Light-Cone 2009: Relativistic Hadronic and Particle Physics

Instituto de Física
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Glueballs at finite temperature from AdS/QCD

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Outline

1. AN OVERVIEW ON THE ADS/CFT CORRESPONDENCE

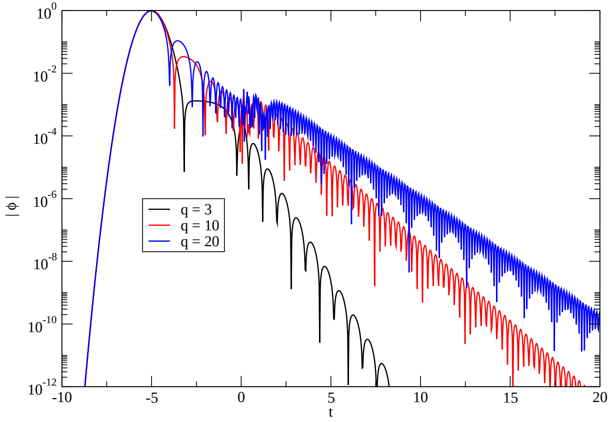
Original formulations [J. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998)].

String theory (or M theory) on AdS background — dual — Conformally invariant field theories (CFTs) on the boundary

- Basis for traditional applications to QCD and QGP:
  - Type IIB string theory in $AdS_5 \times S^5$, which is the near-horizon geometry of a set of N_c D3-branes;
  - $\mathcal{N} = 4$ $SU(N_c)$ Super-Yang-Mills theory in 3+1 dimensions.
- Basis for recent applications to condensed matter physics:
  - M-theory in the presence of N_c M2-branes, which curves the background to $AdS_4 \times S^7$;
  - $\mathcal{N} = 8$ $SU(N_c)$ Super-Yang-Mills theory in 2+1 dimensions.

gravity	gauge theory
string coupling constant $4\pi g_s$	gauge coupling constant g_{YM}^2
dimensionless parameter R^4/l_s^4	't Hooft coupling constant $g_{YM}^2 N_c$
classical (super)gravity	strongly interacting theory
scalar field φ	operator $\mathcal{O} = \text{Tr}\, F_{\mu\nu}F^{\mu\nu}$
mass of the field	scaling dimension of the operator
isometries of the AdS space	conformal symmetries
black hole (brane)	thermal equilibrium system
parameters $(M,\ J,\ Q)$	energy, momentum, charge
$T_{Hawking},\ S_{Bekenstein-Hawking}$	temperature, entropy
partition function	generator of Green's functions
gravitational perturbations	fluctuations of the tensor $T_{\mu\nu}$
electromagnetic perturbations	fluctuations of the current J_μ
quasinormal modes (QNMs)	poles of the correlation functions
real part of the QN frequencies	energy of the collective excitations
imaginary part of the QN freq.	thermalization time scale

What are the black hole (brane) quasinormal modes?



C. V. Vishveshwara, *Nature* **227**, 936 (1970).

2. TOP-DOWN AND BOTTOM-UP HOLOGRAPHIC MODELS

Some examples of holographic models

- Top-down models:
  - D3/D7 branes: N_c colors and, usually, $N_f \ll N_c$ flavors;
    A. Karch and E. Katz, *JHEP* **06**, 043 (2002).
  - D4/D8 branes: flavor degrees of freedom from D8-$\bar{\text{D8}}$ pairs.
    T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005).
- Bottom-up models:
  - Hard wall (HW): introduction of an IR cutoff at AdS space.
    J. J. Polchinski and M. J. Strassler, *PRL* **88**, 031601 (2002).
    H. Boschi-Filho and N.R.S. Braga, *JHEP* **05**, 009 (2003).
    G.F. de Téramond and S.J. Brodsky, *PRL* **94**, 201601 (2005).
  - Soft wall (SW): presence of a dilatonlike field $\Phi(z) = cz^2$.
    J. A. Karch et. al., *PRD* **74**, 015005 (2006).
    - Meson spectrum: $m^2_{n,S} = 4c(n+S)$;
    - Glueball spectrum: $m^2_n = 4c(n+2)$.

