Treacherous Points in DVCS

Revealing some conceptual problems

Ben Bakker

Vrije Universiteit
Faculty of Sciences
Department of Physics and Astronomy

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Outline

Issues: Kinematics, Completeness of GPD formulation

$$\mathcal{A} = \int_{-1}^{1} \mathrm{d}x \, T(x, \zeta, Q^2) \, \mathcal{F}(x, \zeta, t)$$

Reveal treacherous points in the GPD formulation

Treacherous points

- Zero modes
- Arc contributions
- LF divergences

Study DVCS in a simple analytic model: the relativistic quark model, scalar DVCS.

Hadron kinematics: BDH

LFD Kinematics from S.J. Brodsky, M. Diehl and D.-S. Hwang, Nucl. Phys. B **596**, 99 (2001)

$$
\begin{aligned}
p^\mu &= \left(p^+, \mathbf{0}_\perp, \frac{m^2}{2p^+}\right), \\
q^\mu &= \left(0, \mathbf{q}_\perp, \frac{(\mathbf{q}_\perp + \Delta_\perp)^2}{2p^+} + \frac{\zeta m^2 + \Delta_\perp^2}{2(1-\zeta)p^+}\right) \\
p'^\mu &= \left((1-\zeta)p^+, -\Delta_\perp, \frac{m^2 + \Delta_\perp^2}{2(1-\zeta)p^+}\right), \\
q'^\mu &= \left(\zeta p^+, \mathbf{q}_\perp + \Delta_\perp, \frac{(\mathbf{q}_\perp + \Delta_\perp)^2}{2\zeta p^+} + \frac{\zeta m^2 + \Delta_\perp^2}{2(1-\zeta)p^+}\right)
\end{aligned}
$$

$q^2 = -q_\perp^2 = -Q^2 \rightarrow |\mathbf{q}_\perp| = Q$, take $\mathbf{q}_\perp = (Q, 0)$ and $\Delta_\perp = 0$.

Kinematics

Reminder 1: LF boosts of momenta

$$B_{\mathrm{LF}}(\mathbf{v}_\perp;\chi) = \begin{pmatrix} e^{\chi} & 0 & 0 & 0 \\ v_x & 1 & 0 & 0 \\ v_y & 0 & 1 & 0 \\ \frac{\mathbf{v}_\perp^2}{2} e^{-\chi} & v_x e^{-\chi} & v_y e^{-\chi} & e^{-\chi} \end{pmatrix}$$

$B_{\mathrm{LF}}(\mathbf{v}_\perp;\chi)$ boosts a particle with velocity 0 to one with finite velocity:

$$B_{\mathrm{LF}}(\mathbf{v}_\perp;\chi)(m/\sqrt{2}, 0, 0, m/\sqrt{2}) = (p^+, p_x, p_y, p^-),$$

There does not exist an LF boost that changes the plus momentum of a particle from 0 to a finite value or *vice versa*.

Question: Is the $q^+ = 0$ kinematics necessary, consistent?

Polarization Vectors

Reminder 2: LF Polarization Vectors

$B_{\rm LF}(\mathbf{v}_\perp;\chi)$ boosts the polarization vectors from a frame where the momentum $\mathbf{q}$ is aligned with the z-axis to one in which the momentum is $q^\mu = (q^+, q_x, q_y, q^-)$. For a virtual photon take $m^2 = q^2$.

$$\begin{aligned}
\epsilon(q;+1) &= \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, -\frac{q_x + iq_y}{q^+}\right) \\
\epsilon(q;0) &= \left(\frac{q^+}{m}, \frac{q_x}{m}, \frac{q_y}{m}, \frac{q_x^2 + q_y^2 - m^2}{2mq^+}\right) \\
\epsilon(q;-1) &= \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, \frac{q_x - iq_y}{q^+}\right)
\end{aligned}$$

Treacherous point 1

If q^+ vanishes, the polarization vectors can only be defined in a frame where q_x and q_y also vanish.

Full Kinematics in Hall A of TJNF

lepton ℓ + hadron $k \to$ lepton ℓ' + hadron k'+photon q'

IFD

initial	final
$\ell^\mu = (\ell, 0, 0, \ell)$ $k^\mu = (m, 0, 0, 0)$	$\ell'^\mu = (\ell', \ell' \sin\theta_{\ell'}, 0, \ell' \cos\theta_{\ell'})$ $k'^\mu = (E', k' \sin\theta_{k'}, 0, k' \cos\theta_{k'})$

