

Non-perturbative QCD effective charge: theory and phenomenology

Arlene C. Aguilar

Federal University of ABC, Brazil

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Based on:

A.C.A., D. Binosi and J. Papavassiliou, *Phys. Rev. D* **78**, 025010 (2008)

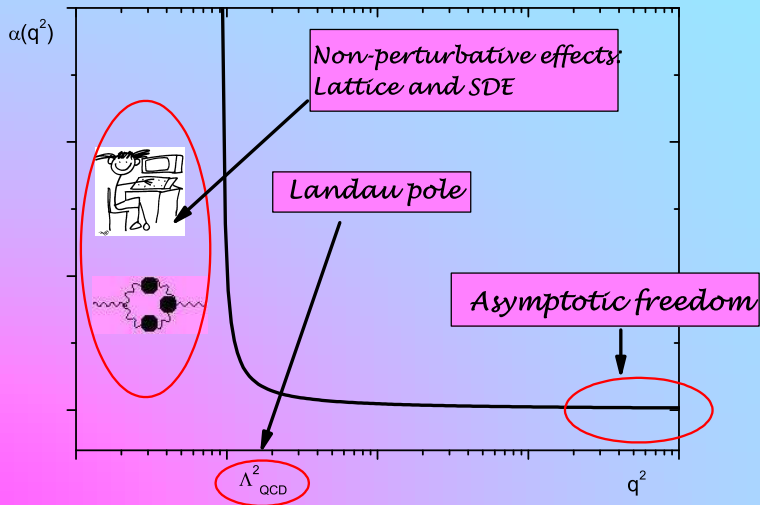
A.C.A., D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, arXiv:0906.2633 [hep-ph] .

A.C.A., D. Binosi and J. Papavassiliou, arXiv:0907.0153 [hep-ph]

Outline of the talk

- Motivation
- The pinch technique effective charge
- The effective charge from the ghost-gluon vertex
- Comparing the two definitions
- Conclusions

The QCD perturbative coupling



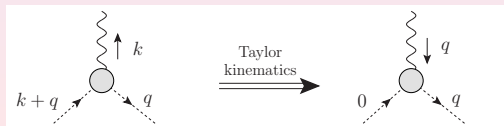
We will study **two** different **definitions** of the **QCD effective charge**

- 1 The first is based on the **pinch-technique gluon self-energy**, in close analogy to QED.

$$\widehat{\Pi}_{\mu\nu}(q) = \text{[Diagram 1]} + \text{[Diagram 2]}$$

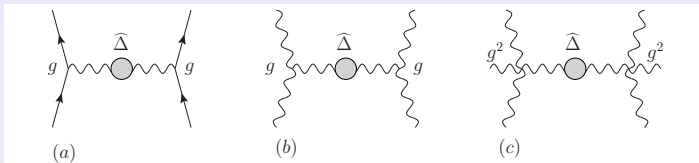
The equation shows the definition of the pinch-technique gluon self-energy $\widehat{\Pi}_{\mu\nu}(q)$ as the sum of two diagrams. The first diagram is a gluon self-energy loop (represented by a wavy line) with external wavy lines carrying momentum q . The second diagram is a ghost loop (represented by a dashed line) with external wavy lines carrying momentum q .

- 2 The second is based on the definition of **ghost-gluon vertex**, in a particular kinematic configuration.



The pinch technique effective charge

The properties of the PT charge



- The PT construction involves a **combination of two point functions** (no explicit reference to any of the full vertices)
- As in QED, **the effective charge** so obtained is universal (**process independent**),
- It **depends** naturally on a **single scale**, namely the physical momentum exchange of a given process.

The pinch-technique effective charge

- Due to the fact that the Green's functions obey simple, QED-like Ward Identities

$$\widehat{Z}_1 = \widehat{Z}_2, Z_g = \widehat{Z}_A^{-1/2}$$

where, the renormalization constants

$$\begin{aligned} g(\mu^2) &= Z_g^{-1}(\mu^2)g_0, \\ \widehat{\Delta}(q^2, \mu^2) &= \widehat{Z}_A^{-1}(\mu^2)\widehat{\Delta}_0(q^2), \end{aligned}$$

- we have the **renormalization-group invariant** combination

$$\widehat{d}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{d}(q^2)$$

At large momenta one may extract a dimensionless quantity,

$$\widehat{d}(q^2) = \frac{\overline{g}^2(q^2)}{q^2}$$

$\overline{g}^2(q^2)$ is the RG-invariant effective charge of QCD; at one-loop

$$\overline{g}^2(q^2) = \frac{g^2}{1 + \mathbf{b}g^2 \ln(q^2/\mu^2)} = \frac{1}{\mathbf{b} \ln(q^2/\Lambda_{\text{QCD}}^2)}$$

$$\widehat{d}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{d}(q^2)$$

- is a **non-perturbative relation**, it can serve unaltered as the **starting point** for extracting a **non-perturbative effective charge**.
- we should have **information** on the IR behavior of $\widehat{\Delta}(q^2)$.