    J. P. Colangelo et. al., *Phys. Lett. B* **652**, 73 (2007).

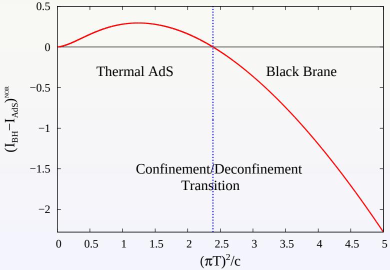
2. THE SOFT-WALL MODEL AT FINITE TEMPERATURE

Action and phase transition [speed of light $= \hbar = k_B = 1$]

- Action for φ in the soft-wall model:

$$I = -\frac{\pi^3 R^5}{4\kappa_{10}^2} \int d^5x \sqrt{-g} e^{-\Phi} g^{MN} \partial_M \varphi \partial_N \varphi \, .$$

- Free energy in the soft-wall model: C. Herzog, *PRL* **98**, 091601 (2007); C. A. Ballon Bayona et. al., *PRD* **77**, 046002 (2008).



- Transition temperature: $(\pi T)^2/c = 2.38643\ldots$

The black brane solution

- Spacetime metric:

$$ds^2 = e^{2A(z)} \left[-f(z)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + f(z)^{-1} dz^2 \right],$$

where

$$f(z) = 1 - (z/z_0)^4$$

and

$$A(z) = -\ln(z/R).$$

- Anti-de Sitter radius: $$R = (-6/\Lambda)^{1/2}.$$

- Hawking temperature: $$T = 1/\pi z_0 .$$

4. THERMAL TWO-POINT CORRELATION FUNCTION

Massless scalar-field perturbations

- Linearized equation for φ on the black-brane spacetime.
- Fourier transformation of φ with $k_\mu = (\omega, 0, 0, q)$.
- In terms of $\psi = e^{-B/2}\varphi$ $(B = \Phi - 3A)$, we have

$$\partial_{r_*}^2 \psi + \omega^2 \psi = V\psi,$$

where $dz/dr_* = -f$ and $V = f\left(q^2 + e^{B/2}\partial_{r_*}^2 e^{-B/2}\right)$.

- Normalizable and non-normalizable solutions:

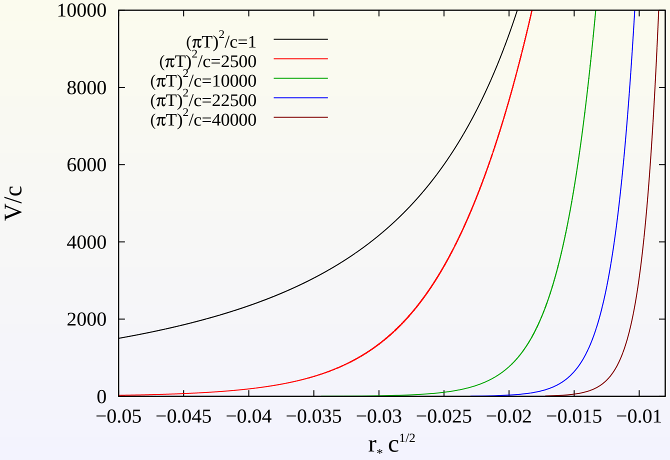
$$\psi_1(z) = z^{5/2}(1+\ldots), \quad \psi_2(z) = z^{-3/2}(1+\ldots) + a\ln(z^2)\psi_1(z).$$

- The ingoing solution at horizon $\psi^{in}(z)$ can be written as:

$$\psi^{in}(z) = \mathscr{A}(\omega, q)\psi_2(z) + \mathscr{B}(\omega, q)\psi_1(z).$$

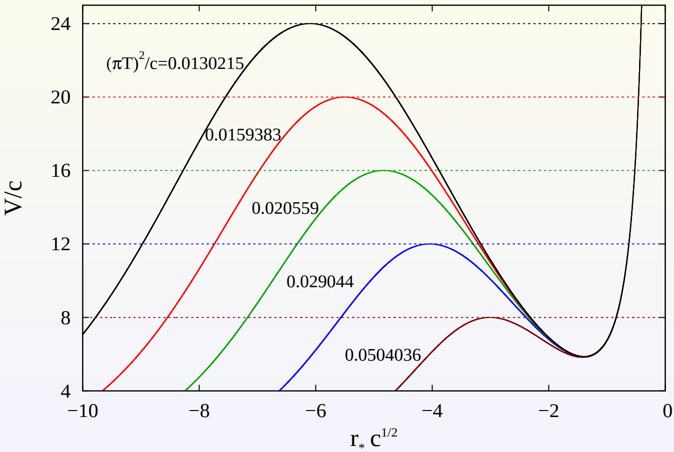
Effective potential for high temperatures

- Potential in the high-temperature regime and $q = 0$.