LFD

initial	final
$\ell^\mu = (\sqrt{2}\ell, 0, 0, 0)$ $k^\mu = (\sqrt{2}m, 0, 0, 0)$	$\ell'^\mu = \left(\ell' \frac{1+\cos\theta_{\ell'}}{\sqrt{2}}, 0, \ell' \frac{1-\cos\theta_{\ell'}}{\sqrt{2}}\right)$ $k'^\mu = \left(\frac{E'+k'\cos\theta_{k'}}{\sqrt{2}}, k' \sin\theta_{k'}, 0, \frac{E'-k'\cos\theta_{k'}}{\sqrt{2}}\right)$

LFD

$$
\begin{aligned}
q^\mu &= \ell^\mu - \ell'^\mu \\
&= \left(\frac{\ell - \ell' + \ell - \ell' \cos\theta_{\ell'}}{\sqrt{2}}, -\ell' \sin\theta_{\ell'}, 0, -\ell' \frac{1-\cos\theta_{\ell'}}{\sqrt{2}}\right)
\end{aligned}
$$

Full kinematics cont'd

The lepton momentum in the final state ℓ' cannot be greater than the beam momentum ℓ, thus

$$q^+ = \frac{2\ell - \ell'(1 + \cos\theta_{\ell'})}{\sqrt{2}} = 0$$

cannot be obtained in the Hall A kinematics for any choice of the scattering angle $\theta_{\ell'}$.

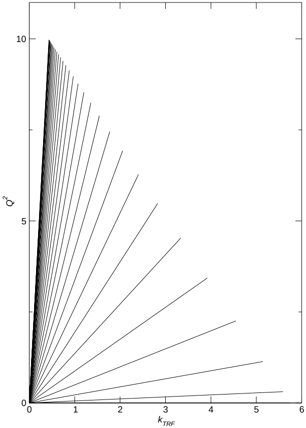
As rotations about the y-axis are not kinematical, we cannot immediately conclude that amplitudes calculated in a kinematics where $q^+ = 0$, are identical to those calculated for the Hall A kinematics if approximations are made.

Momentum scales

In Hall A kinematics, the scale is set by the lepton beam energy $E_{\text{beam}} \equiv \ell$. The photon virtuality is given by

$$q^2 = -2\ell\,\ell'(1 - \cos\theta_{\ell'}) \equiv -Q^2$$

For the Hall A kinematics with $E_{\text{beam}} = 5.75$ GeV we find $Q^2_{\text{max}} = 9.97\ (\text{GeV}/c)^2$ and $k_{\text{TRF max}} = E_{\text{beam}}$.
The lines correspond to different CM electron scattering angle.



Q^2 *vs* k_{TRF}

Full kinematics: Conclusion

Treacherous point 2

Only if one could prove that the GPDs calculated in a frame with $q^+ = 0$ and the Hall A kinematics respectively, are the same, one would be justified to analyze the measured observables in terms of the amplitudes calculated in the BDH frame.

Real Compton scattering in scalar QED

IFD Kinematics, CM Frame

$$
\begin{aligned}
k^\mu &= (E_p, -p\sin\theta, 0, -p\cos\theta), \quad q^\mu = (p, p\sin\theta, 0, p\cos\theta), \\
k'^\mu &= (E_p, -p\sin\theta', 0, -p\cos\theta'), \quad q'^\mu = (p, p\sin\theta', 0, p\cos\theta').
\end{aligned}
$$

LFD Kinematics, CM Frame

$$
\begin{aligned}
k^\mu &= \left(\frac{E_p - p\cos\theta}{\sqrt{2}}, -p\sin\theta, 0, \frac{E_p + p\cos\theta}{\sqrt{2}}\right), \\
q^\mu &= \left(p\frac{1+\cos\theta}{\sqrt{2}}, p\sin\theta, 0, p\frac{1-\cos\theta}{\sqrt{2}}\right), \\
k'^\mu &= \left(\frac{E_p - p\cos\theta'}{\sqrt{2}}, -p\sin\theta', 0, \frac{E_p + p\cos\theta'}{\sqrt{2}}\right), \\
q'^\mu &= \left(p\frac{1+\cos\theta'}{\sqrt{2}}, p\sin\theta', 0, p\frac{1-\cos\theta'}{\sqrt{2}}\right),
\end{aligned}
$$

IFD Polarization vectors

$$\epsilon^\mu(q;+1) = \frac{1}{\sqrt{2}}(0,-\cos\theta,-i,\sin\theta)$$
$$\epsilon^\mu(q;-1) = \frac{1}{\sqrt{2}}(0,\cos\theta,-i,-\sin\theta).$$

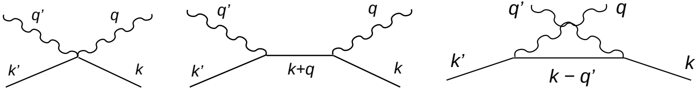
LFD Polarization vectors

$$\epsilon^\mu(q;+1) = \left(0,-\frac{1}{\sqrt{2}},-\frac{i}{\sqrt{2}},-\frac{\sin\theta}{1+\cos\theta}\right)$$
$$\epsilon^\mu(q;-1) = \left(0,\frac{1}{\sqrt{2}},-\frac{i}{\sqrt{2}},\frac{\sin\theta}{1+\cos\theta}\right).$$

Clearly, the singularity in the polarization vectors is an artifact occurring in LFD only due to the choice of the coordinate frame.