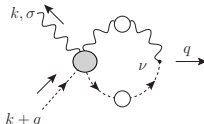
- The *conventional* gluon propagator $\Delta(q^2)$ and $\widehat{\Delta}(q^2)$ are connected by the formal relation:

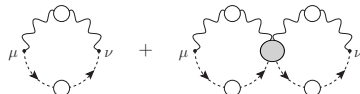
$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$

P. A. Grassi, T. Hurth and M. Steinhauser, *Annals Phys.* **288**, 197 (2001)

D. Binosi and J. Papavassiliou, *Phys. Rev. D* **66**(R), 025024 (2002)

$G(q^2)$ is the $g_{\mu\nu}$ component of the two-point function $\Lambda_{\mu\nu}(q)$

$$H_{\sigma\nu}(k, q) = H_{\sigma\nu}^{(0)} +$$


$$\Lambda_{\mu\nu}(q) =$$


$$\begin{aligned}\Lambda_{\mu\nu}(q) &= -ig^2 C_A \int_k H_{\mu\rho}^{(0)} D(k+q) \Delta^{\rho\sigma}(k) H_{\sigma\nu}(k, q), \\ &= g_{\mu\nu} G(q^2) + \frac{q_\mu q_\nu}{q^2} L(q^2),\end{aligned}$$

- The SDE for $G(q^2)$ is (Landau gauge)

$$G(q^2) = -\frac{C_A g^2}{3} \int_k \left[2 + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k+q).$$

- In addition, the following relation is satisfied

$$1 + G(q^2) + L(q^2) = F^{-1}(q^2)$$

Checking the perturbative behavior of $\widehat{d}(q^2)$

$$g^2 \widehat{\Delta}(q^2) = \frac{g^2 \Delta(q^2)}{[1 + G(q^2)]^2}$$

$$1 + G(q^2) = 1 + \frac{9}{4} \frac{C_A g^2}{48\pi^2} \ln(q^2/\mu^2)$$

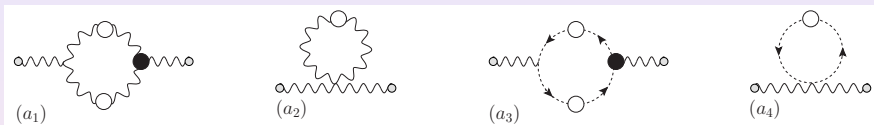
$$\Delta^{-1}(q^2) = q^2 \left[1 + \frac{13}{2} \frac{C_A g^2}{48\pi^2} \ln(q^2/\mu^2) \right]$$

⇓

$$\widehat{\Delta}^{-1}(q^2) = q^2 [1 + b g^2 \ln(q^2/\mu^2)]$$

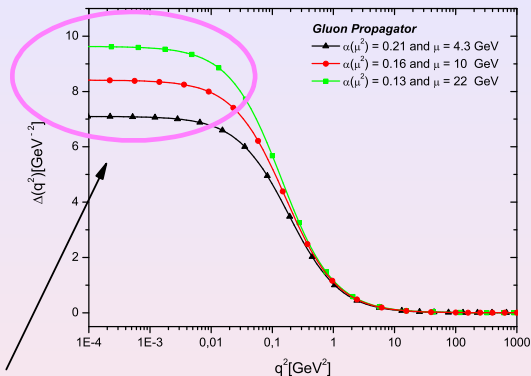
- Enforces β function coefficient in front of UV logarithm ($b = 11C_A/48\pi^2$).

Schwinger-Dyson system: Non-perturbative information for $\Delta(q^2)$ and $G(q^2)$



$$\begin{array}{c}
 \text{---}\circ\text{---}^{-1} \\
 q
 \end{array}
 =
 \begin{array}{c}
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 q
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 +
 \begin{array}{c}
 \text{---}\circ\text{---}^{-1} \\
 q
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 k+q
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 \end{array}$$

