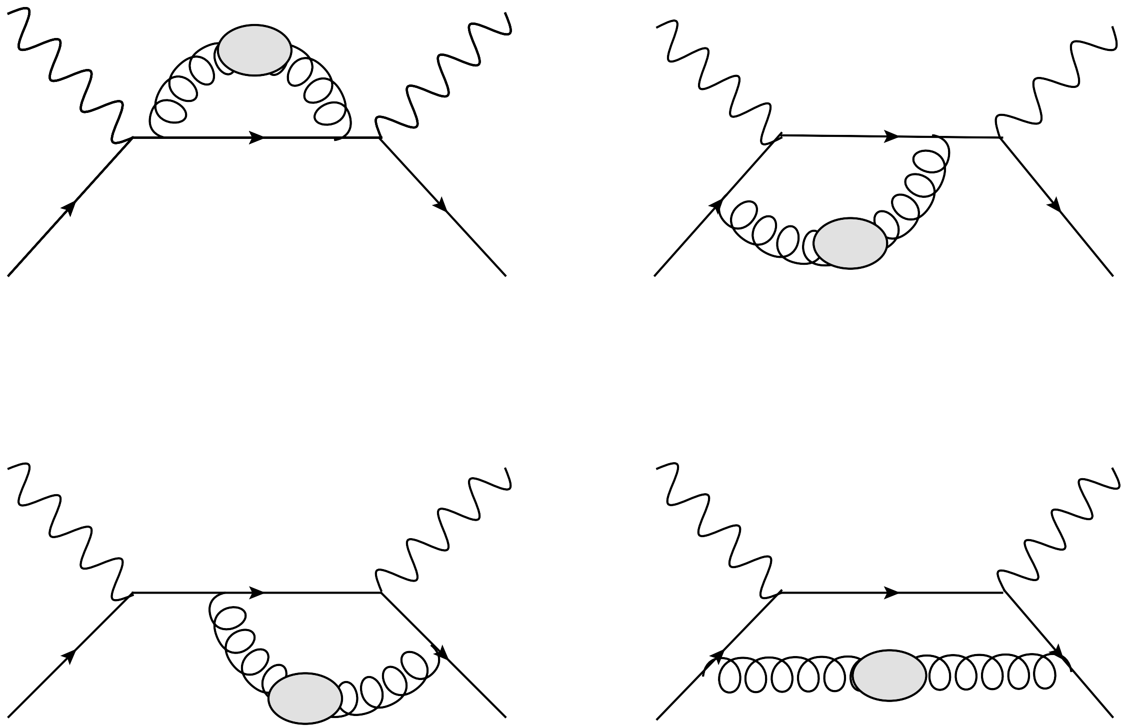

Figure 4: Leading one-loop contributions for the Bjorken sum rule in DPT. The gray blobs indicate the all-order sum of diagrams in the gluon polarization vector.

The Bj sum rule in DPT $\rightarrow$

$$\Gamma^{p-n}(Q^2) = \frac{1}{6}\left|\frac{g_A}{g_V}\right| \Big[1 - \frac{\bar{\alpha}_{sd}(Q)}{\pi} f_1(Q^2, m_g^2, m_q^2) - 3.58 \left(\frac{\bar{\alpha}_{sd}(Q)}{\pi}\right)^2 f_2(Q^2, m_g^2, m_q^2) - 20.21 \left(\frac{\bar{\alpha}_{sd}(Q)}{\pi}\right)^3 f_3(Q^2, m_g^2, m_q^2) + \ldots \Big]$$

The functions f_i are of the form $1 + g(m_g^2/Q^2, m_q^2/Q^2, m_g^2/m_q^2)$, example (first order):

$$\Gamma_1^{p-n}(Q^2) = \frac{1}{6}\left|\frac{g_A}{g_V}\right| \left\{1 - \frac{\bar{\alpha}_{sd}(Q^2)}{\pi}\left[1 + \frac{m_q^2}{3Q^2}\left(\frac{11}{3}\log\frac{m_q^2}{Q^2} - \frac{10}{9}\right)\right]\right\}$$

The effect of dynamical quark mass is $\mathcal{O}(10\%)$ of the massless case (important when compared to next orders...)

The convergence is faster!

New program: SDE + DPT $\rightarrow$ compute all corrections with effect of dynamical masses!