Amplitudes: Definition

In scalar QED, besides the *s*-channel and *u*-channel amplitudes there occurs a seagull, necessary for maintaining gauge invariance.



If the transversality of the polarization vectors to the momenta is taken into account, the amplitudes for an incoming photon with momentum q and helicity h and outgoing photon with momentum q' and helicity h' are given by

$$\begin{aligned} \text{Seagull}: &\quad -2\epsilon^*(q';h')\cdot\epsilon(q;h) \\ s-\text{channel}: &\quad 4\frac{[\epsilon^*(q';h')\cdot k'][\epsilon(q;h)\cdot k]}{s-m^2} \\ u-\text{channel}: &\quad 4\frac{[\epsilon^*(q';h')\cdot k][\epsilon(q;h)\cdot k']}{u-m^2} \end{aligned}$$

Amplitudes: Results in IFD

$$
\begin{aligned}
\text{Seagull}: \quad & A_{\text{Sea}}(+,+) = A_{\text{Sea}}(-,-) = -\frac{1}{2}(1+\cos(\theta'-\theta)), \\
& A_{\text{Sea}}(+,-) = A_{\text{Sea}}(-,+) = -\frac{1}{2}(1-\cos(\theta'-\theta)), \\
s-\text{channel}: \quad & A_{\text{s}} \equiv 0, \\
u-\text{channel}: \quad & A_{\text{u}}(+,+) = A_{\text{u}}(-,-) = \frac{p\sin(\theta'-\theta)^2}{E_p + p\cos(\theta'-\theta)} \\
& A_{\text{u}}(+,-) = A_{\text{u}}(+,-) = -A_{\text{u}}(+,+)
\end{aligned}
$$

As could be expected, neither of these amplitudes is singular. Moreover, they are all separately rotational invariant. This is related to the fact that rotations are kinematical in IFD.

Amplitudes: Results in LFD

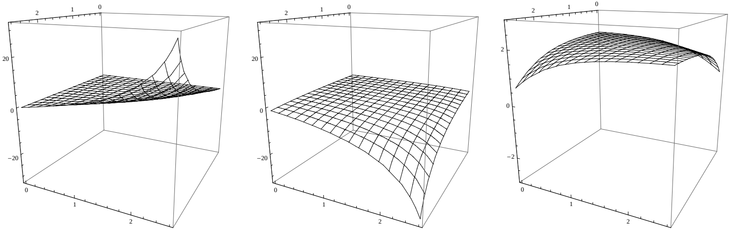
In LFD the expressions are (i) more complicated and (ii) the *s*- and *u*-parts are not rotationally invariant by themselves. As an example we display the $(+,+)$ amplitudes only.

$$
\begin{aligned}
A_{\text{Sea}}(+,+) &= -1 \\
A_{\text{s}}(+,+) &= \frac{\sin\theta'}{1+\cos\theta'}\,\frac{\sin\theta}{1+\cos\theta'}\,\frac{E+p}{p} \\
A_{\text{u}}(+,+) &= -\frac{1}{1+\cos\theta'}\,\frac{1}{1+\cos\theta'}\,\frac{1}{p(E+p\cos(\theta-\theta'))} \\
&\quad \times (E\sin\theta' + p\sin\theta + p\sin(\theta-\theta')) \\
&\quad \times (E\sin\theta + p\sin\theta' + p\sin(\theta'-\theta))
\end{aligned}
$$

The *s*- and *u*-amplitudes are singular at $\theta = \pi$ and $\theta' = \pi$.
However, the sum of the three amplitudes is invariant and equal to the result calculated in IFD.

Amplitudes: Results in LFD, cont'd

CM frame: momentum $p = 1$ GeV$/c$; as a function of θ and θ'.



Left: s-part (singular), middle: u-part (singular), right: total = seagull + s-part + u-part (regular)
The plot of the total amplitude is rescaled by a factor of 10 compared to the other two.

The singularities at $\theta = \pi$ and $\theta' = \pi$ of the s- and u-parts cancel.