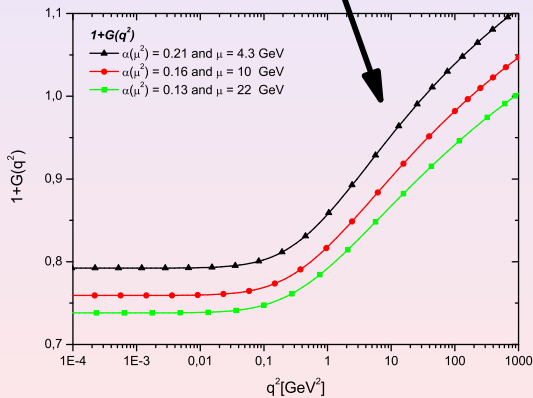
SD results for the gluon propagator



*Different freezing values
Dependence on μ*

SD result for the auxiliary function $G(q^2)$

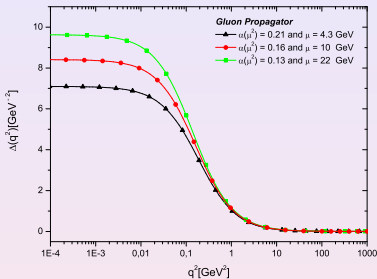
Dependence on μ



A.C.A., D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, arXiv:0906.2633 [hep-ph] .

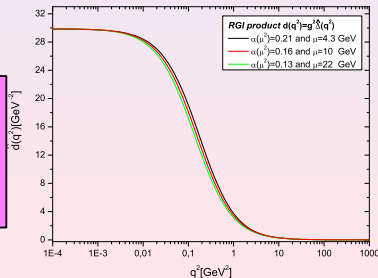
Computing $\widehat{d}(q^2)$

$$\widehat{d}(q^2) = \frac{g^2(\mu)\Delta(q^2)}{[1 + G(q^2)]^2}$$



*All curves on top
of each other
No μ - dependence !*

$\times \frac{g^2(\mu)}{[1 + G(q^2)]^2}$



Effective charge from massive propagator

- The dimensionful $\widehat{d}(q^2)$ can be written as

$$\widehat{d}(q^2) = \frac{4\pi\alpha_{\text{PT}}(q^2)}{q^2 + m^2(q^2)}.$$

where the gluon mass has power-law running

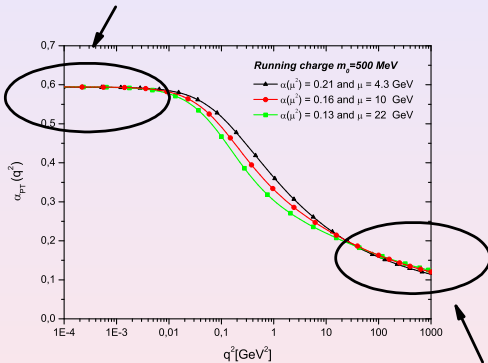
$$m^2(q^2) = \frac{m_0^4}{q^2 + m_0^2} \left[\ln \left(\frac{q^2 + 2m_0^2}{\Lambda^2} \right) / \ln \left(\frac{2m_0^2}{\Lambda^2} \right) \right]^3$$

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)

M. Lavelle , Phys. Rev. D **44**, 26 (1991)

A. C. Aguilar and JP , Eur. Phys. J. A **35**, 189 (2008)

Infrared fixed point

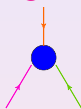


Asymptotic freedom

*Effective charge
from the
ghost-gluon vertex*

The effective charge from vertices

- A **definition** for the **QCD effective charge** can be obtained starting from the various **QCD vertices** .
- It **involves** more than **one scales** , and further assumptions are introduced, to **express** the charge as a function of a **single variable** .



$$Z_g = Z_V (Z_1 Z_2 Z_3)^{-1/2}$$

- The combination

$$\hat{r}(q_1, q_2, q_3) \equiv g^2 V^2(q_1, q_2, q_3) \Delta_1(q_1) \Delta_2(q_2) \Delta_3(q_3)$$

is a RG-invariant quantity.

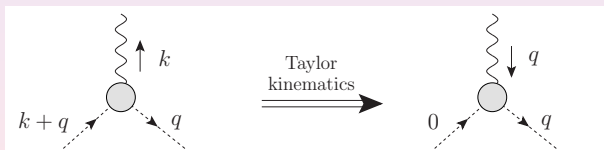
Additional assumption: $q_1^2 = q_2^2 = q_3^2 = q^2$ (and therefore $q_1 \cdot q_2 = q_1 \cdot q_3 = q_2 \cdot q_3 = -q^2/2$)

Effective charge from the ghost-gluon vertex

$$\begin{aligned}\Delta(q^2, \mu^2) &= Z_A^{-1}(\mu^2)\Delta_0(q^2), \\ F(q^2, \mu^2) &= Z_c^{-1}(\mu^2)F_0(q^2), \\ \Gamma^\nu(k, q, \mu^2) &= Z_1(\mu^2)\Gamma_0^\nu(k, q), \\ g_0 &= Z_g(\mu^2)g\end{aligned}$$

- For the ghost-gluon vertex

$$Z_g = Z_1 Z_A^{-1/2} Z_c^{-1}$$



- In the Taylor kinematics (incoming ghost momentum is zero), $Z_1 = 1$

$$Z_g = Z_A^{-1/2} Z_c^{-1}$$

$$\widehat{r}(q^2) = g^2 \Delta(q^2; \mu^2) F^2(q^2; \mu^2) = g_0^2 \Delta_0(q^2) F_0^2(q^2)$$