Jlab data of $\Gamma_1^{p-n}(Q^2)$ (up to order $\bar{\alpha}_{sd}^4$) was used to obtain m_g

The result is shown in Fig.(5), which is α_s fitted with $m_g = \mathcal{O}(300-400)$ MeV, (compatible with previous phenomenological determinations of m_g)

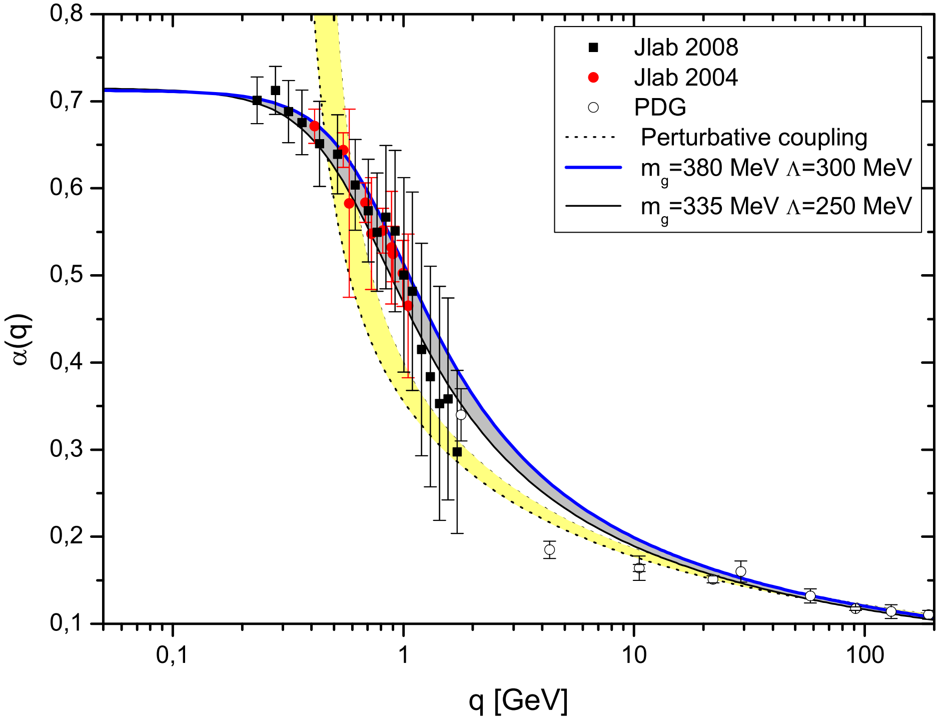

Figure 5: Effective coupling constant extracted from the experimental data of $\Gamma_1^{p-n}(Q^2)$.

The use of DPT improves the behavior of QCD perturbative series

This procedure also faces the problem of the choice of renormalization scale μ and scheme (BLM). How to deal with this problem?

Recall that any physical quantity $\mathcal{O}_i$ can be computed perturbatively and be described by a series in α_s as

$$\mathcal{O}_i = \alpha_s(\mu)\left[1 + A_i(\mu)\frac{\alpha_s(\mu)}{\pi} + B_i(\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + \ldots\right] \ ,$$

Any physical quantity like $\mathcal{O}_i$ can be used to obtain the coupling constant α_s

To do so $\rightarrow$ invert the equation to obtain a function of the form

$$\alpha_s(\mu) = f(\mathcal{O}_i, A_i(\mu), B_i(\mu), \ldots)$$

$\Gamma_1^{p-n}(Q^2)$ assumed as "favorite measurement of α_s" $\rightarrow$ **any $\alpha_s(\mu)$ obtained in the $\mathcal{O}_i$ measurement can be related to $\bar{\alpha}_{sd}$**

SDE + DPT + procedure to deal with the scale ambiguity of any other physical quantity

1) Take a physical quantity, labeled $\mathcal{O}_j$ at 1-loop order, computed in DPT;

2) Invert this quantity in order to determine $\alpha_s^j(Q)$ from it (i.e. $\alpha_s^j(Q) = f(\mathcal{O}_j(Q), A_j)$);

3) Find a commensurate scale relation (Brodsky and Lu) that maps $\alpha_s^j(Q)$ into $\bar{\alpha}_{sd}(Q)$ (with m_g obtained from $\Gamma_1^{p-n}(Q^2)$

$$\bar{\alpha}_{sd}(Q) = f(\mathcal{O}_j(Q^*), A_j^*)$$

4) Higher order contributions can be computed systematically always adjusting the value of the dynamical gluon mass.

Concluding Remarks

- **IR finite gluon propagator and coupling constant: natural cutoff (no renormalons)**

- **Tree level calculations: Use DPT $\equiv$ use dressed quantities. Phase space and sum over gluon polarizations**
(there is always some wave function, distribution function,...)

- **Loop level calculations: Pinch technique in order to apply DPT**
(approximations in the SDE solutions...)

- **IR fixed point: Use it to fix the renormalization scheme!**