Tensor structure

Numerator tensors

$$\begin{aligned} s\text{−channel}: &\quad \epsilon^*(q')_\mu(2k'^\mu + q'^\mu)\epsilon(q)_\nu(2k^\nu + q^\nu) \\ \epsilon(q) \perp q,\ \epsilon(q') \perp q' \to &\quad 4\,\epsilon^*(q')_\mu\epsilon(q)_\nu\, k'^\mu k^\nu \\ u\text{−channel}: &\quad \epsilon^*(q')_\mu(2k^\mu - q^\mu)\epsilon(q)_\nu(2k'^\nu - q'^\nu) \\ \epsilon(q) \perp q,\ \epsilon(q') \perp q' \to &\quad 4\,\epsilon^*(q')_\mu\epsilon(q)_\nu\, k^\mu k'^\nu \end{aligned}$$

In sQED the dominant tensor components originating from the hard momenta q and q' do not contribute, as they are filtered out by the polarization vectors.

The singularities in BDH kinematics ($q^+ = 0$) are introduced by the polarization vectors.

DVCS at tree level in singular kinematics

Special $q^+ = 0$ kinematics

We consider the special case where the hadrons, which have soft momenta, are aligned with the z-axis and the photons have hard momentum Q in the xz-plane.
In the $A^+ = 0$ gauge the plus-components of the polarization vectors vanish, so the amplitudes effectively reduce to

$$A_s = \frac{4\epsilon^*(q')^- \epsilon(q)^- k'^+ k^+}{s - m^2},$$

$$A_u = \frac{4\epsilon^*(q')^- \epsilon(q)^- k'^+ k^+}{u - m^2}.$$

In the DVCS limit $Q \to \infty$ the denominators become

$$s - m^2 = (x - \zeta)\frac{Q^2}{\zeta},$$

$$u - m^2 = -x\frac{Q^2}{\zeta}.$$

DVCS at tree level in singular kinematics, cont'd

Numerators in singular kinematics

$$4\epsilon^*(q')^-\epsilon(q)^- k'^+ k^+ = 4x(x-\zeta){p^+}^2 \left(-\frac{Q}{\sqrt{2}q^+}\right)\left(-\frac{Q}{\sqrt{2}\zeta p^+}\right)$$

Trick: Change q^+ to δp^+ to isolate the singular parts and keep track of them.

This gives, after making further changes for consistency,

$$\begin{aligned} q^\mu_\delta &= \left(\delta p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+} + \frac{\zeta m^2}{2x(x-\zeta)p^+}\right) \\ q^2_\delta &= -\frac{\zeta}{\zeta+\delta}Q^2 + \delta\frac{\zeta m^2}{x(x-\zeta)}. \\ \epsilon^\mu_\delta(q;+) &= \left(0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{Q}{\sqrt{2}\delta p^+}\right). \end{aligned}$$

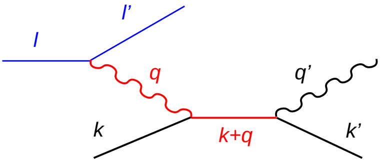
DVCS at tree level in singular kinematics, cont'd

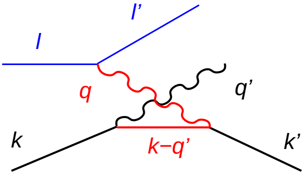
We find for large Q and in the limit $\delta \to 0$

$$A_s(+,+) = \frac{1}{\delta} 2x(x-\zeta)\frac{1}{x-\zeta}, \quad A_u(+,+) = \frac{1}{\delta} 2x(x-\zeta)\frac{-1}{x}.$$

Treacherous point 3: The tree-level amplitudes have a $1/q^+$ singularity. The singularities of the s- and u-amplitudes do not cancel.

As the physical process does not 'know' that we have taken $q^+ = 0$, there must be something missing. So we look at the amplitude for the 'full' process.





Full DVCS at tree level

Add the leptonic part of the physical amplitude and sum over all polarizations of the virtual photons

The lepton momenta are

$$
\begin{aligned}
\ell^\mu &= \left(\ell^+, Q, 0, \frac{Q^2}{2\ell^+}\right), \\
q_\delta^\mu &= \left(\delta p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+} + \frac{\zeta m^2}{2x(x-\zeta)p^+}\right) \\
\ell'^\mu &= \ell^\mu - q_\delta^\mu .
\end{aligned}
$$

The value of ℓ^+ is determined by the on-shell condition $\ell^2 = 0$ (massless lepton).