Effective potential for low temperatures

- Potential in the low-temperature regime and $q = 0$.



The retarded Green function

- Definition:

$$G^R(\mathbf{x}-\mathbf{y}) = -i\theta(x^0-y^0)\langle[\mathscr{O}(\mathbf{x}),\mathscr{O}(\mathbf{y})]\rangle.$$

- We study the function G^R in the momentum space.
- Real-time AdS/CFT prescription: [D. T. Son and A. O. Starinets, *JHEP* **09**, 042 (2002).]

$$G^R(k) = \frac{\pi^3 R^5}{2\kappa_{10}^2} \lim_{z\to 0} \sqrt{-g}\, g^{zz} e^{-\Phi} \varphi_k^*(z)\partial_z\varphi_k(z)\,,$$

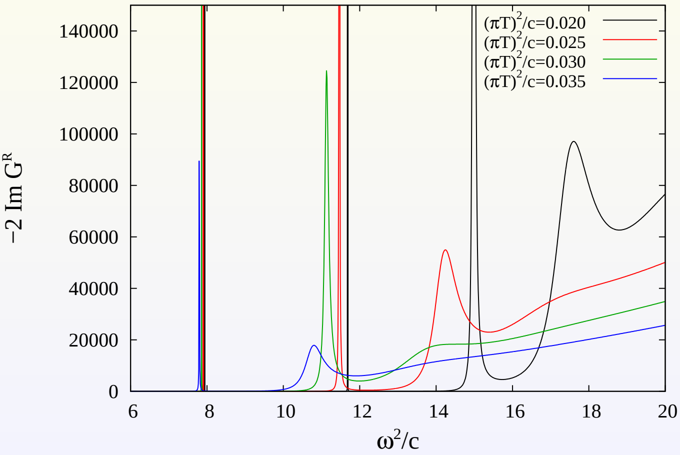
where $\varphi(k,z) = \varphi_k(z)\varphi_0(k)$.

- From that follows the Green function:

$$G^R(k) = \frac{\pi^3 R^8}{2\kappa_{10}^2}\left[4\mathrm{Re}\frac{\mathscr{B}}{\mathscr{A}} - i\,\mathrm{Im}\frac{\mathscr{B}}{\mathscr{A}}\right] + \dots\,.$$

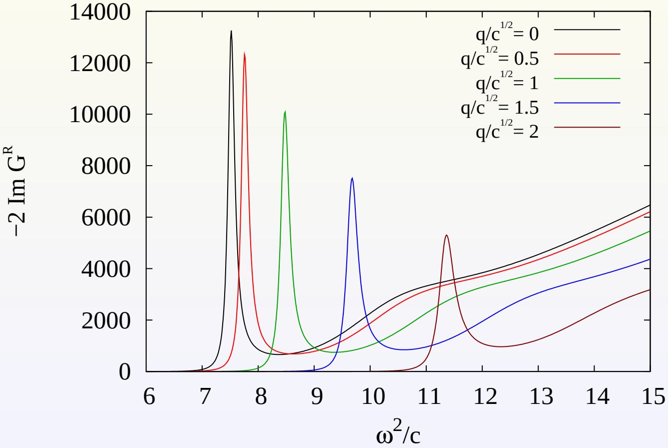
The spectral function of scalar glueballs I

- Numerical results for $q = 0$ and selected values of temperature



The spectral function of scalar glueballs II

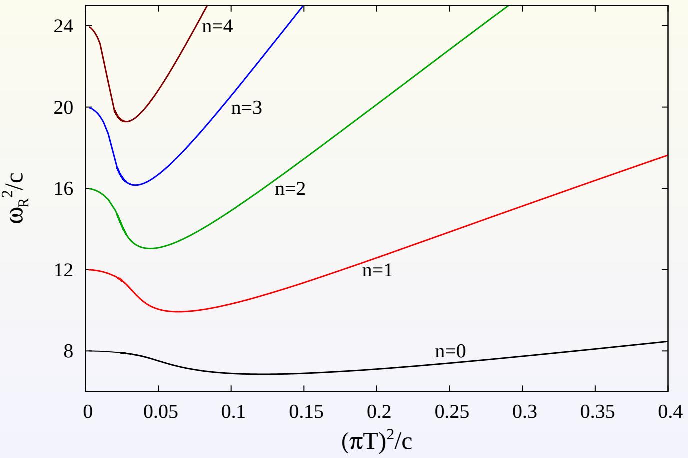
- Results for $(\pi T)^2/c = 0.05$ and selected values of $q/\sqrt{c}$.