- The above combination is a **dimensionful μ -independent**
- For **asymptotically large q^2** ,

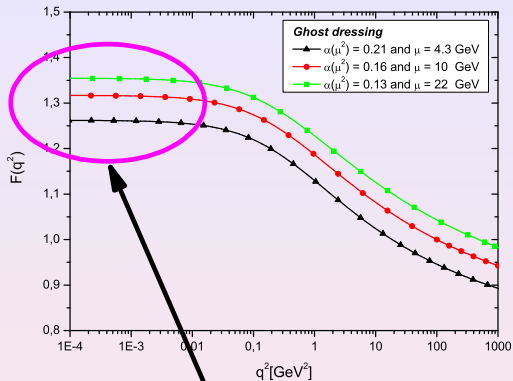
$$\widehat{r}(q^2) = \frac{\overline{g}_{\text{gh}}^2(q^2)}{q^2}.$$

Using the fact that

$$F^{-1}(q^2) = \left[1 + \frac{9}{4} \frac{C_A g^2}{48\pi} \ln \left(\frac{q^2}{\mu^2} \right) \right]$$

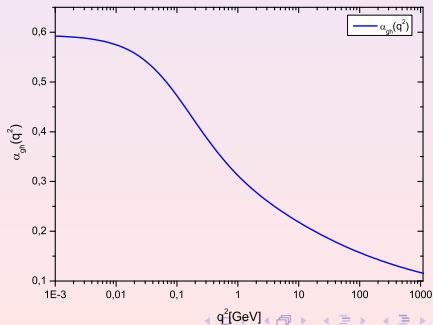
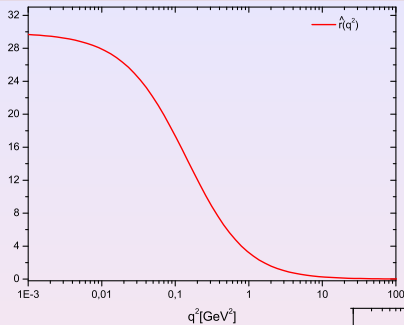
we verify that $\overline{g}_{\text{gh}}(q^2)$ has the right one-loop behavior.

SD results for the ghost dressing

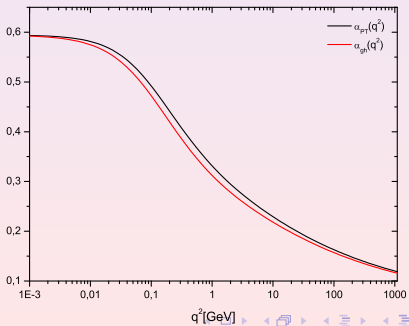
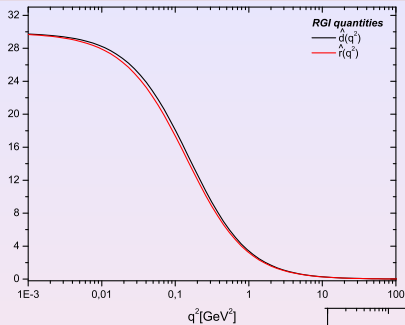


Different freezing value
Dependence on μ

The effective charge from the ghost-gluon vertex



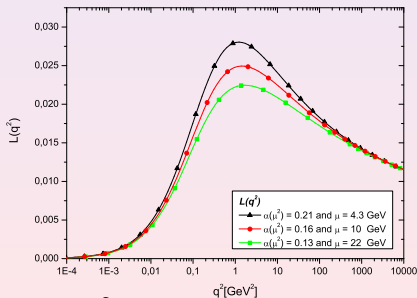
Comparison of the two definition



Comparison of the two definition

- Due to the relation $1 + G(q^2) + L(q^2) = F^{-1}(q^2)$
- Both effective charges are related by

$$\alpha_{\text{PT}}(q^2) = \alpha_{\text{gh}}(q^2) \left[1 + \frac{L(q^2)}{1 + G(q^2)} \right]^2$$



$L(q^2)$ is a small function.

Conclusions

- We have presented a nonperturbative comparison between the two QCD effective charges, $\alpha_{\text{PT}}(q^2)$ and $\alpha_{\text{gh}}(q^2)$.
- They were obtained within two completely different frameworks: **the pinch-technique gluon self-energy** and the **ghost-gluon vertex** .
- A common ingredient is the **infrared finite gluon propagator** $\Delta(q^2)$.
- By virtue of a **special identity** relating their non-common ingredients, in the deep infrared they **coincide** at the **same finite value** .