Full DVCS at tree level: Results

Lepton amplitudes

$\{\lambda, \lambda'\}$	h	Amplitude $L(\{\lambda, \lambda'\}, h)$
$\{\uparrow, \uparrow\}$	$+1$	$-Q\left(1 - \frac{\delta}{4\zeta} + \frac{2\zeta}{\delta}\right)$
$\{\downarrow, \downarrow\}$	$+1$	$Q\left(1 - \frac{3\delta}{4\zeta} - \frac{2\zeta}{\delta}\right)$
$\{\uparrow, \uparrow\}$	0	$-i\,2\sqrt{2}Q\left(\frac{\zeta}{\delta}\right)$
$\{\downarrow, \downarrow\}$	0	$-i\,2\sqrt{2}Q\left(\frac{\zeta}{\delta}\right)$
$\{\uparrow, \uparrow\}$	-1	$-Q\left(1 - \frac{3\delta}{4\zeta} - \frac{2\zeta}{\delta}\right)$
$\{\downarrow, \downarrow\}$	-1	$Q\left(1 - \frac{\delta}{4\zeta} + \frac{2\zeta}{\delta}\right)$

We have expanded the amplitude in Q and δ.
Clearly all amplitudes are singular.

Full DVCS at tree level: Results cont'd

Hadron amplitudes

$\{h, h'\}$	Amplitude $H(\{h, h'\})$
$\{+1, +1\}$	$2 + \frac{2\zeta}{\delta}$
$\{\ 0, +1\}$	$i\sqrt{2}\left(1 - \frac{\delta}{4\zeta} + \frac{2\zeta}{\delta}\right)$
$\{-1, +1\}$	$-\frac{2\zeta}{\delta}$
$\{+1, -1\}$	$-\frac{2\zeta}{\delta}$
$\{\ 0, -1\}$	$-i\sqrt{2}\left(1 - \frac{\delta}{4\zeta} + \frac{2\zeta}{\delta}\right)$
$\{-1, -1\}$	$2 + \frac{2\zeta}{\delta}$

We have expanded the amplitude in Q and δ.
These are also all singular.

Full DVCS at tree level: Results, final

Full amplitudes

The full amplitude is obtained by a convolution over the virtual photon helicity h of the lepton and hadron amplitudes

$$A(\{\lambda\lambda'\}, h') = \sum_{h=-1,0,1} L(\{\lambda, \lambda'\}, h) \, \frac{1}{q^2} \, H(\{h, h'\})$$

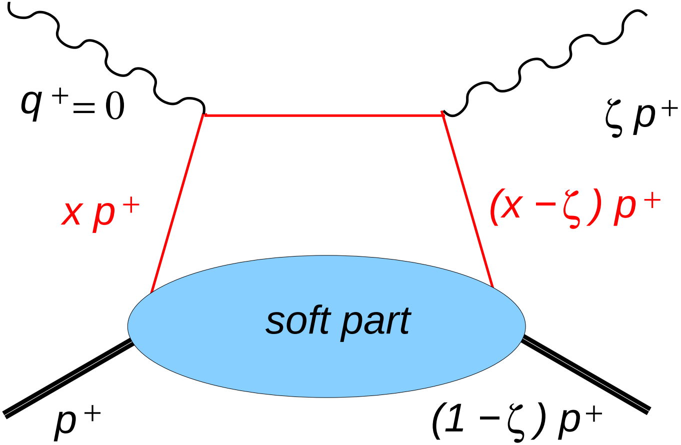
For massless leptons spin flip does not occur.

$\{h, h'\}$	Amplitude $A(\{\lambda, \lambda'\}, h')$
$\{\uparrow, \uparrow\}, +1$	$-Q\left(4 - \frac{2\delta}{\zeta}\right)$
$\{\uparrow, \uparrow\}, -1$	0
$\{\downarrow, \downarrow\}, +1$	0
$\{\downarrow, \downarrow\}, -1$	$Q\left(4 - \frac{2\delta}{\zeta}\right)$

We have expanded the amplitude in Q and δ.

All these amplitudes are regular, as should be expected.

Beyond tree level, the hadronic part changes; a soft part is involved.



Conclusions concerning tree-level results

Conclusions so far

1. The $q^+ = 0$ kinematics is singular, because the polarization vectors are singular.
2. The missing piece is the convolution with the lepton amplitude.
3. The singularity-killing part is provided by the longitudinally polarized virtual photon.
4. Analyzing the numerator tensor neglecting the effect of the polarization vectors is misleading.
5. In no way do these conclusions depend on the structure of the soft part that must be added to complete e.g. the handbag diagram, because the soft part does not depend on q^+.