6. THE SPECTRUM OF QUASINORMAL FREQUENCIES

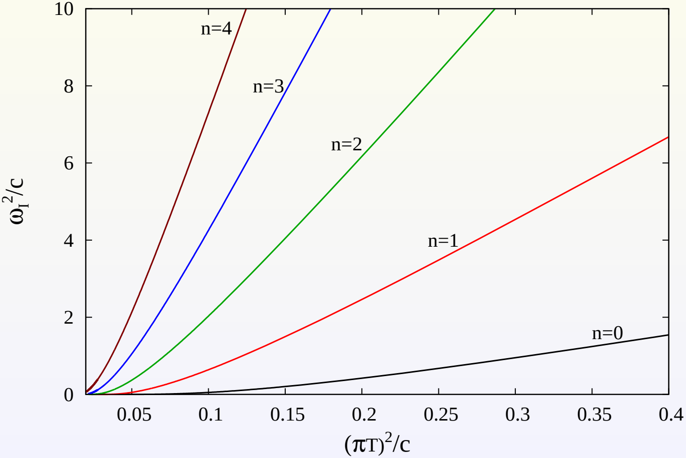
Real part of the frequencies

- Results for the first five quasinormal frequencies and $q = 0$



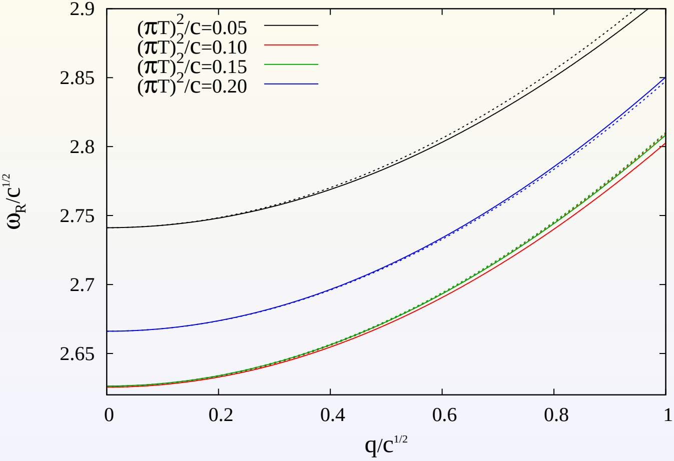
Imaginary part of the frequencies

- Results for the first five quasinormal frequencies and $q = 0$



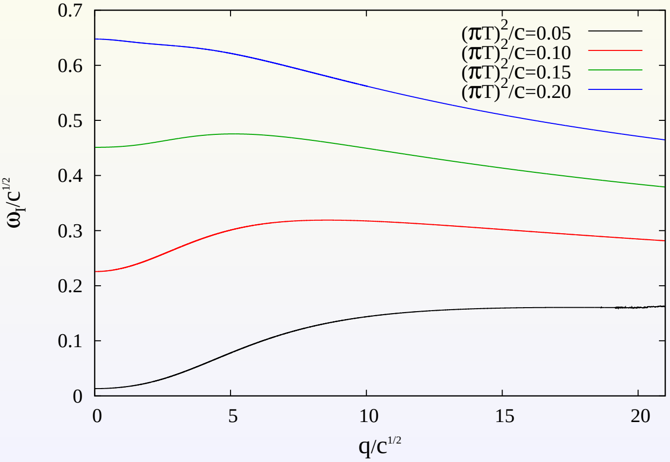
The QNM dispersion relations

- Results for the $n = 0$ mode and selected values of temperature



Momentum dependence of the width

- Results for the $n = 0$ mode and selected values of temperature



6. CONCLUDING REMARKS

Overlook and perspectives

- Spectral function is featureless in the plasma phase;
- Presence of well-defined peaks in the metastable phase;
- Strong indications of glueball formation at low temperatures;
- Similar results for J/ψ mesons in the SW model;
  M. Fujita et. al., arXiv:0903.2316 [hep-ph]
- Similar metastable-phase behavior appeared in the D3/D7-brane model; R. C. Myers et. al., *JHEP* **11**, 091 (2007).
- Indication of a qualitative universal behavior!?
- More studies are necessary, including modifications of the basic SW model;
  W. de Paula et. al., *PRD* **79**, 075019 (2009).
  U. Gursoy and E. Kiritsis, *JHEP* **02**, 032 (2008).
- A derivation of the soft-wall model from string/M theory should be welcome!