Model parameters

Model

ϕ_i bosonic 'quark' fields of masses μ_i

Φ bound state with mass m

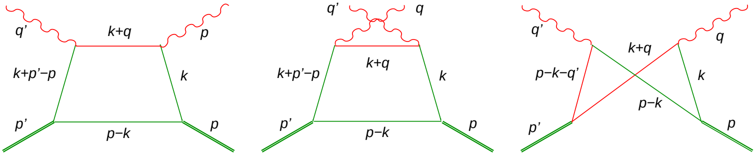
$$\begin{aligned}\mathcal{L} &= \sum_{i=1}^{2}[(D^i_\mu \phi_i)^\dagger (D^{i\mu}\phi_i) - \mu_i^2|\phi_i|^2] \\ &-\frac{1}{2}(m^2\Phi^2 - \partial_\mu\Phi\partial^\mu\Phi) + g\Phi(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)\end{aligned}$$

Parameter values $m = 0.14$ GeV$/c^2$, $\mu_i = 0.25$ GeV$/c^2$

Calculation in $q^+ \neq 0$ kinematics

$q^+ \neq 0$ kinematics

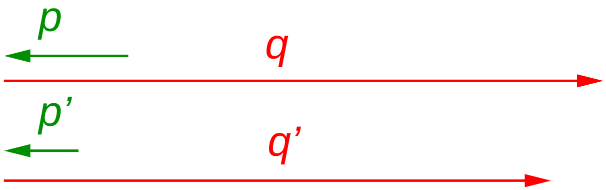
Study DVCS in a simple analytic model: the relativistic quark model 3+1D, scalar QED.



Handbag, crossed handbag, and cat's ears diagrams showing hard and soft momenta.

Kinematics

z-axis



$$p = \left(p^+, \mathbf{0}_\perp, \frac{m^2}{2p^+}\right), \qquad p' = \left((1-\zeta)p^+, \mathbf{0}_\perp, \frac{m^2}{2(1-\zeta)p^+}\right)$$

$$q = \left(-\zeta p^+, \mathbf{0}_\perp, \frac{Q^2}{2\zeta p^+}\right), \; q' = \left(0, \mathbf{0}_\perp, \frac{1}{2p^+}\left(\frac{-\zeta m^2}{1-\zeta} + \frac{Q^2}{\zeta}\right)\right)$$

$$k = (xp^+, \mathbf{k}_\perp, k^-),$$

DGLAP: $\zeta < x < 1$, ERBL: $0 < x < \zeta$

Covariant calculation

Amplitude $$\mathcal{A} = \int \frac{\mathrm{d}^{\mathrm{D}} k}{(2\pi)^{\mathrm{D}}} \, \frac{N}{D}$$

Denominator

$$D = (k_1^2 - m_q^2 + i\epsilon)(k_2^2 - m_q^2 + i\epsilon)(k_3^2 - m_q^2 + i\epsilon)(k_4^2 - m_q^2 + i\epsilon)$$

Using Feynman parameters $D \to (k^2 - M_{\mathrm{cov}}^2)^4$. For the handbag diagram

$$M_{\mathrm{handbag}}^2 = m_q^2 - \alpha_1(\alpha_2 + \alpha_4)\, m_\pi^2 + \alpha_2\alpha_3\, Q^2 - \alpha_1\alpha_3\, s - \alpha_2\alpha_4\, t$$

Similar formulas can be found for the crossed handbag and the cat's ears diagram.
N is given in terms of the contractions of the photons with the currents carried by the constituents. In the present model $N = 4k'^{\mu} k^{\nu}$.

LF time-ordered diagrams

The LF-time ordered diagrams are obtained by an integral over k^-. Using the LF gauge, $A^+ = 0 \to \epsilon^+ = 0$, guarantees that the minus components of the external momenta do not occur in the numerator.

$$D = \prod_i (k_i^2 - m_i^2 + i\epsilon) = \prod_i 2k_i^+ (k_i^- - H_i)$$

$$k_1 = p - k,\ k_2 = k,\ k_3 = k + q,\ k_4 = k + p' - p$$

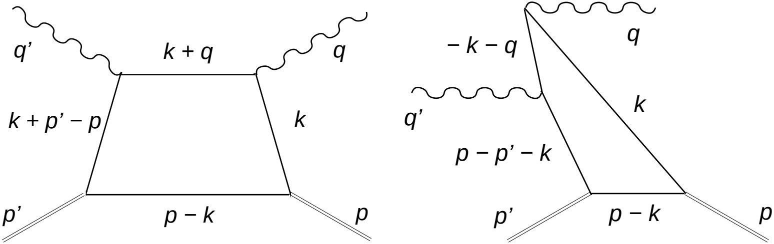
Domains

$$\begin{array}{lll} \text{I}: & p^+ - p'^+ \leq k^+ < p^+, & \text{DGLAP} \\ \text{II}: & 0 \leq k^+ < p^+ - p'^+, & \text{ERBL} \end{array}$$

or

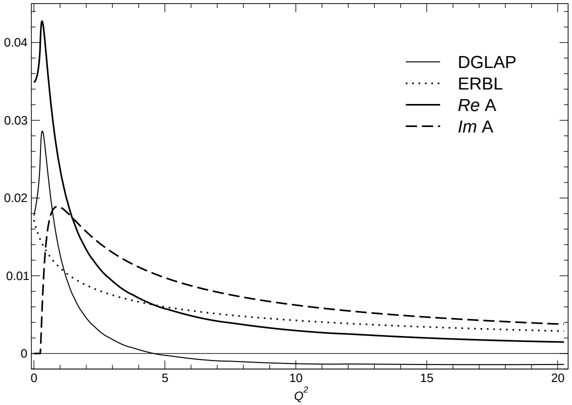
$$\begin{array}{lll} \text{I}: & \zeta \leq x < 1, & \text{DGLAP} \\ \text{II}: & 0 \leq x < \zeta, & \text{ERBL} \end{array}$$

Handbag DGLAP and ERBL diagrams



Valence (DGLAP, left) and nonvalence (ERBL, right) contributions to the handbag.

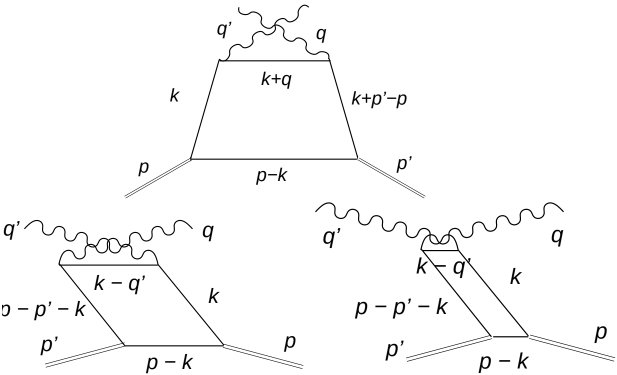
Handbag diagram, results



LF contributions and covariant handbag amplitude Re $A + i$ ImA for $\zeta = 0.5$ and polarizations $(+1, +1)$.

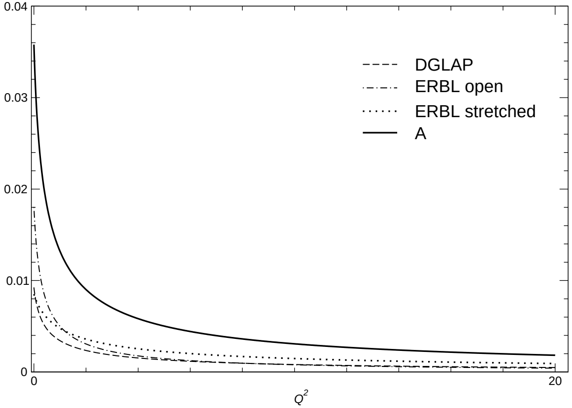
For small values of Q^2 we see the threshold behaviour of the amplitude.

Crossed handbag DGLAP and ERBL diagrams



Valence (DGLAP, above) and nonvalence (ERBL, below) contributions to the crossed handbag.

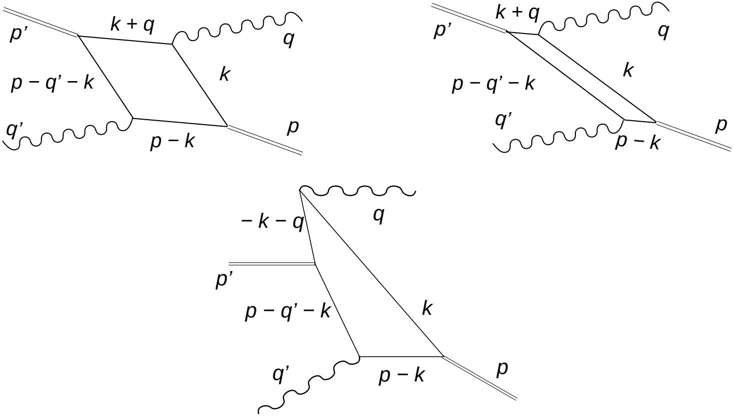
Crossed handbag diagram, results



LF contributions and covariant amplitude A crossed handbag for $\zeta = 0.5$

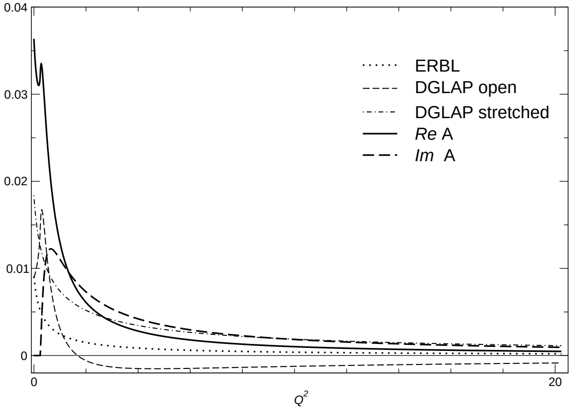
The crossed handbag is real for negative values of the Mandelstam variable t.

Cat's ears DGLAP and ERBL diagrams



Valence (DGLAP, above) and nonvalence (ERBL, below) contributions to the cat's ears.

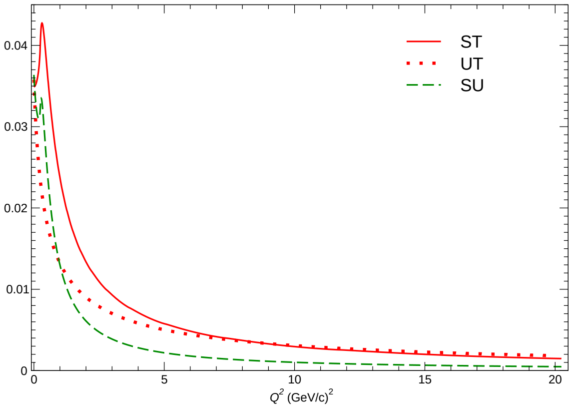
Cat's ears diagram, results



LF contributions and cat's ears covariant amplitude A for $\zeta = 0.5$

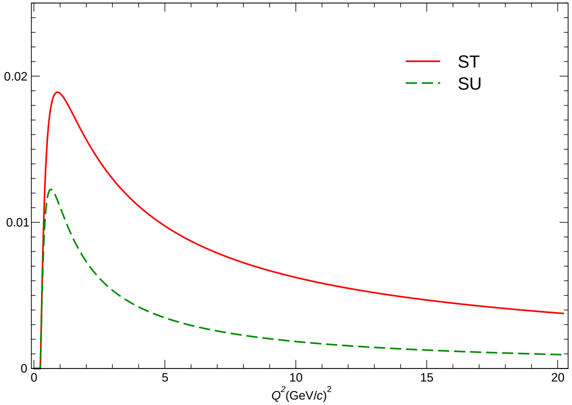
For small values of Q^2 we see the threshold behaviour of the amplitude.

Real parts of diagrams, Q^2-dependence



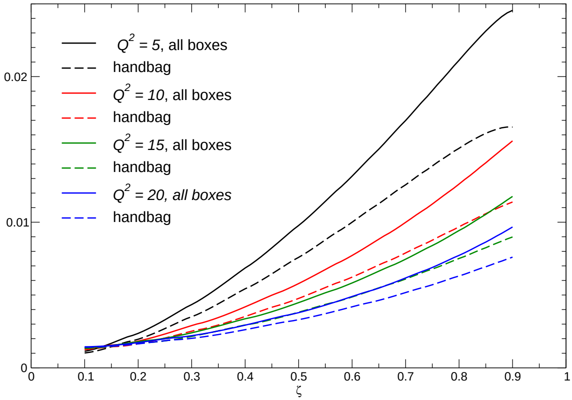
Real parts of the covariant amplitudes for the three boxes. $\zeta = 0.5$

Imaginary parts of diagrams, Q^2-dependence



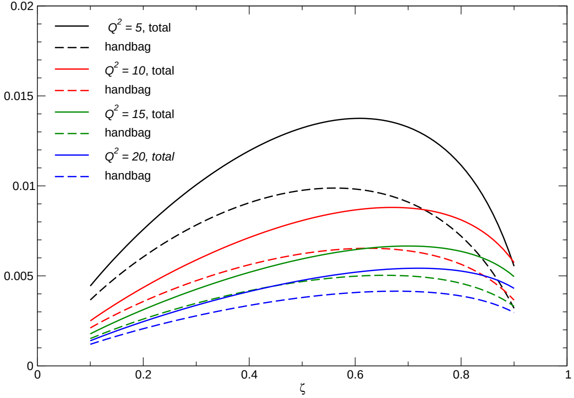
Imaginary parts of the covariant amplitudes for the three boxes. $\zeta = 0.5$

Real parts of diagrams, ζ-dependence



Real parts of the covariant amplitudes as a function of ζ for several values of Q^2.

Imaginary parts of diagrams, ζ-dependence



Imaginary parts of the covariant amplitudes as a function of ζ for several values of Q^2.

Summary

Summary of the model calculation

1. The three box diagrams were calculated covariantly and using light-front methods; the two methods give identical results.
2. The imaginary parts of the amplitudes are due to the valence diagrams only.
3. The nonvalence LF diagrams give important contributions as they are responsible for the ERBL part of the amplitudes.
4. The relative magnitudes of the LF diagrams depend strongly on the value of Q^2.
5. For large values of Q^2 the handbag diagram does dominate at the level of $\sim 70\%$. One must be careful to extract the GPD from the data.
6. In our model the soft part has point vertices; for realistic vertices the numerical results can be expected to change.